A

TREATISE

ON

ASTRONOMY

THEORETICAL AND PRACTICAL.

BY

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PART I. VOL. I.

CONTAINING THE

THEORIES OF THE FIXED STARS.

A NEW EDITION.

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ERRATA ET ADDENDA.

P. 7. l. 13. for 'greater,' read 'greatest.'

P. 9. l. 15. for 'more,' read 'move.'

P. 10. l. 3. for *bPb*, read *bPa*.

P. 16. last line, read *kWE'* and *ESk*.

P. 17. l. 8. from bottom, for 'notions,' read 'nations.'

P. 18. l. 4. for 'is first,' read 'it is first.'

P. 30. l. 2. for 'night,' read 'day.'

P. 40. l. 14. for $\frac{360°}{10}$, read $\frac{360°}{1°}$.

P. 194. l. 6. for 'exacted,' read 'exact.'

P. 344. l. 19. instead of $\frac{1}{2}(50''.1)^2$, &c., read $\frac{1}{2}$ sin. $1''(50''.1)^2$.

P. 697. the value of the obliquity $I_{,}=23° \, 27' \, 35''.1$, was taken from the N. A. of 1812, but all the values of I therein expressed are wrong to the amount of 8 seconds and upwards. The value of I on Nov. 12, 1812, ought to have been 23° 27′ 43″.6: in which case, the resulting latitude would have been 4° 58′ 27″.6: the value of the longitude will be very slightly affected by the change in the value of the obliquity.

P. 703. the two last figures in the logarithmic value of 106° 6′ 20″.73, instead of 06 ought to have been 10. If these and the following figures be corrected, the complement of the latitude will be 86° 13′ 29″.2. But the longitude in p. 704, is derived from the latitude: and if, in the calculation of the longitude, the above altered value of the computed latitude be substituted, the resulting value of the longitude will be 10ˢ 1° 45′ 13″.3. The observation of Nov. 12, 1812, was made with the new mural circle: but those of Sept. 27, and 28, 1811, with the brass quadrant, which, it is now known, has, since it was first put up, changed its figure. The *changes* have not been accounted for in pp. 701, &c. these amount to +7″.3, +6″.6, corrections additive to the north polar distance of the 27th and 28th, and, if the calculations be made with the north polar distances so corrected, the resulting latitudes and longitudes on the 27th and 28th, will be respectively,

3° 46′ 23″.4, 2° 41′ 1″,

10ˢ 1° 45′ 11″.7, 10ˢ 16° 47′ 12″,

and the errors of the Tables in latitude −15″.4, −14″.5,

in longitude + 3.5, + 6.

ERRATA ET ADDENDA.

The results now agree much more nearly with those printed by order of the Board of Longitude. The *elements* of the latter results are now, by the kindness of the Astronomer Royal, in the Author's possession: they differ, however, in some small respects, from what he has used.

The above Errata are few in number, and not of much moment: others, no doubt, will be detected: the Author, however, does not anticipate the detection of many, relying on the careful and intelligent superintendence which the Work, during its progress, has received from the Rev. Dr. French, Master of Jesus College.

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* By mistake Chap. V. repeated.

Chap. VIII.

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PREFACE.

It may be necessary briefly to state the arrangement of the present Treatise.

In the first Chapters, I have explained, in a general way, certain of the obvious Phenomena of the Heavens: then, with a view of affording the Student the means of distinctly apprehending the methods, by which, those Phenomena are observed, and their quantities and laws ascertained, I have described, although not minutely, some of the principal instruments of an Observatory. By an attentive consideration of the means, by which, in practice, right ascensions and latitudes are estimated and computed, a more precise notion of those quantities may, perhaps, be obtained, than either from the terms of a definition, or from their representation in a geometrical diagram.

But, an observation expressed by the graduations of a quadrant, or the seconds of a sidereal clock, cannot be immediately used for Astronomical purposes. It must

previously be *reduced* or *corrected.* To the theories, then, of the necessary *corrections*, I have very soon called the attention of the Student: since, without a knowledge of them, he would be unable to understand the common process of regulating a sidereal clock, or that, by which, the difference of the latitudes of two places is usually determined.

The corrections are five; Refraction, Parallax, Aberration, Precession, and Nutation. The two latter, although they may be investigated on the principles of Physical Astronomy, are yet, in the ordinary processes of Plane Astronomy, equally necessary with the preceding.

To the Theory of the fixed Stars, which includes, as subordinate ones, the theories of the corrections that have been enumerated, succeed, the Solar, Planetary, and Lunar Theories. Of these, the last is, by many degrees, the most difficult. And, since, in its present improved state, it is not made to rest solely on observation, I have been compelled, in endeavouring to elucidate it, slightly to trespass on the province of Physical Astronomy.

The *Equation of Time,* which, essentially, depends on the Sun's motion, is placed immediately after its Theory.

On the same principle of arrangement, Eclipses are made to succeed the Solar and Lunar Theories. The method of computing them is that, which M. Biot has, in the last Edition of his Physical Astronomy, adopted, probably, from a Memoir of Delambre's* on the passage of Mercury over the Sun's disk. The traces of this method, may be discerned in a Posthumous work†, of the celebrated Tobias Mayer, on Solar Eclipses.

The method just noticed is as extensive as it is simple. For, it equally applies to Eclipses, Occultations of fixed Stars by the Moon, and the Transits of inferior Planets over the Sun's disk. And this circumstance has determined the places of the two latter subjects, which are immediately after that of the former.

In the last Chapters are discussed, the methods of computing Time, Geographical Latitude and Longitude, and the Calendar.

Such is the arrangement of the present Treatise. And, since it could not be entirely regulated by the necessary connexion of the subjects, it has, occasionally, been so, by certain views, of what seemed, their proper and natural sequence. It so happens, therefore, that

* Mem. *Inst.* tom. III. p. 392. (1802).

† Mayer, *Opera Inedita.* vol. I. p. 23.

the more difficult investigations are not invariably preceded by the more easy. The methods, for instance, of computing the Time, Geographical Latitude and Longitude, follow the Lunar Inequalities, Eclipses, Occultations, and Transits; but, since they do not follow by strict consequence, the latter, if it so suits the convenience of the Student, may, in a first perusal, be omitted.

I have been solicitous to supply every part of the Treatise with suitable Examples. For, they are found to be in Astronomy, more than in any other science, the means of explanation.

They become the means of explanation for reasons different from those which operate in other cases. For, Astronomical Examples are not always the mere translations of a rule, or of an algebraical formula, or of a geometrical construction, into arithmetical results. But, frequently, they are of a different description, and require the aid of certain subsidiary departments of Astronomical Science not then the subjects of consideration.

For instance, the difference of the latitudes of two places is equal to the sum or the difference of the zenith distances of the same Star. This rule cannot be applied according to its strict letter; for, when we descend into its detail, we may be obliged to reduce the observed zenith distances by four corrections. Consequently, we

ought either to have previously established, or we must proceed to investigate, the theories of those corrections. This instance will also serve to shew, what frequently happens, that a rule shall possess a seeming facility in its general enunciation, which vanishes when we become minute and are in quest of actual results.

There is, in fact, scarcely any thing in Astronomical science single, or produced, at first, perfect by its processes. No series of propositions, as in Geometry, originating from a simple principle and terminating in exactness of result. But, every thing is in connexion; when first disengaged, imperfect, and advanced towards accuracy only by successive approximation.

Consider, for instance, the Sun's Parallax. That essential element is determined by no simple process, but is, as it were, extricated by laborious calculations from a phenomenon in which, at first sight, it does not seem involved. Again, the common method of determining the Longitude at Sea rests on whatever is most refined in theory and exact in practice: on Newton's system in its most improved state, and on the most accurate of Maskelyne's observations.

The preceding remarks, besides their proper purpose, may perhaps serve to shew that an Astronomical Treatise, with any pretensions to utility, cannot be contained

within a small compass. It ought to teach the Principles of Astronomy; but it cannot well do that, except by detailing and explaining its best methods: that is, by explaining methods such as are practised, and as they are practised. Now, the methods of Astronomy are very numerous, and the details of several of them very tedious.

Some methods are merely speculative; such as cannot be practised, although founded precisely on the same principle as other methods that are practised. For instance, the separation of the Sun from a Star, in a given time, is equally certain and of the same kind, as the separation of the Moon from a Star, but since, in practice, it is not so *ascertainable*, it cannot be made the basis, as the latter is, of a method of finding the Longitude.

The exclusion then of methods merely curious, and of no practical utility, has been one mean of contracting the bulk of this Treatise. Another I have found, in omitting to explain the systems of Ptolemy and of Tycho Brahe. These do not now, as formerly, require confutation. The spirit of defending them is extinct. They are not only exploded but forgotten. And, were they not, it would be right to divert the attention of the Student, from what is foreign, fanciful, and antiquated, to real inventions and discoveries of more modern date, and purely of English origin.

The present Treatise is not intended to explain Physical Astronomy and the system of Newton. But, the discoveries and inventions of Bradley and Halley are within its scope. Their numerous and accurate observations and their various Astronomical methods, would alone place them in the first rank of illustrious Astronomers. But, they have an higher title to pre-eminence. In point of genius, they are, after Newton, unrivalled. The first, for his two Theories of Aberration and Nutation: the last, for his invention of the methods of determining the Sun's Parallax from the transit of Venus, and the Longitude from the Lunar motions.

This Lunar method of determining the Longitude was not reduced to practice by its author. That it has been since, is owing to Hadley and Maskelyne. The first, by his Quadrant, furnishing the instrumental, the latter, by the Nautical Almanack, the mathematical means.

This last-mentioned Astronomical Work, for such it is, and the most useful one ever published, is alone a sufficient basis for the fame of its author. Besides its results, it contains many valuable remarks and precepts. It is a collection of most convenient Astronomical Tables, and should be in the hands of every Student who is desirous of learning Astronomy; and who, for that end, must be conversant with Examples and Tables.

But, mere precepts and instances will not effect every thing. In order to remove the imperfection necessarily attached to knowledge acquired solely in the closet, instruments must be used and observations made. The means of doing this, however, are not easily had; and, it is to be regretted, they are not afforded to the Students of this University. An Observatory is still wanting to its utility and splendor.

PREFACE

TO THE SECOND EDITION.

The present Edition is, in its plan, like the former. In matter and manner, however, it is so different that the Author, instead of calling it a new Edition, might have called it a new Work.

It is not worth the while to point out the changes which the Work has undergone. Few of its readers will trouble themselves on that point. The fact worth enquiring about is whether the Work be a good Work, not whether it be better than that it comes after.

That it is better may be presumed from the very circumstance of its coming after. Nor can there be any arrogance in attributing its improved state to the change that, during the two Editions, has taken place in the Author's knowledge. The usual effect of time, in this respect, has not been counteracted. The other cause which ought to improve a treatise, namely, the improved state of the science treated of, has, of late, but slowly operated. Astronomical Science is now, nearly, the same as it was ten years ago. Having reached a

kind of *maximum* state of excellence, its changes are minute and must continue to be so. All great changes ended with Bradley. He swept the ground of discovery, and left little to be gathered by those that follow him.

Yet, during the 60 years that have elapsed since Bradley, it cannot be said, but that Astronomy has greatly advanced, although not by the aid of discoveries, such as those of Aberration and Nutation. The aid has come partly, indeed, from the Observatory, but principally from Physical Astronomy: which, originating with Newton, has, under his successors, Mayer, Clairaut, Euler and Laplace, grown up into an exceedingly great science.

Of the benefits thence accruing to Astronomy, the most excellent, by many degrees, are the Lunar Theories of Mayer and Laplace; or, as it may be stated, the Lunar Tables deduced from those Theories, and the Observations of Bradley and Maskelyne. If we go back to Halley's time, the improvement in such Tables will appear most striking. Halley states that, in his time, the differences between Observations and the results of Newton's Theory amounted frequently to 5 minutes, which differences now (if we speak of their mean states) do not much exceed as many seconds.

Navigation has been made more safe by means of these Lunar Tables: which, perhaps, is the only prac-

tical good that Astronomy has conferred on Society. Its other benefits are philosophical and intellectual. Should these be held to be of no moment, we might, perhaps, at the present time, shut up our Observatories, and live upon the hoards of Astronomical Science. We are now possessed of sufficient means, as far as Astronomy is concerned *, for determining the place of a vessel at sea; and if we would enable the mariner on the Atlantic or Indian Ocean to determine his place, to within less than 10 miles, we must provide him with better means of observation: with an Instrument more excellent than the Sextant.

But, such is the present ardour for philosophical pursuits, the duties of an Observatory, instead of ceasing, are likely to become more arduous. Within a few years from the present date, an Astronomical Society has been formed in the Metropolis, and an Observatory nearly established here. These Institutions indicative, as we have said, of the spirit of the times, can hardly fail to augment Science: they will do some good although perhaps not all the good that is intended to be done by them.

* The words of the commission that appoints the Astronomer Royal of Greenwich enjoin him, 'to apply himself with the utmost care and diligence to the rectifying the Tables of the Motions of the Heavens, and the places of the Fixed Stars, in order to find out the so much desired Longitude at sea, for the perfecting the Art of Navigation.'

As the latter of these Institutions may, in future times, become one not merely of local interest, we shall be excused if we say something farther concerning it.

The good resulting from Observatories, whatever it may be, practical or intellectual, the founders (if we may so call them) of the present Observatory are anxious to secure. Their first and chief object is to have Observations made as good as they can be made. The second, to have as many as possible of such Observations. In order to obtain the first, the best Instruments that Europe can furnish are ordered to be made. To secure the second object, houses are attached to the Observatory for the constant residence of the Observers.

Another object of the Institution is, the instruction of Academical Students in the use of Instruments, and in practical Astronomy: an object, it should seem, not incompatible with the former, but secondary and subordinate. Instruction alone could have been imparted by means much more simple than those which are now put into action.

But good Observations * will not necessarily be made,

* Two circumstances (there may be more) are unfavorable to the Observatory we are speaking of. One is, the not sufficient vicinity to the Artists of London: the other, common to our Island, and the same as that of which Lacaille complains, 'Constans nimis Parisiis tempore hiberno nebularum, imbrium et nubium mora, eorumque tempore æstivo frequens reditus, &c.'

because he, who ought to make them, is obliged to reside in an Observatory furnished with good Instruments. Something else remains to be done: some regulation to be made, or motive supplied, to compel (as it were) Observers to employ, in the duties of an Observatory, the time they must spend there. To effect this, there will not be found, perhaps, any means so simple and efficacious as that of some absolute rule for printing and publishing annually the Observations, and for sending copies thereof to the principal Observatories of Europe. Other Regulations may be suggested to counteract the proneness of Institutions, like the one spoken of, to become worse. But they should be simple and few. Regulations may, indeed, prevent much wrong from being done; but they rarely create a zeal for the performance of duties. The minute detail of the hours, modes, and objects of Observation, would never supply motives to him who should be insensible to his own personal reputation, and the honor of his Country and University.

The augmentations of Astronomical Science have, with scarcely any exception, come from publick Observatories: which fact is to be accounted for, from the excellence of the Observers' Instruments, the constant discharge of their duties, and, above all, the zealous discharge of those duties by the influence of publick opinion. A like moral controul will, probably, operate here, and serve to carry into effect the enlightened in-

tentions of the munificent Patrons of our Observatory. It has not been built merely to prevent its being said that an University, famous for its science, was without such an Institution: nor to add to the title and emolument of an individual; nor to be used as a kind of Astronomical toy, and to become the mere resort of leisurely amateurs and random star-gazers: nor, which is indeed a better but still a subordinate object, to confirm or correct results elsewhere obtained, to see, for instance, that Observations have been rightly made at Paris and Palermo. The chief object of the Observatory is, by its own means, to enlarge the boundaries of Science; to extend the fame of the University that founds it, is a secondary one, or rather, will be a sure consequence, if the first shall be obtained.

Caius College,
Dec. 27, 1822.

A

TREATISE ON ASTRONOMY,

IN TWO VOLUMES.

AN

ELEMENTARY TREATISE

ON

ASTRONOMY.

CHAP. I.

Certain Phenomena of the Heavens explained by the Rotation of the Earth.

In an Elementary Treatise on Plane Astronomy, two objects are required to be accomplished: 1st, The description and general explanation of the heavenly phenomena. 2dly, The establishment of methods for exactly ascertaining and computing such phenomena. Our attention will be first directed to the former of these two objects.

If, on a clear night, we observe the Heavens *, they will appear to undergo a continual change. Some stars will be seen ascending from a quarter called the East, *or rising;* others descending towards the opposite quarter the West, *or setting*. In some intermediate point, between the East and West, each star will reach its greatest height, or, will *culminate:* The greatest heights of the several stars will be different, but they will all appear to be attained towards the same part of the Heavens; which part is called the South.

If we now turn our backs to the South and observe the North, the opposite quarter, new phenomena will present themselves. Some stars will appear, as before, rising, reaching their

* *Exposition du Système du Monde, p. 2.*

greatest heights, and setting; but, besides these phenomena, other stars will be seen that never set, and that move with different degrees of velocity; and there are some stars that, to appearance, are nearly stationary. About one of these stationary stars, the other stars that never set appear to revolve, and to describe circles. Such stationary star is called the *Polar* Star: and the stars revolving round it, *Circumpolar*.

The *Polar* Star, that which is usually so denominated, is not, when accurately observed, or observed by means of instruments, strictly stationary. It is not, therefore, to be held as the place of the *Pole*, which is indeed an imaginary point, always, however, as we shall hereafter see, ascertainable by theory and observation. In such point or pole, a star, if we suppose it there placed, would appear stationary.

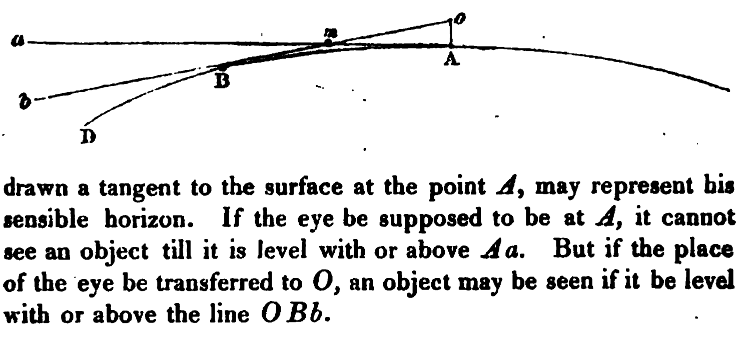
Almost all the stars in the Heavens retain towards each other the same relative position; no mutual approach or recess takes place between them: and accordingly they are called *Fixed* Stars. There are, however, certain stars, called *Planets*, not under the above conditions, and which continually change their places. The Sun and Moon also, the two celestial objects of the greatest interest, are from day to day changing their places in the Heavens.

A spectator at sea, or placed in a level country, may imagine himself in the centre of a plane, extended equally on all sides, and bounded by a circular or curved line apparently separating the sky and sea, or the sky and land. The plane so extended and bounded is called the spectator's *Horizon*, and sometimes the *sensible Horizon*. It is the boundary of the spectator's view, and when stars first appear just above it, they are said to *rise*: when they sink beneath it, they are said to *set*. On this imaginary plane the concave heavens, or the hemisphere of the heavens, may be fancied to rest.

The surface of the sea is not strictly plane; a few simple observations are sufficient to shew that it is a convex surface, the convexity being towards the heavens, and the spectator being placed on its summit. The preceding definition, therefore, of the horizon, must be slightly altered: it must now be defined to be a plane which, at the summit just mentioned, (the place indeed of the spectator) is a tangent plane to the earth's convex surface, extended on all sides till it is bounded by the sky.

The convex surface of the Earth is nearly spherical; more nearly spheroidical: the Earth being, (as it appears probable from various reasons) a spheroid of small eccentricity. The plane of the horizon, therefore, is a tangent plane to the spheroid at the place of the spectator, and a perpendicular to the plane at such place passes very nearly through the centre of the Earth. The perpendicular line just mentioned tends, if produced upwards, to a point in the Heavens called the *Zenith*. The opposite point in the line's direction continued downwards is called the *Nadir*.

If the eye of the spectator were in that plane which has been defined to be the plane of the *horizon*, stars would not appear to have *risen* whilst they were beneath that plane. But it is otherwise, if the spectator be elevated above the horizon either by being on a tower, or eminence, or on a ship; indeed, as mere elevation above the horizon is the circumstance that modifies the preceding statement (see p. 2. l. 28.) his own stature will cause stars to appear to have risen before they are above the horizontal plane. Thus if, on the Earth's surface *ABD*, *A* be the place of the spectator's feet, or the bottom of a tower on which he is, *Aa*

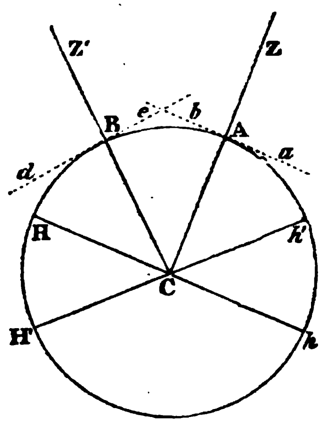


drawn a tangent to the surface at the point *A*, may represent his sensible horizon. If the eye be supposed to be at *A*, it cannot see an object till it is level with or above *Aa*. But if the place of the eye be transferred to *O*, an object may be seen if it be level with or above the line *OBb*.

The line *OBb* is a line drawn from the spectator's eye at *O* and touching the earth's surface at *B*; and the horizon, were it supposed to be composed of such lines as *OBb* would be a conical surface having its apex in *O*.

The depression of *b* below *a*, measured by the angle *amb* is technically denominated, the *Dip:* which, from *OA*, the eye's elevation, and the radius of the Earth, may easily be computed.

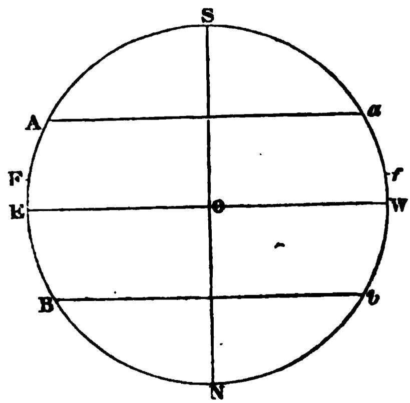
The tangent plane in which aAb lies, has been called by Astronomers (as we have seen), the *Sensible Horizon:* but they have also imagined, for the purposes of calculation, another horizon the plane of which, parallel to the former, passes through the Earth's centre, and is denominated the *Rational Horizon.* HCh, parallel to aAb, may represent this latter plane. It is plain that both the *Sensible* and the *Rational* horizon are merely relative: in other words, they must change with a change in the spectator's place. Of a spectator at A, ab perpendicular to CAZ is the



sensible, and Hh, parallel to ab, the *rational* horizon; and Z is his zenith. Of a spectator at B, ed perpendicular to CBZ' is the sensible, and $H'h'$, parallel to ed, the rational horizon: and Z' is his zenith.

Let us consider a little farther the appearances that would take place, were a spectator stationed at sea, or in the midst of a level country. Suppose then O (see fig. p. 5.) to represent his station and $SENW$ the imaginary circular boundary of a plane extended beneath his feet to be his horizon. If a star *rose* at A it would describe a curve above the horizontal plane, and sink beneath it, or *set*, at some point a. In like manner another star rising at B would describe a curve above the plane of horizon, and *set* at some point b. But this circumstance also, wherever the stars A, B were, would always take place; namely, the equality of ab, the distance of the points of setting with AB with the distance

of the points of rising. If the arc AB equals the arc ab, then the chord Aa is parallel to the chord Bb: and a diameter such as SON drawn perpendicularly to Aa, and consequently bisecting



it, will be perpendicular to and will bisect all other chords such as Bb: and will moreover bisect the arcs ASa, BSb, &c. The points S and N determined after the preceding manner are the *South* and *North* points of the horizon, or (as it is called) the *Azimuth* circle $SENW$. EOW drawn perpendicularly to SON determines E and W, the *East* and *West* points, which together with the two preceding form the four *Cardinal points*. $SENW$ has been called the *Azimuth* circle, and azimuth distances are measured from the South and North points. SA is the azimuth of the star rising at A, Sa of its setting at a.

The complement of the azimuth of a star is its *Amplitude*: and amplitude is accordingly measured from the East and West points. Thus the *Amplitude* of the Star's rising at A is EA; the *Amplitude* of its setting at a is Wa.

A star rising at A will gradually ascend above the plane of the horizon till it attains its greatest height; it will then decline, by like degrees, until it sets or disappears at a. If we conceive a plane passing through S and N and perpendicular to the plane of the horizon, then a star rising at A and ascending after the manner just described will be at its greatest height above the horizontal plane when it reaches the perpendicular plane. The same will happen to every other star. The greatest heights of different

stars will be different, but they will all be attained to in that plane which, passing through *S* and *N*, is perpendicular to the plane of the horizon. The perpendicular plane above described is called the plane of the *Meridian;* because, the middle of the day happens when the Sun in his ascent above the horizon reaches it. It is usual to suppose this plane bounded by a circle passing through *S* and *N*, and having therefore the same radius as the horizon or azimuth circle *SENW:* which, in fact, is to suppose these circles to be the *great* circles of the same sphere.

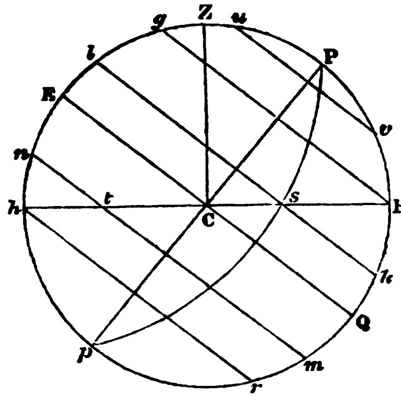
The meridian intersects (see l. 2.) the horizon in *S* and *N* the South and North points: it must also pass through the zenith (see p. 4.) and through the pole (see ll. 1, 2 &c.) Every circle, the plane of which is perpendicular to the plane of the horizon, is denominated a *Vertical* circle. The meridian, therefore, is a vertical circle. The vertical circle, which passes through *E* and *W* the East and West points, is distinguished by the name of the *Prime Vertical.*

We have spoken of the risings and settings of stars, such as they will appear to be to a spectator placed at *C* the centre of the plane of the horizon, but, hitherto, we have said nothing, of the intervals of time elapsed between the respective risings and settings. Now a spectator in our northern climate, looking towards *S* the south, cannot fail to remark that a star between its rising at *F* and setting at *f* is longer above the horizon than a star which rises at *A* and sets at *a:* which kind of inequality takes place, and in a greater degree, with every star successively placed between *A* and *S*. But he may also note that every star takes the same time in passing from its rising through its setting to its rising again. A star therefore at *A* is longer below the horizon than a star at *F*, and still much longer than a star at *E*. But a star rising at *E* the East point has this peculiarity: namely, that it is above the horizon exactly as long as it is below. On this account the great circle in which such star moves is called the *Equator.*

The phenomena that have been described may be explained by supposing the concave Heavens, in form like an hollow sphere, to revolve round an axis passing through the pole and the centre of the Earth, and in a time equal the interval between two successive risings of a star.

Thus, let *PCp* be the axis, *HCh* the rational horizon: then *CZ* drawn perpendicularly to *Hh* (see p. 4. l. 10.) determines *Z* the

spectator's zenith *. *EQ* is perpendicular to *Pp* and *vu*, *Hg*, *kl*, *mn*, *hr*, (representing the projections of circles to the planes of which *Pp* is perpendicular) parallel to *EQ*. If we conceive the plane of



this diagram to be placed perpendicularly on the plane of the former (see p. 6.) which was meant to represent the horizon, so as to be adapted to northern latitudes, then the plane *PEpl* will be the plane of the meridian, *E* will be the point of the greatest ascent of a star rising at *C*, and if we suppose *t*, *s* to be the orthographical projections on the plane of the meridian of the points *A*, *B*, then *n* and *l* will be the points of the greatest ascents of stars rising at *A* and *B*. Now suppose the figure to revolve round *Pp*: then *tn* will be proportional to the star's ascent from *t*, the place of rising, to *n* its greater elevation, and *tm*, every point of which is below the horizon, will be proportional to the time from the star's greatest depression (at *m*) beneath the horizon to its rising at *t*, and *tm*, as it is evident, is greater than *tn*: again, since $CE = CQ$, the time that a star is above the horizon is exactly equal to the time of its depression beneath that plane (p. 6. l. 29). A star rising at *s* will be above the horizon during a time proportional to $2Sl$, and below it during a time proportional to $2Sk$: and, as it is evident, $2Sl$ is greater than $2Sk$ (p. 6.)

Suppose a star to be exactly at *H*, then it can never set, but it will be a *circumpolar* star (see p. 2.): and such will be all stars

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* For the rational horizon is parallel to the sensible.

situated between H and P. A star placed at P would appear to be at rest.

Each of the stars of which we have spoken must (from the very nature of the scheme intended to explain their phenomena,) consume an equal portion between two of its successive risings: which portion of time may be called a *Sidereal* day, and which it is usual to divide into 24 equal parts, or *hours of Sidereal time.*

The preceding scheme is intended to shew, that the hypothesis of the revolution of the sphere of the Heavens round an axis passing through the poles, will adequately account for all those common phenomena relative to the risings, settings, ascents, &c. of stars which will present themselves to a spectator situated as we have described him to be. The hypothesis, therefore, is, at the least, a probable one. There is, however, another hypothesis equally probable with the former or rather more so, as being more simple, which hypothesis makes the concave Heavens to be at rest, but the globe of the Earth to revolve within them, round an axis, and in a direction from West to East.

Each hypothesis equally explains such pheuomena as have been already described: and since also to each hypothesis the same mathematical explanations and reasonings are applicable, we will adhere to the one already made use of and its connected diagram, and deduce some farther results.

The line EQ is intended to represent the Equator, lk, nm, vu, &c. which, from the supposition of the revolution of the figure round Pp, must be parallel to EQ, are called *Parallels of Declination.* The *declination* of a star is its angular distance from the Equator. The declination, therefore, of a star, which appears to move in the parallel kl is kQ (which is the measure of the angle subtended by kQ at the centre of the sphere); the declination of a star whose parallel is mn, is mQ or nE: of the *circumpolar* star at v, vQ is the declination; vP is its distance from the pole, or, as it is called (P being the north pole) its *north polar distance: mp* is the *south polar distance* of a star at m, the complement, as it is plain, of mQ the star's south declination. A *secondary* is a great circle passing through the poles of that other great circle to which it is a secondary. Thus $HphP$, the meridian, is a secondary to the horizon Hh. The circle Psp &c. is a secondary to the Equator EQ. The *prime Vertical* (see p. 6.) a secondary to the horizon, as indeed

is every great circle passing through Z and a point in Hh: a great circle, however, of this latter description, is farther distinguished by being called a *Vertical Circle,* since its plane, perpendicular to that of the *horizon,* is, in other words, vertical.

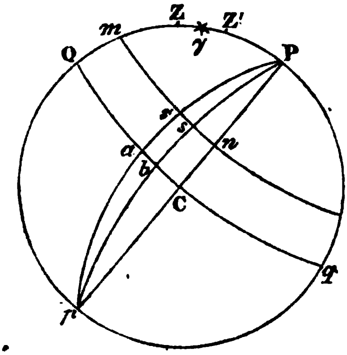
What Declination and its complement Polar distance are with respect to the Equator, *Altitude* and its complement, *Zenith distance,* are with regard to the horizon. The former is the star's angular distance from the spectator's horizon measured on a vertical circle: the latter is the distance from the zenith of the same spectator. The altitude, for instance, of the Equator, or of a star therein situated, is Eh: its zenith distance is ZE: the altitude of a star at n, is nh; its zenith distance is Zn.

Since the sphere, with all the stars supposed to be fixed in its surface, revolves in 24 hours of sidereal time, the stars situated in different parallels will appear to more with different velocities. A star near to P will appear scarcely to move: the velocity of a star describing vu will be as much less than the velocity of a star situated in the Equator, as uv is less than EQ: but uv has to EQ the same proportion as its radius has to the radius of the Equator: or that proportion which the sine of the angle PCu has to Cu: but

$$\begin{aligned}\text{sin. } PCu &= \text{sin. } Pu = \text{sin. North polar distance,}\\ &\text{or} = \text{cos. declination.}\end{aligned}$$

If therefore we call V the velocity of a star or point in the Equator, the velocity of any other star $= V$. cos. star's declination.

The *Hour-angles* are those angles at the Pole which, contained



between two secondaries to the Equator, intercept the space passed

over by a star, in any assigned time, either on the Equator or on a parallel. Thus if a star move from s to s', the *hour-angle* is said to be sPs' or bPa, which is measured by ab, ab being an arc of the equator. Now ab, or the angle bPa, must be proportional to the time, for since the point b is, by reason of the sphere's revolution, transferred from C to Q or through an arc of $90°$, in 6 hours, it must be transferred from C to b and from b to a, by reason of the sphere's *uniform* revolution, in times which bear, respectively, that proportion to 6 hours which Cb, ab, estimated in degrees, bear to 90 degrees. If ab, therefore, contains $1°$, the time through ba, or the hour-angle $aPb = \frac{1}{90}$th of 6 hours, or $\frac{6}{90}$ths of an hour, or the value of the *horary* angle aPb, or sPs' is $0^h.06666$ &c. or 4^m.

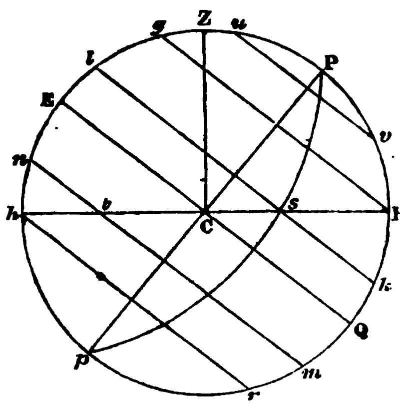
The *Poles* and the *Equator*, that have hitherto been described, belong to the celestial sphere; but the Earth also has its Equator, Poles and Axis. Conceive an interior sphere, in the figure of p. 7, described round C to represent the Earth, then the plane of its Equator and axis will be such parts of the Equator EQ and axis Pp as are contained within the sphere representing the Earth and are terminated by its surface. Or, we may reverse the process and give to the Celestial Sphere its Equator and Axis, by extending to the Heavens the Earth's Equator and Axis.

Places situated on the Earth's surface are said to have *Latitude*, which is to be defined, distance from the Earth's Equator. But the *Latitude* of a place in its astronomical meaning, or with reference to its astronomical measure, is an arc of the meridian intercepted between the zenith of the place and the celestial Equator, or, which is the same thing, it is the complement of the arc which lies between the zenith of the place and the pole: which latter arc, therefore, may be called the *Co-latitude* of the place.

If the Pole Star, that which is usually so called, were exactly situated in the Pole, the method of determining the latitude of a place, by means of that arc which is its complement, would be a very simple one: since the plumb-line determines the zenith. But the *Pole Star* being, in fact, a circumpolar star, its angular distance from the zenith will vary with the time of observation. Its distance, therefore, cannot give the true value of the co-latitude, or its distance requires a *correction* in order to give the co-latitude

truly. Such method then of determining the latitude cannot be a very simple one, and can only be practised subsequently to, and by the aid of, an improved or refined state of astronomical science. It is now mentioned for the purpose of giving the student some general notion of latitude and of the means of measuring it.

If a circumpolar star can be made subservient to the finding the latitude, any other known star may, and for like reasons. It is sufficient to measure the angular distance from such star, when on the meridian, and the zenith, which latter point, as we have already said, the plumb-line determines; that is, it is that point in the heavens to which the plumb-line, if we imagine it to be continued upwards, is directed. The polar distance of the star is known (since that condition is implied in the expression of *known star*), therefore the co-latitude of the place (PZ in the figure of p. 9.) is either the star's polar distance minus the meridional distance of the star from the zenith, or the star's polar distance plus the star's distance from the zenith. For instance, if the star be at l.



$$PZ = Pl - Zl$$

if at u, $PZ = Pu + Zu$.

It is easy to shew, on grounds like those that have been laid down, that the *difference* of the latitudes of two places may be determined simply from the distances of the same star from their

repective zeniths. Thus, if the star γ (fig. p. 9.) should lie between Z, Z', the two zeniths,

$$ZZ' = Z\gamma + Z'\gamma$$
$$\text{but } ZZ' = PZ - PZ'$$
$$= (90 - PZ') - (90 - PZ)$$
$$= \text{lat. of } Z' - \text{lat. of } Z$$

in which operation it is not necessary to know the declination of the star γ.

Suppose the star γ Draconis should be $2' \, 4''.9$ North of the Greenwich Observatory, and $19' \, 23''.3$ South of the Observatory at Blenheim, then $ZZ' = Z\gamma + Z'\gamma = 2' \, 4''.9 + 19' \, 23''.3 = 21' \, 28''.2$ the *difference* between the latitudes of the two Observatories: consequently if the latitude of one Observatory were known, that of the other might be determined: for instance, if the latitude of Greenwich be taken at

$$51^\circ \, 28' \, 40'',$$

that of Blenhein must equal

$$51^\circ \, 28' \, 40'' + 21' \, 28''.2 = 51^\circ \, 50' \, 8''.2.$$

As a second instance, if the zenith distance of γ Draconis from the Dublin Observatory on January 1, 1818, be $1^\circ \, 52' \, 20''.7$, then the difference of latitudes between the two Observatories of Greenwich and Dublin is

$$1^\circ \, 52' \, 20''.7 + 2' \, 14''.9 = 1^\circ \, 54' \, 35''.6,$$

supposing the distance of γ Draconis from the zenith of Greenwich to be, at the same time, $2' \, 14''.9$. *

There are other methods explicable, as to their general nature, even in this early stage of our progress, that may be used in deter-

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* We have taken what were, nearly, the *mean* or *reduced* zenith distances of γ Draconis from the two Observatories at the beginning of 1818. It will appear, and fully, during the progress of the work, why the zenith distance of a star does not always remain the same at the same place. The star γ Draconis is continually approaching the zenith of Greenwich, and receding, by equal quantities, from that of Dublin.

mining the latitudes of places. For instance, if we determine the respective zenith distances of two known stars at two places, we may deduce the difference of latitude of those places. In point of theory it matters not where the stars, relatively to the zeniths of the places of observation, are situated: but the excellence of the practical method depends on this circumstance, that the star observed should be near the zenith of the place of observation: for, in such a case, one great cause of inequality, namely, the refraction of the air, would be nearly rescinded, and the accuracy of determining the difference of latitudes would rest on the ascertained or ascertainable difference of the declinations of the two stars.

In this first chapter we have advanced, very little beyond the general description of the ordinary appearances of the Heavens, and their explanation on the hypothesis of the revolution of the starry sphere. The revolution of that sphere (the *Primum Mobile* as it was called) from East to West, with the supposed quiescence of the Earth, will account for the risings, settings, durations of ascent and descent of the stars equally well (and we may add, on the same principle), as the rotation of the Earth round its axis from *West to East*, the Heavens being supposed quiescent. The first is the most obvious hypothesis, the latter, when more closely viewed, the most simple hypothesis. The stars *seem* to move round us; but when we consider the prodigious velocity with which, by reason of their immense distance (a point easily made out) they must revolve, we are disposed to search out for and to adopt some other hypothesis that is free of so revolting a circumstance. There is, indeed, no summary proof to be given of the truth or falsehood of either of the hypotheses. One, for several reasons that will hereafter appear, is much more probable than the other. Indeed the hypothesis of the revolution of the sphere is inadequate, as astronomical science now stands, to solve all the phenomena.

We must, however, be content, at present, to take for granted the truth of the hypothesis of the Earth's rotation. If it continues to explain simply and satisfactorily, other astronomical phenomena than those already noted, the probability of its being a true hypothesis will go on increasing.

We shall never indeed arrive at a term when we shall be able to pronounce it absolutely *proved* to be true. The nature of the subject excludes such a possibility.

We will now proceed to notice some other phenomena different from those that have preceded and not explicable solely on the hypothesis of the Earth's rotation. They need not, however, be considered as overturning that hypothesis. It will be more simple to consider that hypothesis to be established, and the new phenomena as indicating the necessity of some additional hypothesis, or the existence of certain circumstances of motion and translation that take place contemporaneously with the Earth's rotation and consistently with it.

CHAP. II.

On the proper Motions of the Earth, Moon, and Planets.

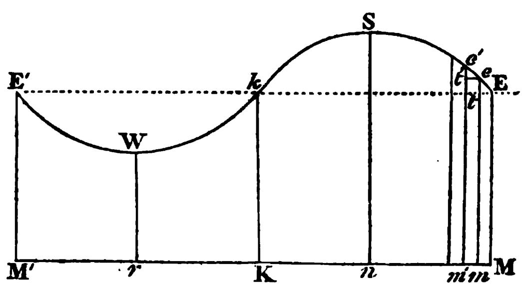
In the preceding Chapter the phenomena described and explained are chiefly phenomena of stars called, from their preserving the same invariable distance from each other, *Fixed Stars.* Their risings, settings, the times of their elevation above the horizon, of their depression beneath it, are easily explicable, as we have seen, on the hypothesis of the Earth's rotation round an axis inclined, in our latitude and in every habitable latitude, to the horizon.

There are other heavenly bodies, the Sun, the Moon, and the planets, that assume only in part, or nearly, those appearances that belong to the fixed stars. The Sun, for instance, if he should rise at the same point in the horizon, which a fixed star rises in, would set in the evening, nearly where the star sets. The length of day would not seem to differ from the time of the star's ascent above the horizon: and his meridian height, would, to common observation, appear to be the same as that of the greatest elevation of the star above the horizon. The same circumstances would appear to take place with the Moon and Planets. But minute differences are not to be detected by common observation. The Sun and star, if they rose exactly at the same point of the horizon, would not pass the meridian exactly at the same point. On any day between the middle of winter and the middle of summer, the Sun rising where the star rises would pass the meridian in some point *above* the star's passage: during the other half year, in some point *below*. But in order to distinguish these circumstances some nicety of observation is requisite. If, however, we examine a star and the Sun, or a star and one of the planets for a longer interval than a day, their separation or their approach, which is perpetually taking place, will become manifest even without the aid of instruments.

Suppose, for instance, at the beginning of March, that we observed the Sun and a star to rise at the same point F (fig. p. 5.) of the horizon: they would set nearly at the same point f and

cross the meridian nearly at the same point. The next day the star would still rise at the same point F, but the Sun would rise at some point between F and E, would set at some point between f and W, and would pass the meridian above the point of the star's passage. The like would happen on each succeeding day. The Sun would rise nearer and nearer to the east, would set nearer to the west, and pass the meridian more and more above the Star. In about 20 days from the time of the first observation, the Sun would rise in the east (at E) set in the west at W, and reach a meridional height equal to the co-latitude of the place of observation. After that time the Sun would rise between the east and north points of horizon (E and N) and set between the west and north (W and N) till about the end of June, at which time, having, reached his extreme intermediate point of rising between E and N, and his greatest meridional height, he will begin to reiterate his course of risings and meridional heights, and passing the term from which we began (see l. 1.) to date them, he will reach, between E and S, his farthest point of rising from E, will ascend to his least meridional height*, and again begin to regress.

If we take a line MM' and erect on it perpendiculars ME, me, $m'e'$, &c. to represent the Sun's meridional heights on successive days, ME representing the height on the day when the



Sun rose in the east, nS his greatest height on the day when he rose on a point of the horizon nearest the north, &c. then the curve passing through the meridian Sun, during the year, will be of the form $ESkWE'$, the part KWE' being similar to ESK.

* Either on that day or on the preceding.

If we go no farther than the preceding instance, it is clear, if the stars be supposed to be fixed and their phenomena accounted for by the rotation of the Earth, that the phenomena just described as appertaining to the Sun cannot be so accounted for; they plainly indicate the Sun to have a proper and peculiar motion, or, which we shall find to be the same thing, the place of observation (meaning thereby the Earth) to have a proper motion, or, one distinct from the *conversion* of the Heavens. But it is easy to make other observations that shall plainly indicate a proper motion in the Sun, and shew the necessity, if we would explain the phenomena, of correcting or of adding to the hypothesis of the Earth's rotation: which cannot be the sole hypothesis.

As a second instance, leading to the same inference as the former, let us take that of the Sun and a star when they set nearly together. Suppose, on a particular day, that we observe a certain star to set a little after the Sun. On the following day and on each successive day, the star's *setting* will follow more closely that of the Sun: till their proximity will become so close as to cause the light of the former to fade away and to be extinguished by the effulgence of the latter: the star, therefore, for some time, will disappear; but, if, after a few days, we direct our view to the *rising Sun*, we shall perceive the star emerging, as it were, from its beams, and, after this, on succeeding mornings, preceding, by still greater and greater intervals, the Sun in its rising.

The latter part of the phenomenon, which we have just noticed, namely, that of the star's rising just before the Sun, is technically called the *Heliacal* rising of the star. There are only certain stars that can so rise, and that only at particular times of the year. Their heliacal risings, therefore, must be indicative of those times. It was by such observations that the rude notions of antiquity recognised the seasons, and regulated the labours of the year *.

The phenomenon which we have last described indicates, like the former, the Sun to have a proper motion among the fixed

* The Egyptians looked for the inundation of the Nile at the time of the heliacal rising of Sirius, or, as they called it, of Thoth the Watch-Dog.

stars: *towards* those stars that set after him and *from* those stars that rise before him: which are circumstances of the same kind, or indicate the same direction of the Sun's motion. The Sun's motion, however, although, as it has been described, is first *towards* a certain star, and then, having passed it, *from* it, is not made in a direction either the same as that of the *star's parallel* (see p. 8, l. 26, &c.) or parallel to it, but in some oblique direction: which indeed may easily be collected from those circumstances which were described in pages 15, 16. as belonging to the first phenomenon. For it was there shewn, by noting the points of the horizon at which the Sun rose on successive days, that the Sun has an horizontal motion, or, as it is technically called (see p. 5.) a motion in *azimuth;* and, also, by noting his meridional heights on those days, that the Sun has a motion perpendicularly to the plane of the horizon: which two motions so detected must be the parts of a compound oblique motion.

The apparent motion of the fixed stars is from east to west: the real motion of the Earth (according to the preceding supposition, (see p. 13.) which causes the former apparent one, from west to east, and, in our hemisphere, to a spectator looking towards the south, from the right hand to the left: and in the same direction, that is, from the right towards the left, or from the west towards the east, is the Sun's proper motion.

The fact of a motion of the Sun from the west to the east is sufficient to explain why certain remarkable stars and groups of stars, called *Constellations,* are seen in the south at different hours of the night during the year. For, the hour depends solely on the Sun: it is noon, when he is in the south. Stars directly opposite to him are, therefore, by the rotation of the Earth, brought on the meridian at midnight. But the stars on the meridian at 12 one night, cannot again be there, at the same hour, on the succeeding night: for, the Sun having shifted his place a little to the east, the stars before opposite to him are now opposite a part of the Heavens to the west of the Sun: that is, they must come on the meridian a little before midnight: and on succeeding nights more and more before midnight. It thus happens then that every star is, during the year, on the meridian at all the hours of the four and twenty. There are some stars indeed that may be on the meridian, and yet, by reason of the Sun's brightness, may not be discerned there.

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If we apply to the Moon the same kind of observations that have been described to be used for detecting the Sun's motion, we shall find the Moon to move, by a proper motion, amongst the fixed stars towards the same parts, (that is, in a general way of speaking, from west to east) as the Sun, but with greater rapidity and not by similar and regular changes of place, whether we consider the *azimuthal* or the *meridional* changes, (see p. 18.)

For instance, the Sun's annual path traced out in p. 16. will be nearly the same every year. But a path so traced out for the Moon, during one of her revolutions, would not be her path in her next revolution round the Earth. The Moon, therefore, has a proper motion of her own and not similar to the Sun's: we may go farther and state that, as far as we can judge from common observations, the two motions are unconnected, or there is no single principle which will account both for the one and the other.

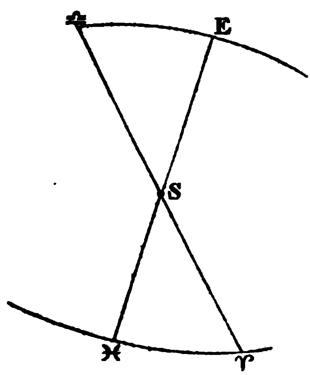
Besides the Sun and Moon there are certain other stars which have their proper motions: and motions so peculiar and irregular as to have procured to the stars possessing them the denomination of *Planets*. They sometimes appear to move, like the Moon, towards the east: at other times, however, towards the west; and there are conjunctures, when, during several successive nights, they appear nearly stationary. It will be seen hereafter that there is no real difference between the direction of the planets' motions and that of the Earth.

If the spectator be supposed to have taken his stand at the Sun, he will view the Earth as one of the planets, and, then, all the planets constantly moving in the same direction. That they sometimes appear stationary, and, at other times, *retrograde* (that is, moving in a direction contrary to their usual one) is to be attributed to the motion of the Earth, which motion combined with that of the planets, causes them, under certain circumstances, to appear to move otherwise than they are really moving. The *retrogradation* of a planet is a phenomenon partaking somewhat of the nature of an illusion.

The motions from west to east that we have spoken of, take place and must be combined with that diurnal motion from east to west, which arises from the rotation of the Earth. This latter

motion is so great, that, as it were, it overpowers the former, and, with an inattentive spectator, prevents it from being observed. Even the Moon, which of all the planets has the swiftest proper motion towards the east, shifts her place in the course of a day by not more than 13^0; whilst, by the rotation of the Earth, she is seemingly carried in the same time through 360^0. There are, however, conjunctures when we cannot but recognise her proper motion; when, for instance, the Moon is near a star previously to an occultation: for moving over a space equal to her diameter in an hour she then visibly approaches the star.

As the stars which are fixed seem to move by reason of the Earth's rotation, so the Sun, which is, in fact, stationary, seems to move by reason of the Earth's revolution round him. But it makes no difference either in the explanation of phenomena, or in the deduction of such results as belong to the subject; whether we suppose the Earth to move round the Sun, or the Sun to move round the Earth. A spectator at *E* sees the Sun *S* in the



heavens at the place ♓. Transferred to ♎ he sees the Sun in ♈. The Sun appears to him to have moved from ♓ to ♈: the same appearance as that of a real translation of *S* from ♓ to ♈.

Of the *Solar System*, composed of the Earth, the Moon, the Planets, their Satellites, and certain stars more erratic than the planets, and called Comets, the Sun, the chief body, occupies the centre. Round the Sun, in their order, at different distances, and in different periods, revolve Mercury, Venus, the Earth,

Mars, Vesta, Juno, Ceres, Pallas, Jupiter, Saturn, the Georgium Sidus.

These planets Astronomers have distinguished (as they have also the Sun and Moon) by appropriate symbols : thus

The Sun	☉	Ceres	⚳
Mercury	☿	Pallas	⚴
Venus	♀	Jupiter	♃
The Earth	⊕	Saturn	♄
Mars	♂	The Georgium Sidus or Herschel.	♅
Vesta	⚶		
Juno	⚵	The Moon	☽

Mercury, Venus, Mars, Jupiter and Saturn, are what are called the *old Planets*, discernible by the naked eye, and consequently known to the antients *. The Georgium Sidus, (or in order to give it what the others have, a mythological denomination, Uranus) was discovered in 1778 by Dr. Herschel, and therefore, it is frequently called by Foreigners, the *Herschel*. The other four planets Vesta, Juno, Ceres, Pallas, (at first fantastically called *Asteroids*) have been discovered since 1801, the first and fourth by Olbers, the second by Harding, and the third by Piazzi. The latter *new* planets are extremely small and cannot be seen without a telescope, which is the case also with the Georgium Sidus, not indeed by reason of his small size, but of his great distance.

The system which has been briefly described is sometimes called, from its author *Copernicus*, the *Copernican*. The characteristical point, it must be noted, in his system is the placing the Sun, as an immoveable and the chief body, in the centre of it.

In the next Chapter we will consider whether, on the proposed hypotheses and the established facts, we are able to account for the vicissitudes of seasons and the different durations of day and night. The only thing aimed at will be something of

* Maxume vero sunt admirabiles motus earum quinque stellarum, quæ falsò vocantur errantes, nihil enim errat, quod in omni æternitate conservat progressus et regressus reliquosque motus constantes et ratos. Cic. *de Nat. Deorum*, Lib. II. 19, 20.

the nature of a popular explanation, probably accounting for the phenomena, on hypotheses that are simple and consistent with themselves. Independent and rigorous demonstrations belong not to the present subject of enquiry: as far indeed as the establishment of systems and the verification of hypotheses are concerned. The purely mathematical demonstrations which are subsidiary are, indeed, as true in Astronomy as in any other science: but the theory they have acted in aid of, they may have vainly propped, and it may be false. A theory if false may be proved to be so by one instance: whereas the truth of a theory can hardly ever be easily or soon established.

CHAP. III.

On the Vicissitude of Seasons, and of Day and Night.

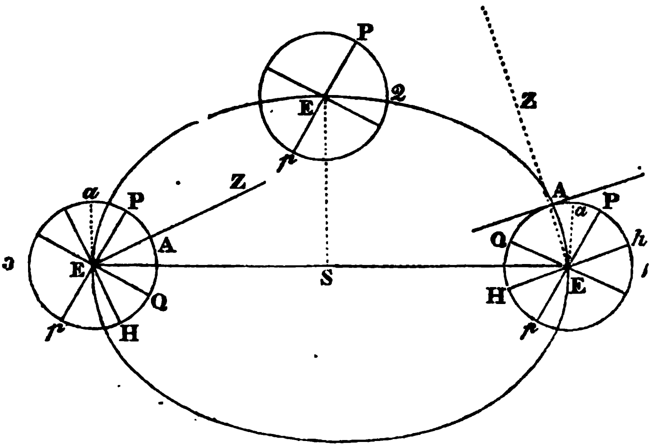
THE daily rotation of the Earth round its axis, and the annual revolution of the Earth round the Sun, are the two hypotheses which, in the preceding Chapters, have been found adequate to explain several of the ordinary phenomena of the Heavens. A condition attending the former hypothesis is that the axis of the Earth always preserves its parallelism. For the polar star is always (to common observation at least) quiescent, and the circumpolar stars always describe circles of the same magnitude. A condition attending the second hypothesis is that the path of the Earth's circuit, or its orbit, lies in one plane: since the points of the Heaven in which the Sun, during the year, is successively seen, lie in one or the same plane.

If the Earth's axis of rotation were perpendicular to the plane of its orbit, the planes of the equator and of the orbit would be coincident. The Sun would always describe the same parallel of declination; if he rose once at the east point E, (see fig. pp. 5 and 7.) he would always rise there, his apparent diurnal course would be always in the equator, and his annual course would be amongst those fixed stars which lie in the celestial equator. But we have seen (pp. 15, &c.) that this is not the case; his annual course is made obliquely to the equator, or, since it is made in the same plane, the plane of his orbit is inclined to the plane of the equator, and (which is only to repeat the same thing in different words) the axis of the Earth is inclined to the plane of the Earth's orbit.

This point enables us at once to explain the vicissitudes of the seasons, and the different durations of day and night, as dependent on the combined circumstances of the time of the year and of the latitude of the place.

Let S be the Sun, E the Earth in three positions 1, 2, 3, of her orbit; let also Pp be the Earth's axis, EQ the equator, and $PAQp$ must be conceived to be a section of the Earth perpendicular to the plane passing through the orbit EEE; so that

PAQp will be opposite to the Sun, and to a spectator at *A*



will be a meridian *. The axes *Pp* are drawn parallel to each other in the three positions.

Let us first consider the position marked 1, in which the angle *SEP* is the greatest possible. The spectator, situated in a northern latitude, is supposed to be at *A* : his zenith at *Z*, and his rational horizon will be *Hh*, *Hh* being perpendicular to *EZ*. The Sun in this position, as it is plain, is the most below the equator *EQ* and least above the horizon when on the meridian;

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* Diagrams in Astronomy are not only imperfect representations, since solids are to be represented *in plano*, but, with regard to proportion, preposterous representations of the things they ought to exhibit. The first is a real evil, the latter a very slight one; for, the demonstration in the text is equally clear whether *E2* be the half or the double of what it is in the above Figure, it is in fact, independent of the represented relative proportion of *E2* to *SE*; yet, the former is to the latter, in fact, as 1 to 22984, and not, as in the Figure, as 1 to about 3 or 4. The first evil, however, if we do not recur to schemes of *solid* representation, admits of no remedy, except from the student's attention. The orbit 1, 2, 3, must be conceived as viewed obliquely, and then *PAHp* to be perpendicular to it. Or, if the orbit be conceived coincident with the plane of the paper on which it is drawn, then the plane passing through *PAHp* is perpendicular to the paper.

and, according to the construction of the diagram, the Sun is on the meridian of the spectator A. The position 1, corresponds to that case of p. 16, in which the Sun rose between the east and south points at his farthest point from the east.

In this position of the Earth, a plane drawn perpendicular to SE, at the point E, would divide the Earth into two hemispheres, one illumined, the other in darkness as it is represented in Fig. of p. 29: the south pole (p) being in the former, the north pole (P) in the latter. In this case, since the boundary (df) of light and darkness falls between A and P, it is clear that the spectator at A would, by the rotation of the Earth round Pp, be transferred from A to c in a less time than he would be transferred from c to a: but $2Ac$ is proportional to his day, $2ca$ to his night. Again, since $2Ak$ is proportional to 12 hours, the duration of the day ($2Ac$) would be less than 12 hours, and the duration of the night ($2ca$) greater than 12 hours, and the difference would be measured by $2ck$.

This difference is easily computed in any given latitude: through c draw Pcm, a quadrant of a *secondary* to the equator, then, by similar figures, mE bears to QE the same proportion as ck bears to Ak: now, in the right-angled spherical triangle cEm, we have

$$cm = QA = \text{(see p. 10.) the latitude of the place,}$$

$$\angle cEm = 90^0 - SEQ = \text{co-declination of the Sun,}$$

whence by Naper's Rule, (see *Trig.* ed. 3. p. 146.)

$$\text{rad.} \times \sin. mE = \text{co-tan.}\ cEm \times \tan.\ cm$$

$$= \tan. \odot\text{'s dec.} \times \tan.\ \text{lat.}$$

Suppose, for instance, the latitude of the place to be $51^0\ 52'$ and the Sun's declination (which must be his greatest south declination) to be taken equal to $23^0\ 28'$, then we shall have

$$\log.\ \tan.\ \text{lat.}\ (51^0\ 52') \dots\dots\dots\ 10.10510$$

$$\log.\ \tan.\ \text{dec.}\ (23^0\ 28') \dots\dots\dots\ 9.63761$$

$$\therefore\ 10 + \log.\ \sin.\ mE = 19.74271$$

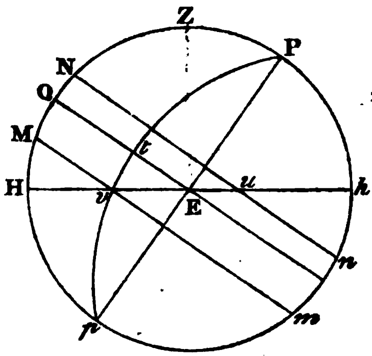
$\therefore$ (by the Tables) mE . . . . $= 33^0\ 34'\ 20''$, nearly,
and (15^0 being equal 1 hour) in time. . . $= 2^h\ 14^m\ 17^s$, nearly;

consequently, the length of the day $= 12^h - 4^h\ 28^m\ 34^s$

$= 7^h\ 31^m\ 26^s$,

and the length of the night $= 16\ \ 28\ \ 34$.

We may also arrive at this result by means of a diagram



like the present. Thus, let Z be the zenith of the place, take $ZQ = 51^0\ 52'$ and draw QE, representing the equator, through the centre; also $QM = 23^0\ 28'$, and draw Mvm parallel to QE: then Mvm (see p. 8.) is the Sun's parallel of declination: and (Hh being the horizon drawn perpendicularly to ZE) $2Mv$ measures the time the Sun is above the horizon (Mm measuring 12 hours), or $2Qt$ measures the length of the day, and $2QE + 2tE$ the length of the night, $2QE$ measuring 12 hours: in order to find $2tE$, the difference of day and night, we have by Naper's Rule,

$$\text{rad.} \times \sin. tE = \text{cotan.} \angle tEv \times \tan. tv$$
$$= \tan. ZEQ \times \tan. QM$$
$$= \tan. \text{lat.} \times \tan. \odot\text{'s dec.}$$

as before (see p. 25.)

At the point v, which is the intersection of the parallel of declination Mvm and of the horizon Hh, the Sun rises. In the case which we have taken, it is the point of the Sun's rising that is farthest from the east towards the south (see p. 16.). Its distance from E may be computed by solving the right-angled triangle tEv thus:

By Naper*, $r \times \sin. tv = \cos.\ \text{lat.} \times \sin. Ev$,

$$\begin{array}{lr} \log. r + \log. \sin. 23^0\ 28' \ldots\ldots\ldots & 19.60011 \\ \log. \cos. 51^0\ 52' \ldots\ldots\ldots & 9.79063 \\ \hline \log. \sin. Ev = \ldots\ldots\ldots & 9.80948 \\ \hline \end{array}$$

$\therefore Ev = 40^0\ 9'\ 25''$.

In the position (3), which is diametrically opposite to (1), the Sun, (since the axes Pp, Pp are parallel to each other) is as much *above* the equator as he was below in the position (1). If therefore we were to draw, as before, a plane passing through E and perpendicular to SE, it would separate the Earth into two hemispheres, one illumined by the Sun, the other deprived of his light: but, in this latter case, the north pole P would be as much within the illumined part as the southern pole p was in the position (1).

The length of the day, therefore, will be what the length of the night was in the position (1), and *vice versa:* and the Sun in rising will now rise between the east and the north points, and as much towards the north, as in the position (1) it rose towards the south. This scarcely needs any proof; a proof, however, if required, might easily be had by the aid of the diagram already used. Thus take NQ equal the Sun's greatest northern declination, and draw Nun parallel to the equator QE: then the Sun will rise at u, and, in order to find Eu, we have (supposing a secondary to the equator to pass through u),

$$\text{rad.} \times \sin. \odot\text{'s dec} = \sin. Eu \times \sin. QEH,$$

$$\text{or rad.} \times \sin. \odot\text{'s dec.} = \sin. Eu \times \cos.\ \text{lat.}$$

the same equation as that given above, for determining Ev; $\therefore$ since $NQ = QM$, $Eu = Ev$, and consequently the arc Eu, or the *Sun's amplitude*, (see p. 5.) equals $40^0\ 9'\ 25''$.

The instances taken have been those, in which the Sun is most below and most above the equator: but the scheme will serve for other positions of the Earth: and, the computations for the lengths of day and night, and for the distance from the east,

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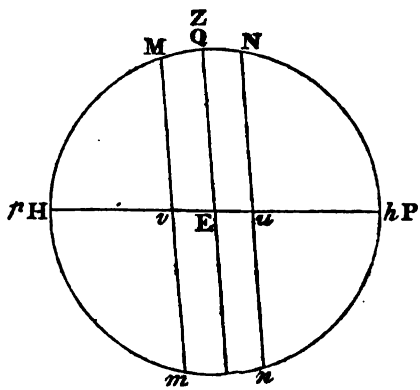
* *Trigonometry*, p. 146.

will be similar: since, instead of 23° 28′, we have only to substitute some other number of degrees, representing the declination.

In the position (2), the Sun is neither above nor below the equator, but in its plane produced. In the preceding diagram, Q would be the Sun's place: and the parallel described in 12 hours would be Qq, and, EQ being $= Eq$ *, the days and nights would be equal. The position (2) represents the Earth in spring. In the preceding instances we have supposed the spectator situated in some northern latitude between P the north pole and Q the equator. If we suppose him transferred from A (see fig. of p. 29.) towards Q, the zenith Z, which is always in EA produced, will descend towards the equator, and the point h (Hh being always perpendicular to EA) will approach to P. When A reaches Q, or when the spectator is at the equator, h and P will coincide, and the axis of the Earth will lie in the spectator's horizon. The diagram, therefore, of p. 26, will now assume the following appearance, in which the parallels of declination

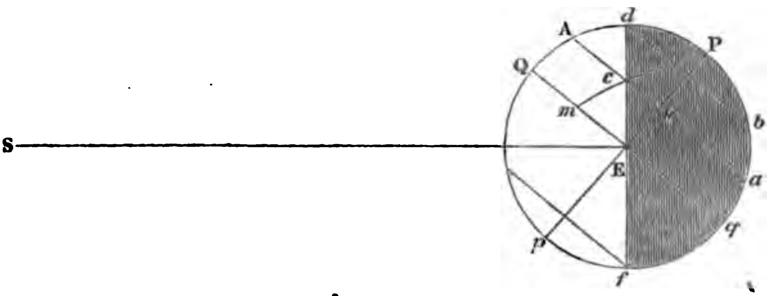


Mm, Nn, always bisected by Pp, are now bisected by Hh. In other words, the Sun (if Mm, Nn represent his parallels of declination) will, whatever be his declination, remain as long above as below the horizon: or the days and the nights of a spec-

* q is omitted (Fig. p. 26.) in the point where QE produced cuts the circle.

tator at the equator consist, whatever be the season, each, of 12 hours. If Mm, Nn represent the parallels of declination belonging to stars, then the inference is that every star is as long above as below the horizon, and that there are no *circumpolar* stars.

If the spectator, instead of moving towards Q, move towards P, the arc Ac which represents, or relatively measures, half his

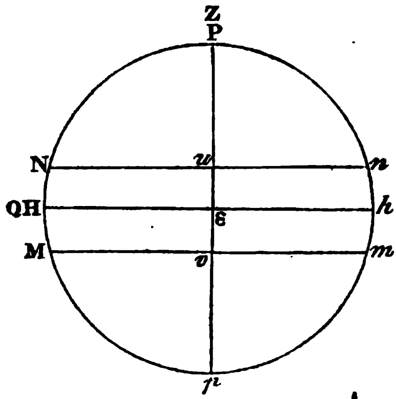


day, will decrease: At the point d, the spectator will be in darkness during the 24 hours *: but, since the figure is constructed for the greatest southern declination of the Sun, the above circumstance, namely, that of a night's duration of 24 hours, cannot take place either on a preceding or a following day: since, in either case, the Sun's declination, being less than his greatest declination, will cause the boundary of light and darkness to fall a little within the point A (the place of the spectator) and P.

Between d and P the spectator will be always within the darkened hemisphere, and, at P, the zenith and pole will coincide, as will the equator and horizon: the following diagram will represent the circumstances of the spectator's situation, which it will represent not only when it corresponds to fig. 1, (see p. 24,) that is, for the greatest southern declination of the Sun, but for any other declination. Thus, it must be continual night whilst

* If $SEQ = 23° \; 28'$, $PEd = 23° \; 28'$, which angle, if the spectator be at d, is the complement of his latitude. Consequently, in latitude 66° 32', on the *shortest day*, there is no direct light from the Sun. He would at noon just appear on the south point of the horizon.

the Sun describes any parallel beneath QHh, or whilst the Sun is



to the south of the equator; and continual ~~night~~ day, when the Sun's declination is northern. A spectator then, if we imagine him in such an extreme situation, would, during one half of the year, experience continual day, and, during the other half, continual night.

We have spoken (see p. 26.) of the Sun's describing a *parallel* of declination, which expression is not strictly correct: since the Sun's declination, which is perpetually changing, will bè a little different at the end of 24 hours from what it was at their beginning. If the Sun is ascending from the equator towards the north, he will be higher above the horizon of the spectator at the north pole at the end of 24 hours than at the beginning. Instead, therefore, of describing a *parallel* to the horizon (the horizon and equator in this instance are coincident) he will describe a spiral, and, in such a curve, he will appear continually ascending above the horizon till he has reached his greatest northern declination. From that summit he will, by like steps, descend, during a quarter of a year, or thereabouts, to the horizon and equator.

But if the Sun does not describe an exact parallel to the horizon of a spectator situated at the pole, a fixed star does. Every star, in fact, that is then visible, is a circumpolar star: equally elevated above the horizon wherever viewed; a spectator in fact, placed exactly in the pole has neither a meridian nor any east and west points.

Whatever be the circumstances relating to the durations of light and darkness which a spectator experiences in a northern latitude, when the Sun has a *south* declination, the same will a spectator, situated in a corresponding southern latitude, experience when the Sun has a corresponding *north* declination. Or the durations of day and night, when the Sun has a certain declination, will become reciprocally the durations of night and day when the Sun has an equal *contrary* declination. Thus, the Earth occupying the position (3) (in which the Sun is supposed to be at his greatest northern declination) the length of the day to a spectator in north latitude $66^0\ 32'$ (see Note to p. 29.) would, on his longest day, be just 24 hours. The Sun, at midnight, would just cease to be visible on the north point of the horizon.

It has appeared (see p. 26.) that $PEd = SEQ = 23^0\ 28'$ when the Sun is at his greatest northern declination. Draw from d (fig. p. 29.) a parallel db to the equator, and also a similar parallel from the point f: the parallels or small circles thus determined are denominated respectively the *Arctic* and *Antarctic* circles, or generally the *Polar Circles*. The distance of the former from the north pole, and of the latter from the south pole, is equal to the Sun's greatest declination.

The vicissitudes of seasons, inasmuch as they depend on the durations of day and night, have been explained from the revolution of the Earth round the Sun, and from the rotation of the Earth round an axis constantly inclined at the same angle to the plane of the Earth's orbit. If the Sun be the source of heat as well as of light, then heat will be imparted to an inhabitant of a northern latitude, during a less time in the position (1) than in the position (3). But, besides this circumstance, the Sun's rays fall more obliquely on A in the position (1) than in the position (3),

$$\text{for in (1)}\ \angle\ SEA = \angle\ AEQ + \angle\ SEQ,$$
$$\text{and in (3)}\ \angle\ SEA = \angle\ AEQ - SEQ.$$

This, in some degree, will account for the differences of temperature experienced by the same spectator at different seasons of the year; and one of the causes previously assigned, namely, the degree of obliquity of the Sun's rays, will explain why the regions near the equator are, *cæteris paribus*, hotter than the more remote. The distinction of the Earth's surface into climates and *zones* has

been long made. Within two parallels of declination, each distant from the equator 23° 28′, and called *Tropics*, the *Torrid Zones* lie: the *Frigid Zones* lie within the arctic circle and north pole, and the antarctic circle and south pole. The *Temperate Zones* are included within the tropics and the polar circles.

The above must be viewed merely as general and arbitrary divisions. We cannot affirm a place not to be cold solely because it is within the temperate zone. Local causes have vast influence. The temperature of the air at a place is not proportional solely to the place's latitude and the Sun's declination and distance*.

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* We have not supposed hitherto the Sun's distance to be variable, which it is.

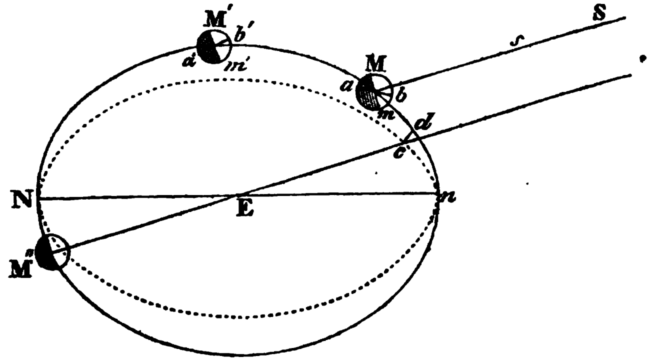
CHAP. IV.

On the Phases and Eclipses of the Moon.

If, in arranging the heavenly phenomena, we had purposed to give precedence to those which were either more obvious or which excited greater curiosity, we ought to have considered the Moon previously to the Sun and the planets. The proper motions of the latter, and their other phenomena, do not obtrude themselves so forcibly on our notice, as those of the Moon. Venus, to unassisted vision, always appears to shine with a full orb: but viewed through a telescope she assumes, like the Moon, her several *Phases*, and shines with an orb more or less *deficient*.

The Earth, as it was stated in p. 20, moves round the Sun. The Moon also (such is the doctrine to be laid down) moves round the Earth but, in an orbit, the plane of which is not coincident with, or parallel to, the plane of the Earth's orbit. If to these we add another condition, namely, that the Sun illuminates the Moon, and that the inhabitants of the Earth perceive the effects of such illumination, we shall possess the means of explaining why, at some times, the whole face or disk of the Moon is luminous, whilst, at other times, only portions of it are: we shall, in other words, be able to explain the Moon's *Phases*.

Let M, M', M'' be three different positions of the Moon in



her orbit, and let the dotted curve line represent the outline of a

portion of the plane of the ecliptic, which plane we must suppose inclined to that of the Moon's orbit. *E* is meant to represent the Earth, and the Sun is supposed to be so far distant that lines from it to *M*, *M'*, *M''*, &c. and *E* may, for small portions near those points, be held as parallel. *Nn* is the *line of the nodes*, that is, the intersection of the plane of the Moon's orbit with the plane of the ecliptic, or the plane of the Earth's orbit round the Sun. Now *Ss* is the direction of light issuing from the Sun to illuminate the Moon: suppose the Moon to be a sphere; then a plane, passing through its centre and perpendicularly to *Ss*, would divide the Moon into two hemispheres, the convex surface of the one being bright, that of the other dark. But, except in certain positions, a spectator at *E* will see only part of the illumined hemisphere. Divide the Moon into two hemispheres by a plane passing through the Moon's centre, and drawn perpendicularly to a line joining that centre and the spectator, then the hemisphere, which is towards the spectator, is the one he views. *Mm* (in the figure of p. 33.) perpendicular to *Ss* is the projected boundary of light and darkness: *ab*, perpendicular to a line drawn from *E* to the centre of the Moon, is the projected boundary of vision: a spectator at *E*, therefore, views only that illumined part of the Moon's disk, of which *mb* and two lines, drawn from the Moon's centre to *m* and *b*, form the projected boundary. If the Moon, therefore, were at *c* between the Sun and Earth, *ab*, and *Mm* coinciding, no portion of her illumined disk would be visible: but, at *M''*, the whole illumined disk would be visible, (supposing the planes of the Earth's orbit and of the Moon's to be so inclined, that the Earth impede no light from falling on the Moon); at *M'*, (in which position it is intended that the lines *M'm'*, *a'b'* should be perpendicular to each other) the Moon will shine with half a face.

There are several technical denominations given to the Moon in the above positions. At *c*, the Moon is a *new Moon;* at *M''*, a *full Moon;* at *M'*, supposing half of her disk to be luminous, the Moon is said to be *dichotomized*. In the course of her circuit, which occupies a period of about 29 days, the Moon must, it is plain, exhibit all her *Phases:* the narrowest crescent near to *d:* an *half Moon* at *M'*, a full orb at *M''*: past that state, her orb becomes *deficient*, and the Moon *wanes*, till reach-

ing a line joining the Earth and the Sun she turns her dark side entirely to the spectator.

In the position *c* when the Moon is *new*, she passes the meridian at the same time the Sun does, or, in other words, she is on the meridian at noontide. In the position *M*, she must, since the Earth's rotation is from west to east, pass the meridian after the Sun, and it is her western limb which appears illuminated. At M'', the Moon, *at her full*, comes on the meridian at midnight: and past M'' and beginning to *wane*, she becomes *deficient* on her western side.

The Moon's orbit, as it has been already remarked, is inclined to the ecliptic. The line *Nn* is meant to represent the intersection of their two planes. Now the line *Nn*, technically denominated the line of the nodes, is found to be continually changing its position. If during these changes it should occupy the position $M''Ec$, whilst the Moon were either at *c* or at M'', then the Moon, Earth and Sun would be situated in the same right line, and give occasion to the phenomenon of an *eclipse*.

Suppose, in the first place, the Moon to be at *c*, and the Sun to be in the line *Ec* produced. Then a spectator at *E* would either see the Moon as a dark spot, or dark circle, concentric with the Sun's disk and within it, or, if we choose to conceive the Moon sufficiently large, the spectator would be unable to see the Sun by reason of the Moon's interference. The phenomenon, in the first of these predicaments, is called an *Annular Solar Eclipse*, in the latter, a *Total Solar Eclipse*.

In the second place, if the Moon be at M'', the Earth, being interposed between the Moon and Sun, must intercept some of the Sun's light in its passage to the Moon. It may (if we argue the matter independently of the actual magnitudes of the Sun and Earth) intercept the whole; and, under any consideration, it must cause the Moon to be less illuminated than it would be, did it not intervene. In fact, the Earth being a sphere or nearly so, its shadow will be conical and towards the Moon. We may, *hypothetically*, assign such dimensions to the Earth that the vertex of its shadow shall fall within the Earth and the Moon, in which case the Moon's disk would be only dimmed but not eclipsed; but, according to the actual dimensions of the Earth and its distance from the Moon, the shadow of the former always extends beyond the latter and causes it to be *eclipsed*

From the preceding account of the causes of eclipses, we may easily infer a material distinction between a lunar and a solar eclipse. When the former happens, the Moon is deprived of the Sun's light, and is darkened by the Earth's shadow; and every spectator on the Earth, that can see the Moon, sees her eclipsed. In the case of a solar eclipse, the Sun is not darkened but concealed, either entirely or partially, by the intervention of the Moon. The Sun may appear, on its rising, eclipsed to one inhabitant of the Earth, whilst, at the same time, to another inhabitant, in a different region, he may set with a full and bright orb. It will require the aid of computations to point out the exact circumstances of eclipses: that matter is reserved for a future Chapter. We will close the present by observing that the Earth's shadow, at the Moon, is sufficiently large to eclipse the whole of the latter body. The section of the Moon's shadow, on the contrary, at the Earth, is a round spot, of no great dimensions, that rapidly passes over the parts of the Earth's surface which it successively eclipses.

We have, in the present Chapter, supposed the Earth to be either spherical, or nearly so, and to cast a conical shadow. In the next Chapter we will briefly examine the grounds on which such supposition is built.

CHAP. V.

On the Earth; its Figure and Dimensions.

ONE of the proofs of the spherical form of the Earth is drawn from the phenomena of the preceding Chapter. In all lunar eclipses, the boundary of the Earth's shadow on the Moon's disk is apparently circular: such as ought to be the section of a conical shadow of a sphere. A considerable *defect of sphericity* might, however, exist in the Earth's figure without its being detected by this phenomenon.

There are, besides, other circumstances that render probable, and with like nature and degree of evidence, the globular form of the Earth. A ship, viewed as it approaches us, first comes in sight by shewing us the tops of her masts: next, more and more of the masts are seen, and, lastly, the hull. And, this phenomenon is the same, whatever be the *quarter*, be it the east, west, north, or south, that the ship approaches from.

Again, on a rock or mountain surrounded by the sea, such as is the Peak of Teneriffe, the sea appears, as it were, depressed, and equally on all sides of the spectator. On the mountain just alluded to, the angular distance between the zenith and any point of the horizon is nearly 92 degress. The Sun, therefore, must there rise sooner and set later (by about 12^m in the case before us) than to an observer on the plain: and, which is the same phenomenon or one immediately following from it, the summit of the mountain will be illuminated 12 minutes before Sun-rise and 12 minutes after Sun-set. The same phenomenon, modified solely with regard to time, and consistently with the hypothesis of the sea's spherical surface, is always found to take place in mountains of less or greater height.

The preceding circumstances shew that the Earth is round, and that it is neither flat like a plane, nor concave like a bowl: but they will not serve, not being of a sufficiently precise nature, to found thereon a proof of the Earth's sphericity. That the

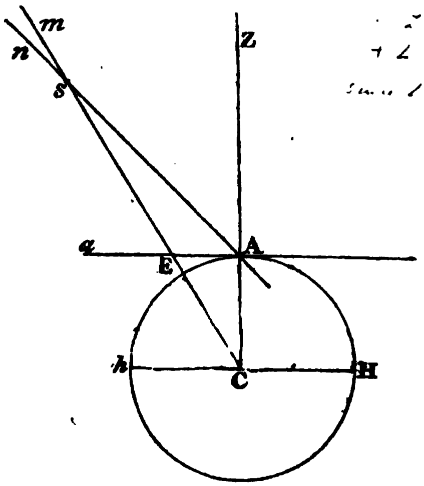
Earth cannot be a perfect sphere it is indeed easy to shew, although it is not easy to shew what is precisely its figure. The disposition of mankind to believe in the existence of simple and regular bodies first suggested a sphere, and then a spheroid, as the Earth's figure. And the labours of mathematicians have been directed, these last hundred years, to ascertain the truth of the latter suggestion. It is a matter, not unworthy of notice, that the Moon which, by one of the circumstances of her eclipses, (see p. 37, l. 4.) proves the *roundness* of the Earth, in another way (by one of her *inequalities*) proves its *non-sphericity* and the degree thereof.

We have not yet mentioned an argument, an analogical one, indeed, and not a very strong one, by which it is inferred that the Earth, one of the planets, is round, because Venus, Jupiter, &c. appear to be so. If we argue *similarly* with respect to the nature of the Earth's deviation from a spherical form, we ought to infer that the Earth resembles an oblate spheroid bulging out at its equator and flattened at its poles, because Jupiter is so formed. Indeed, if the Earth be not a rigid mass, such ought to be its figure. It is easy to see, on mechanical principles, that a fluid globe revolving like the Earth round an axis would become protuberant in its equatoreal parts.

What has preceded relates to the figure of the Earth; but its dimensions are an object of enquiry. If the Earth be a sphere, what is its radius? if a spheroid, what is (as it technically is called) its *Ellipticity*? These are questions about which Astronomers have been busied from the earliest times.

If we look to all the curious apparatus of methods, instrumental as well as computative, by which modern science has attempted to measure the Earth, there cannot well be a wider interval than that which exists between the rude Essay of Erastothenes made more than 2000 years ago, and what is now practised. The methods, however, rest on a common ground. At Syene, in the Thebais, the Sun on the meridian, at the time of the solstice, was vertical. It illuminated the bottoms of wells, and the highest buildings cast no shadow. On the same day the Sun's distance from the zenith of Alexandria was observed to be $7^0\ 12'$. Let C be the centre of the Earth, s the Sun vertical to

E (Syene), and let the angle sAZ, the angular distance at A



(Alexandria) of s from Z the zenith of A, be $7^\circ\ 12'$: then

$$\begin{aligned} \text{we have the } \angle ECA &= \angle ZAs - \angle CsA \\ &= \angle ZAs = 7^\circ\ 12', \end{aligned}$$

if we neglect, by reason of its smallness, the $\angle CsA$, the angle which the Earth's radius, in this case, subtends at the Sun.

Hence the Earth's circumference equals $\frac{360^\circ}{7^\circ\ 12'} \times EA$, and is known in linear dimensions when EA is so expressed.

If the distance of Syene and Alexandria be assumed equal to 5000 *Stadia* * and those places be supposed to lie (which they do not) in the same meridian, then

$$\begin{aligned} \text{the Earth's circumference} &= \frac{360^\circ}{7^\circ\ 12'} \times 5000^s = 50 \times 5000^s \\ &= 250000 \text{ stadia.} \end{aligned}$$

and the number of stadia contained in 1 degree would be

* According to Lalande, the Egyptian Stade = 114.13 toises, and a French toise = $6^{ft}\ 4^{in}.34$.

$694^s.444$, &c. $\left(=\frac{25000}{36}\right)$.

It is not necessary to stop here to shew the various sources of inaccuracy, in the above method. Let us attend to the modern way of proceeding. If we advance towards the north, the pole star approaches our zenith, or, if we proceed along the same meridian, the star which we at first observed in our zenith, recedes from it. Suppose between two stations of our progress that the pole star has become 1^0 nearer to the zenith, or (which is the same thing) that the star, which was vertical at the first station, is 1^0 distant from the zenith of the second station; then, if the actual distance between the two stations should be $69\frac{1}{2}$ miles, the Earth's circumference, which contains 360 degrees or 360 such *differences of latitude* as are equal to 1^0, would equal $\frac{360^0}{1^\circ} \times 69\frac{1}{2}$ and would be about 25020 miles: and its diameter would be about 7960 miles.

This method, it is plain, is founded on the same principle as that of the Astronomer of Alexandria; and, if it be pursued, it must needs furnish a proof of the Earth's *ellipticity*, or rather, of the defect of its figure from perfect *sphericity*. For, were the Earth a perfect sphere, the same linear distance ($69\frac{1}{2}$ miles for instance) ought always to be found between any two places on the same meridian and differing in their latitude by 1°. This, however, is not the case. In latitude 66^0 the linear distance between two places, under the above predicaments, is found to be 122457 yards. But, near the equator, such distance is found to be 121027 yards. The former distance being $69\frac{1}{2}$ miles $+137$ yards, the latter $69\frac{1}{2}$ miles -1293 yards. And it is established as a fact, by means of observations and measurements, that degrees (by which we mean their linear values) increase as we move from the equator to the pole.

If the Earth be supposed to be a spheroid, its measurement is to be conducted, as in the hypothesis of its being a sphere, by finding the difference of latitudes between two places, and by measuring and computing the linear distance between them. The axes of the spheroid cannot, it is plain, be determined by so simple a process as that which gives the radius of the sphere.

It is a question of pure mathematics to assign, from two degrees, one measured at the equator, the other near the pole (or any two other places), the eccentricity of that ellipse, which, by revolving round its minor axis, shall generate the spheroid to which it is believed the Earth is like. If all meridians were similar, and all measurements equally to be relied on, the same eccentricity ought to result, wherever the two degrees, the data of the problem, should have been measured. But the case is otherwise. One mathematician by comparing a degree measured in Lapland, with a degree measured in France, assigns $\frac{1}{307.405}$ for the Earth's *oblateness;* $\frac{1}{320}$ results from Col. Lambton's measurements in India: who compared (for so may the problem be mathematically solved) a degree of the meridian with a degree perpendicular to it. Lalande thinks $\frac{1}{300}$, Delambre $\frac{1}{309}$ to be its true value. In fact the question, whether we look to its theoretical or to its practical part, is a very difficult one, and likely, for many years, to remain doubtful, and to be the subject of discussion.

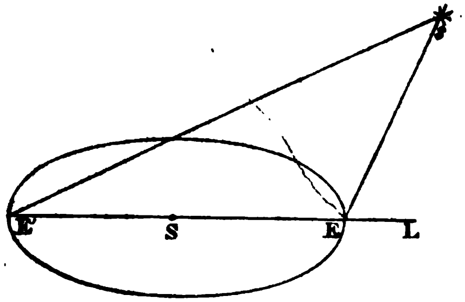
There is another method of determining the Earth's *oblateness,* founded on the different times of vibration of the same pendulum in different latitudes, or rather, on those differences of vibration which depend solely on an augmented or diminished gravity. The variation of gravity, or of the weight of a body, arises from two causes: the *non-spherical* form of the Earth and its rotation. From the first cause, the attraction is not, as in the case of an attracting sphere, the same as if all the matter of the spheroid were collected into the centre, and the resulting force directed to that centre. Two plumb-lines (and the directions of gravity are no other than the directions of such lines) containing, at the pole, an angle equal to 1^{0}, will meet in a point of the polar diameter beyond the centre of the spheroid. At the equator two such lines, so conditioned, would meet in a point of the equatoreal diameter *short* of the centre. In other situations the point of concourse will not be in a diameter passing through one of the extremities of the arc.

The second cause, the Earth's rotation, gives rise to a *centrifugal* force, a resolved part of which, acts in the direction of gravity and diminishes it. This centrifugal force is nothing at the poles and greatest at the equator, and that resolved part of it, which counteracts gravity, varies as the square of the cosine of latitude.

This enquiry, like the former one, is not easy, and, whatever be the mathematical skill bestowed upon it, must always terminate in doubtful results. For it rests on two hypotheses very difficult to be verified, 1st, the spheroidal form of the Earth, and 2dly, an assumed regularity and law in the disposition of its materials.

If we refer to p. 4, we shall find that the rational and sensible horizons are parallel to each other, and distant from each other by an interval equal the Earth's radius. Now that radius, as we have just seen, is about 4000 miles. It is, however, a distance, compared with that of a fixed star from the Earth, of no relative value: from which it follows that, in what regards the fixed stars, we may suppose the two horizons coincident: or, which amounts to the same thing, any calculation, made with respect to a fixed star by a spectator on the surface of the Earth, is precisely the same as if the spectator had been placed in the Earth's centre, to which point, on other occasions, that is, when the Moon or a planet is concerned, it is usual to refer or reduce Astronomical computations.

In order to prove what has been just asserted, let S represent the Sun, s a fixed star, and E, E' two positions of the Earth in



opposite points of her orbit. At these two positions the angles sEL, $sE'L$ can be determined by observation and calculation,

and, on comparing them, they are found to be equal: but $\angle sEL = \angle sE'L + \angle E'sE$, consequently, the angle EsE' has no value, or, the distance sE is so immense that the diameter of the Earth's orbit subtends no angle at s. There is no assignable proportion, therefore, between sE and EE', and, *a fortiori*, none between the Earth's radius and sE: since EE' is to the Earth's radius as 45968 to 1 *.

We have in this, as in each preliminary Chapter, treated its subject in a popular manner. The explanation has been general, and consequently vague, and indeed it is scarcely worth any thing if it were not preparatory to discussions of greater precision. We have spoken (see pp. 38, &c.) of the antient measurement of the Earth as of a rude method: but that which is afterwards described as the modern method may, notwithstanding any thing contained in that description, be equally so. In fact, the superiority of one method over another, cannot be shewn except by entering into their respective details. Those of the first may be comprised, as they have been, in a few lines: the details of the latter are sufficient to fill a large volume.

We have spoken of the zenith distance of the Sun at Alexandria, in the time of the solstice, as being $7^0\ 12'$, and of two places differing from each other in latitude by 1^0; and a student, in the outset of his Astronomical career, may imagine that nothing is easier than to form a notion of these angular distances. It is not likely, indeed, that he should anticipate (for he can only know them till after trial) the difficulties that await him. The angular distance of a star on the meridian from the zenith is the angle contained between a straight line drawn from the star to the spectator, and a line vertical to the spectator (the direction, in fact, of the plumb-line.) Now the first point of enquiry (which Erastothenes did not enter into) is, whether the star is really in the direction of the former line, or whether the direction of the ray of light when it enters the eye coincides with that of the former line. If it does not, then is the angle we see and measure, not

* If s were near the pole of the ecliptic, and $Es = 200000\,ES$, the angle EsE' would be about $2''$: but since no such angle can be detected, or at the utmost, an angle not exceeding $2''$, the ratio of Es to the Earth's radius must be at the least, that of 4569800000 to 1.

the angle we are in search of. We may be able to correct the former angle, and thence find the latter, but then there comes a second point of enquiry, whether or not the correction known for one case will suit all others; whether, for instance, the same quantity of correction which reduced the observed zenith distance (7° 12′) of the Sun at Alexandria, would truly reduce an observed distance at another place, at Rhodes for instance, where, at the solstice, the Sun's zenith distance would be about 13 degrees. If we would answer these questions we must enter into an investigation, which is no other than that of *the Laws of Refraction*.

But the enquiry would not terminate with the settling of those laws. Suppose we knew how much the light of a star would be made to deviate, by reason of the atmosphere, from a line joining the star and the spectator, would the deviation of the same star, to the spectator at the same place, be the same at whatever hour the star passed the meridian? The student, it is probable, would here also feel no hesitation in answering that the star's apparent angular distance must be independent of the time of its transit over the meridian, and that, if refraction were away, a star would always pass the meridian of Greenwich at the same distance from the zenith of Greenwich (such distance being determined by an instrument) whether the hour of transit were 9 in the morning or noon.

The fact, however, is otherwise, and, as it will be shewn hereafter, there is, besides refraction, a cause of inequality which makes the instrumental zenith distance different from, if we may so call it, the true zenith distance: which cause of inequality is connected with the time of the star's transit over the meridian.

But the process of correction would not cease here; there are, at the least, six causes of *inequality*, each of which will render the observed angle, whether it be an angle between two stars, or, between the zenith of the observer and a star on the meridian, unequal to the true one. So hard to be understood then, notwithstanding its apparent simplicity, is the expression; of the *difference of the latitude of two places being* 1°. Erastothenes if he had possessed the most perfect of modern instruments, had he possessed them without modern science, could not have ascertained the Earth's dimensions.

But although this be the case: although it is essential to know

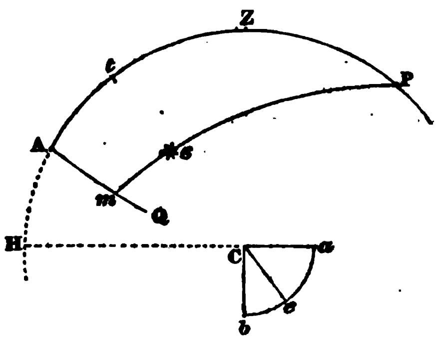
the quantities and the laws of those *corrections*, by which we are to derive the latitude of a place, or any other angle, which may be the object of research, from the observed or instrumental angle; still, it is plain, this latter angle is of primary importance. If we are unable to determine that exactly, the corrections provided by Astronomical Science may be of little or no use. Their sum may be less than the error of observation, which error, in such a case, would vitiate all subsequent processes founded on the observation.

It will, therefore, be following something like a natural order, to describe the instruments by which angular distances are measured, previously to the investigation of methods for *correcting* such distances. And, in pursuit of this plan, we shall not digress into a description of antient instruments nor (however instructive in itself such an enquiry may be) into the history of their successive improvements. We shall be content to describe the instruments which are essentially necessary to determine the places of the heavenly bodies; those instruments which are called, for distinction's sake, the *Capital* Instruments of an Observatory, which, indeed, are few in number, and simple in their construction, each being appropriated to one class only of observations. The tendency (if we may so describe it) of improvement in Astronomical Instruments has been towards simplicity in their construction. In former times Astronomers endeavoured, in their instruments, to imitate the celestial sphere: which were formed in *cœli effigiem;* hence came their Astrolabes and Armillary Spheres. According to modern practice, all important observations are made on stars on the meridian. It is there that Astronomers, with fixed instruments, wait for a star instead of attending on its course from east to west.

CHAP. V.

On Astronomical Instruments.

The position of a point in a plane may be determined, by means of two rectangular co-ordinates (as they are called), that is, of two lines perpendicular to each other and measured from the same point. In like manner, the position of a star on the celestial sphere may be determined by portions of two great circles, perpendicular to each other, and measured from the same point. Thus, let P be the pole, AmQ a portion of the equator, s a star, and Psm a circle of declination: then, if A



should be a known point or known star in the equator, the position of s on the sphere will be determined from Am and At ($= ms$): since we have only to set off Am, on AmQ the equator, to draw the quadrant mP and to set off, on mP, ms equal to At. Now, the *right ascension* of a star is its distance measured on the equator from a fixed point in the equator. If that point be A, Am will be the *right ascension* of the star s, ms its *declination*, Ps its polar distance, and Zt its zenith distance, when s is on the meridian, the position of which is represented by PZA.

We must consider what are the means of measuring At and Am when the star s is on the meridian.

With regard to the first point; we have only (by which term, however, we do not mean to signify the great facility of the

operation) to divide a quadrantal arch such as aeb into a number of equal parts, to place it in the plane of the meridian, and to direct a telescope in the direction eC to the star t: then eb will express, by a certain number of the above-mentioned equal parts, the zenith distance of the star s on the meridian, and ea will express the altitude. Besides the conditions mentioned, it is clear that Cb must be vertical, which it will become by being made coincident with, or parallel to, the direction of the plumb-line.

With regard to the second point: there are no obvious means, and certainly no simple ones, of instrumentally measuring the angular distance between A (even supposing it to be a star) and m the point where the secondary passes through s. Other means, than those of instruments giving angular distances, must be resorted to: and Astronomers have called in *time* to express, *intermediately*, the right ascension of a star: which plan may be thus explained.

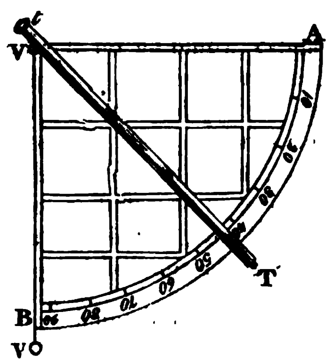
Suppose (for the sake of simplicity) A to be a star, and the point m to be carried, by the rotation of the sphere, in the direction mA: then m and S would be on the meridian at A and t at the same instant, and the arc Am, the measure of the angle sPA, would bear to the whole circle, or to 360 degrees, that proportion which the time elapsed, between the transits of A and s over the meridian, bears to the whole time of the sphere's rotation; and contrariwise, an observed or noted time between the transits would, in terms of time, be the right ascension Am, which, if 24 hours be assumed as the time of the sphere's rotation (or of the Earth's diurnal rotation) would equal $\frac{h}{24} \times 360^{\circ}$.

To enable us, then, to find the right ascension of stars, there are, according to the above plan, two instruments necessary: a telescope in the plane of the meridian to observe A and the star s when on the meridian, and a clock to note the respective times of their being there. The instant of a star's passage cross the meridian being denominated its *transit*, the telescope used for observing the star, at that instant, is denominated the *Transit Instrument*. The three capital instruments then of an Observatory, are the *Astronomical Quadrant*, the *Astronomical Clock*, and the *Transit Instrument*.

In point of theory, or if we regard solely the mere purpose of explanation, the two former instruments are the only ones essentially necessary, because no reason, not suggested by actual experiment, can be assigned why the office of the third instrument should not be performed by the quadrant, which is supposed to be placed in the plane of the meridian, and to be furnished with a telescope capable of being pointed to any part of the meridian. The special use, or the practical convenience of the transit instrument, depends on reasons altogether practical and not yet explained.

We will now proceed to a more particular description of the *Astronomical Quadrant*, which may be considered as representing a class of instruments, known by the names of *Declination Circles*, and of *Mural Circles*, and designed for the measuring of zenith distances and polar distances.

The annexed figure is meant to represent a mural quadrant, or one fixed in the plane of the meridian. *TV* is a telescope



moveable about a centre at *V*: and *Vv* is a plumb-line, which is, in general, a fine thread or wire with a weight attached to it, and, for the sake of steadiness, plunged in water.

The first point to be considered is the division of the quadrantal arc *AB*.

The most usual *graduation* of the arc consists of 90 degrees: but many quadrants (the two 8 feet mural circles of Greenwich, for instance,) have, besides this usual graduation, a second one,

consisting of 96 equal parts*. An observation is to be *read off* (as the phrase is) on each scale, and then, by means of a computed Table, the divisions of the ninety-six scale are to be *reduced* to those of the ninety.

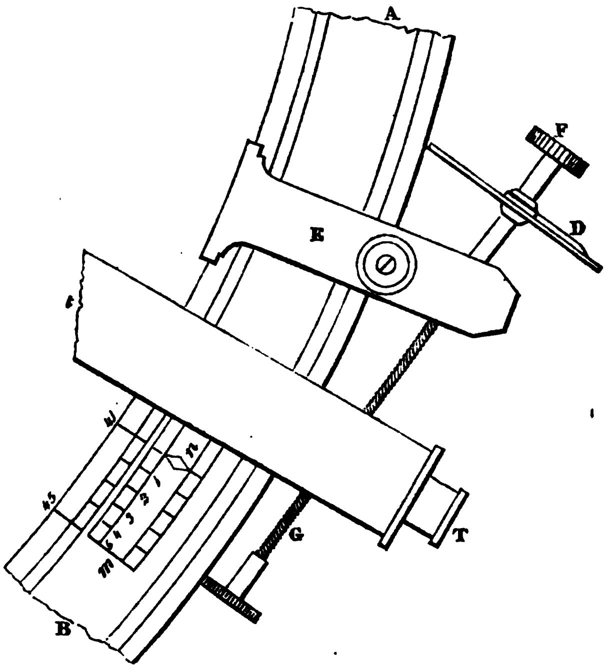
But the graduation is not limited to 90 or 96 parts or degrees. Each degree is itself divided into a number of equal parts, each part containing a certain number of minutes: the number of minutes being the less (we are speaking practically) the greater the instrument. In quadrants and circles of nine inch radius, the smallest division on the *limb* of the instrument contains generally 30 or 20 minutes. Quadrant of 18 inches are divided to 15 minutes. The 8 feet mural quadrants of Greenwich, and the 6 feet mural circle, are divided into equal parts of 5 minutes each.

There are, however, certain little and subsidiary instruments, called *Verniers*, attached to that end of the telescope which moves along the arc of the quadrant, that enable us to *read off* the observations to a greater nicety, and that (if we may so express ourselves) stand in the stead of a minuter graduation of the limb of the instrument. We will now explain the principle and use of the Vernier.

Let AB represent part of the limb of a quadrant (of that, for instance, which was represented in p. 48.), Tt part of the telescope which moves along the limb, and nm a thin plate of

* The graduation of ninety-six degrees was adopted on principles of mechanical convenience; and for the purpose of lessening the great difficulty which attends the graduation of instruments. A chord of 60 degrees, in the common division of the circle, being equal to the radius, a chord of 64 degrees, will be equal to radius, when the quadrant is divided into 96 equal parts, or degrees. Hence, by means of a line equal to the radius of the quadrant, two points can be determined on its arc, containing 64 out of 96 equal parts; and, by the continual bisection of $64 (= 2.2.2.2.2.2)$ a division equal to one of those equal parts is obtained. It is very easy to conceive a circle divided into 360 equal parts or to describe it as so divided; but the practical effecting of the graduation requires a great deal more than mere dexterity of hand, as artists will testify, or, as any one who will make the trial, will soon experience.

metal (the Vernier) attached to the telescope at *n*, and together with the telescope, moveable along the limb of the quadrant.



In the present scheme the vernier is divided into five equal parts, the sum of which is equal to the sum of four equal divisions of the quadrant: and this equality is represented in the figure: in which the *lozenge*, that mark on the vernier to which *b* would correspond, coincides * with the division or mark on the quadrant marked 41, whilst the mark 5 of the vernier coincides with the mark 45 of the instrument. In *reading off* we must first look to

* Instead of *coincides with*, we ought, perhaps to say, is *opposite to*, or in the same *right line with*, the mark 41 of the instrument. The engraver having separated the boundary of the vernier from the circular line on which the division-marks of the limb of the instrument *abut* prevents a *coincidence* from taking place. We may farther note, that one boundary of the *vernier* is the fourth concentrical circular line, reckoned from the left hand: the other is the seventh, reckoned in the same way.

the *lozenge* for the position that is intended by the instrument maker to mark the altitude of the observed star or other object. Thus, as the figure is drawn, if the telescope were properly directed to a star, the altitude of such star would be 41° : and in such a simple case the vernier is of no use. But suppose the telescope were directed to a star a little higher than the former, then the lozenge would be moved from the division 41 towards 45, and let us suppose it just so far moved that the second mark (1) of the vernier coincides with the division of the quadrant next succeeding the 41st (the 42d) *. In this case it is clear the lozenge (to which we are to look in noting the altitude) has been moved through a space equal to the *difference* between one division of the instrument and one of the vernier. The altitude of the star then is 41° + this difference: which difference must now be estimated.

In the figure to which we are at present referring, the divisions of the instrument are intended to represent divisions of one degree each, and, since four of these divisions, or 4°, are equal to five divisions of the vernier, the difference between a division of the quadrant and the vernier is

$$1^\circ - \frac{4}{5}1^\circ = \frac{1^\circ}{5} = 12',$$

so that the altitude of the star is to be *read off* equal to

$$41^\circ\ 12',$$

and this is the most simple illustration of the use and *property* (for such it is) of the vernier.

If a star still higher be supposed to be observed, and the telescope and its attached vernier be so moved, that the mark 2 of the vernier coincides with the 43d of the instrument, then the index or lozenge has been moved from its original place, opposite to 41, through a space equal to *twice the difference* of the divisions of the quadrant and vernier, and consequently, the altitude must now be

$$41^\circ + 2 \times 12', \text{ or } 41^\circ\ 24'.$$

* To avoid confusion, and to lessen the difficulties of the engraver, the divisions which lie between 41 and 45, namely, 42, 43, 44, are not figured.

If the mark 3 of the vernier coincided with the 44^{th} of the instrument the altitude would be

$$41° + 3 \times 12', \text{ or } 41° \; 36'.$$

If the marks 4 and 45 should coincide, the vernier and the lozenge must have moved through a space equal to five times the difference of a division of the instrument and of the vernier, or through a space equal to one division of the vernier; and in such case the altitude would be

$$41° + 4 \times 12', \text{ or } 41° + \frac{4}{5} 1°,$$

each of which equals 41° 48′.

If the mark 5 should be found to coincide with the mark next to the 45^{th} of the quadrant (which mark would be 46); then, it is plain, the vernier and every mark on it and, of course, the lozenge, must have been moved through a space equal to one division of the instrument, or through 1°; and the altitude of the star, if such were the object observed, would be 42°.

In this situation, the vernier would have returned to a position precisely similar to its original one (that in which the lozenge coincided with 41), and any subsequent translations or movements of the vernier, producing *exact* coincidences (or coincidences seemingly such) between any two marks or lines of the vernier and instrument, will be precisely similar to those that have been just explained.

But it is obvious that the motions or translations of the telescope and attached vernier may be less, in degree, than those which have hitherto been spoken of. The spaces through which the telescope moves, may be less than the difference between a division of the instrument and a division of the vernier, in which case, there would be no exact coincidence between any two marks or lines of the respective divisions. If, for instance, the telescope should be moved from the position in which *o* of the vernier coincided with 41 of the instrument, and through a space *less* than the difference of a division of the instrument and the vernier; the mark 1 would not reach 42 of the instrument, and the altitude to be noted would be something between 41° and 41° 12′, and which the observer, should there be no other mechanism belonging to the vernier than what we have described, must estimate by guess and according to the best of his judgment.

In the scheme illustrating the use of the vernier, we have chosen to consider each division of the instrument to be equal to 1°, in which case the vernier will not note smaller angles than twelve minutes: but if each division, instead of 1°, were 1′, the accuracy of the vernier would then extend to twelve seconds: and, generally, when five divisions of the vernier are equal four of the quadrant, the difference between a division of the one and the other will always be equal to $\frac{L}{5}$,

$$\text{since } L - V = L - \frac{4L}{5},$$

L being a division of the quadrant, and V of the vernier.

It is clear then, the smaller the divisions of the instrument are, the more minutely (with regard to degrees and parts of degrees) will an observed angle be noted by means of the vernier. But supposing, in an instrument of a given size, the magnitude of each division to be settled, (and there are practical and mechanical reasons that prevent the instrument from being subdivided beyond a certain point) a question will then arise concerning the *length* of the vernier, or, as the case is stated, concerning the number of its divisions. Instead of five of its divisions being equal to four of the instrument, will it not be better to make ten of its divisions equal to nine of the instrument? or twenty equal to nineteen, or sixty equal to fifty-nine? In fine, if n divisions of the vernier are to equal $n-1$ of the instrument, what is the value which it is most commodious to assign to n?

Let, as before, L denote the value of a division of the instrument, and V that of one of the vernier, then since

$$(n-1)L = nV,$$

$$L - V = L - \frac{n-1}{n}L = \frac{L}{n};$$

consequently, L being given, $L - V$ is less, the greater n is. But n cannot exceed a certain limit, for the magnitude of each division being (see p. 49.) supposed to be assigned, and each division being an aliquot part of a circle, the arc of the quadrant can only contain a certain number of such divisions; for

instance, if each division contains fifteen minutes, the quadrant contains $4 \times 90°$, or 360 of such divisions, and, in such a case, the limiting value of n is 360, and the difference between a division of the quadrant and one of the vernier, with such extreme value of n, would equal

$$\frac{15'}{360} = \frac{15 \times 60''}{360} = \frac{15''}{6} = \frac{5''}{2} = 2''.5.$$

But it is plain that a vernier extending along the whole limb of the instrument would be very incommodious (to say the least): and a like objection would lie against verniers either half or a quarter of the arc of a quadrant: so that there are (in this as in every other case relative to the construction of instruments) certain practical considerations that limit, in a quadrant of a given radius and given number of divisions, the length of the vernier.

It is proper then now to state what are usually the proportions between the length of the vernier and the radius of the instruments.

Quadrants and circular instruments of 9 inches radius, are frequently divided into equal parts, each consisting of 20 minutes, and 59 of such equal parts are made equal to 60 divisions of their verniers. In this case

$$L - V = \frac{L}{60} = \frac{20'}{60} = 20'',$$

so that, with such instruments, you can *read off*, by the aid of their verniers, to an accuracy of 20 seconds. In this case, the vernier must occupy on the limb of the instrument a space, at the least, equal to $19^0\ 40'$.

There are quadrants, of 18 inches radius, divided to every 15 minutes, and in which 14 of such divisions are equal to 15 of the vernier. In these instruments then

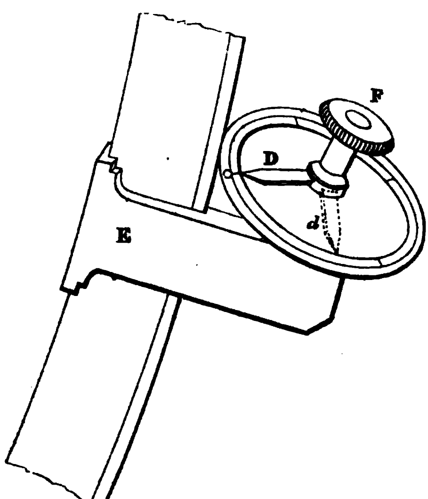
$$L - V = \frac{L}{15} = \frac{15'}{15} = 1',$$

and the space occupied by the vernier, is, at the least, equal to $3^0\ 30'$.

It would appear then that, in this case, we are not able to *read off* so accurately as before, although the instrument is twice the

size of the former. The fact is, that that happens here which we before alluded to in p. 52. The divisions of the instrument and vernier differ so much, that, in taking an altitude, the telescope, will probably occupy a position, in which there is no exact coincidence between a dividing mark of the vernier and one of the quadrant. But, instead of *guessing* what the *defect* between the two nearest coincidences is, the observer is assisted by a piece of mechanism attached to the instrument, which enables him to compute that defect. This we will now briefly explain.

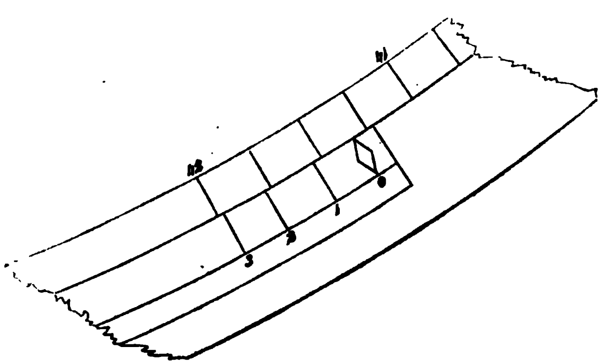
The part *E* can be fastened to the limb of the quadrant by means of a screw. *FG* a screw, (Fig. p. 50.) with a milled head at *F*, works in a collar fixed in the under part of *E*, and in a female screw fixed in the under part of the telescope *Tt*. When the part *E*, then, is fixed or *clamped*, and the screw is turned round by its milled head at *F*, it must communicate a direct motion to the female screw (and, consequently, to the telescope and vernier) in the direction of *FG*. Attached to the male screw, or to the small cylinder on which it is formed, is an index *D* moveable together with the screw and on a thin graduated immoveable plate, the profile only of which is shewn in the Figure



of p. 50. It is more fully exhibited in the above figure, in which *F, D, E,* represent the same parts as in the former figure.

Suppose now the screw to be of that fineness that, whilst it is turned round, or whilst the screw-head and the index D make one complete revolution, the vernier is so far advanced on the limb of the quadrant, that the mark 1 of the vernier is brought into coincidence with 42 of the limb: then, in our scheme of illustration, one revolution of the screw is equal to $12'$. If the circumference of the thin plate then (see Fig. p. 55.) be divided into 60 equal parts, one of such equal parts must be equal to $12''$: and if, in order to make a coincidence between the lozenge of the vernier and any division of the limb, it were necessary so to turn the screw that the index D should be moved from D to d, and 15 graduations should be contained between Dd, then the space moved through by the vernier on the limb would be equal to $15 \times 12''$, or $3'$.

Similar results will take place, if the instrument and vernier be differently divided: thus, if each degree of the quadrant be divided into 4 equal parts, and 14 of such parts be equal to 15 of the vernier, the difference between the respective divisions being $1'$, one graduation of the brass plate would equal $3''$, supposing, now, that three revolutions of the screw move the vernier through a space equal $1'$. In the annexed Figure, in which a degree is



divided into four equal parts, the lozenge or index of the vernier occupies a position between 41° $15'$ and 41° $30'$. The dividing mark 2 of the vernier *very nearly* coincides with the mark of the quadrant which denotes 41° $45'$. If it exactly coincided, then

the lozenge, or index, being advanced beyond the mark next to 41, (the mark denoting 41° 15′) by a space exactly double the difference of a division of the instrument and one of the vernier, the altitude or angular distance denoted by the instrument would be

$$41^\circ\ 15' + 2 \times 1',\ \text{or}\ 41^\circ\ 17',$$

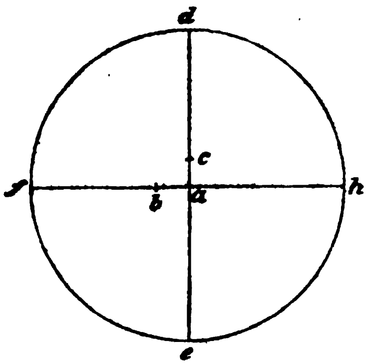
but the angular distance is, obviously, of somewhat greater value. Suppose, in order to carry the vernier so far back as to make its division 2 coincide exactly with that division of the instrument, which is just behind it (the division 41° 45′), that we must so much turn the screw F (see Figure of p. 55,) that the index D should be advanced from o to d, or through a quarter of the circumference, then this quarter, which is $15 \times 3''$ or $45''$, is the value of the space through which the vernier has been moved, or of the distance between 2 of the vernier and 41° 45′ of the instrument: it measures, therefore, the excess of the altitude, which the instrument ought to denote, above 41° 17′; in other words, the altitude is now to be estimated equal to 41° 17′ 45″.

By this contrivance, then, without any inconvenient minuteness of division of the limb of the instrument, or of inconvenient length of vernier, we are enabled to read off angles to as great an exactness as that of 3 seconds. In the Greenwich mural quadrants, by a similar contrivance, the angles may be read off to one second. That part of the vernier which we have been just describing, and which enables us to measure minute differences, is called a *Micrometer*. The two Greenwich mural quadrants, of 8 feet radii, are, as we have said, furnished with such. But the mural circle is furnished with a micrometer of a different construction.

Having now examined the methods of *reading off* the altitude to which the index of the vernier, in a fixed position, points, we will next consider by what means the vernier is brought to such fixed position. The vernier is attached to the telescope, and the telescope is moved, till the star (the altitude of which we are seeking) is seen through it. But, as the field of view is not a mere point, there is not one certain position of the telescope in which only we can see the star. If the star should appear to be *nearly* in the middle of the view, we may move the telescope, a little upwards and a little downwards, and still see the star. It is evident then, since the altitude we are seeking for is a certain and

determinate quantity, that we require some rule for stopping and fixing the telescope. We cannot say that the telescope is in its just position when the star appears in the centre of the field of view, because the eye cannot judge of that circumstance with sufficient precision. We must therefore place some fixed point in the field of view, and in the focus of the eye-glass, which fixed point is to be the centre of the field of view, or to be considered as such, and the telescope is to be judged to be in its proper position, when the fixed point and the star appear to be coincident, or when, as the technical phrase is, the point *bisects* the star.

The intersection of two fixed wires placed in the focus of the object-glass of the telescope, will furnish us with such a fixed point; and one wire may be vertical, the other horizontal. *de* may

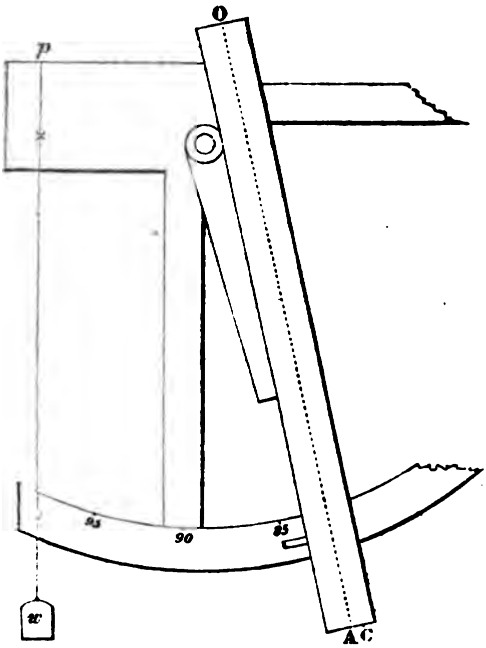


represent the former, *fh* the latter, and then *a* would be the intersection, or their *centre*. These wires, as we have said, are fixed in the principal focus of the object-glass, and then must be viewed with the eye-glass: or, if they are attached to the tube containing the eye-glasses, that tube must be moved so that the wires shall be in the above principal focus: in either of these cases the eye sees distinctly, at the same time, the wires and the image of a star: and the observation is to be held as made when the star is upon, or is bisected by, the point *a*.

We gain, at the least, this advantage by the above method, that all stars are observed according to it, and that any error attached to it must equally affect all stars: in other words, that the error must be a common one, and consequently all observations may be immediately corrected should the quantity of that error be

once detected. We will now consider by what means that error may be detected and valued.

Let the subjoined Figure represent (in part) the Astronomical Quadrant, placed in the plane of the meridian, and with its gra-



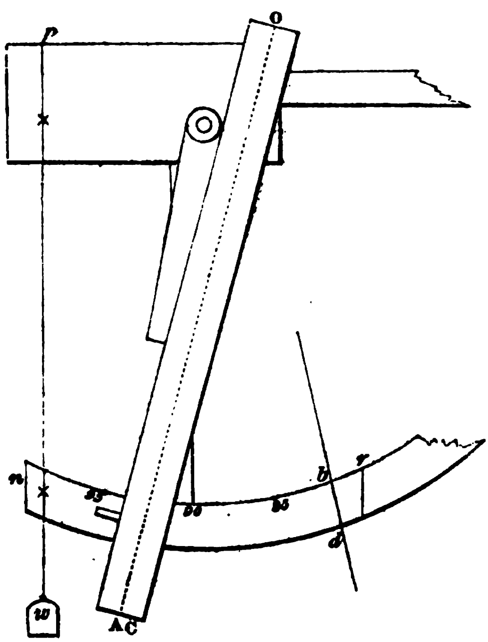
duated face opposite the east, and suppose the telescope to be directed to a star the altitude of which is 85°.

If *A* be the intersection, or centre of the cross wires (what answers to *a* in the Figure of p. 58,) and *OA* be the direction of a ray of light passing through *O* the object-glass and coming to its focus at *A*, then, the image of the star and the centre of the wires being coincident, the observation (see p. 58, l. 6, from bottom) is properly made, and the index of the vernier, being made to coincide with 85° of the quadrant, will properly denote the star's altitude, and also, (the instrument being supposed to be truly graduated) the vernier, in other positions of the telescope, directed to other stars, will justly note their altitudes.

Suppose now from some accident, or, purposely, the system of cross wires to be deranged, so that their centre, instead of

being at *A*, is moved, through a little space, to *C*, so that *A* is between *C* and *p W* the plumb-line, the line passing through *A*, *C*, being supposed to be in the plane of the meridian. In this new position of the cross-wires (the telescope retaining its position) the star is no longer *bisected* by their centre, but will be seen in the field of view, a little to the south of that centre, or towards the plumb-line. In order then to bring the star on the centre, that end of the telescope in which *A*, *C*, are, (the telescope being moveable about a pivot or centre of motion situated near its other end) must be pushed a little to the south and towards the plumb-line, $23''$ for instance, in which case the index of the vernier, moving with the telescope, will point to $85^{0}\ 0'\ 23''$. We have now then to enquire (putting aside the supposition of the star's altitude being exactly 85^{0}) why the altitude, in this case, is not justly indicated.

Suppose we are able to turn the quadrant half round, or that we possess some means or other of placing its graduated face which, in the Figure of p. 59, is opposite to the east, opposite to



the west, and let the above Figure represent the quadrant in this

latter position: in which, OA would be directed as db is, not, as before, to the south of the zenith, but to the north. In order then to bring the star into the field of view, the telescope must be moved past the graduation of 90°, to that of 95°. In this latter position, the image of the star and the point A would be coincident, but C being now the centre of the cross-wires, in order to bring the star upon C, the end of the telescope which contains the eye-glasses and cross-wires must be pushed towards the plumb-line (as the Figure is constructed) or from the division of 90°. It must be pushed also, since the distance AC is supposed to remain invariable, just as much as it was in the former position of the quadrant (the position of p. 59.) that is, through 23″. The index of the vernier now then will point to a graduation of

$$95° \ 0' \ 23'',$$

or, which is the same thing, will indicate a zenith distance equal to

$$5° \ 0' \ 23'',$$

whereas, the altitude in the first position of the quadrant being 85° 0′ 23″, the zenith distance will be

$$4° \ 59' \ 37''.$$

Half the sum of these two zenith distances is 5°, the true zenith distance, and half their difference (46″) is the error caused by the derangement of the cross-wires after they had been once adjusted.

This error or derangement has a technical denomination: the line between O and A, A being the centre of the cross-wires, or the line between O and C, C being the centre of the cross-wires, is called the *Line of Collimation*, and the error, of which we have treated, and shewn the method of detecting and valuing, is called the *Error of the Line of Collimation*, or, more briefly, the *Error of Collimation**.

* This error may be corrected by moving and adjusting the cross-wires, so that C (in the Figure we have used) may be replaced in A. But it is plain we may leave the system of cross-wires untouched, and so alter the index of the vernier, that it shall, the telescope being directed to the star, note its true altitude. On this account the error of collimation is frequently called the *Index Error*.

We have then, in all cases, in which we are able to turn the instrument half way round in azimuth, or through 180° of azimuth, this simple rule for finding the zenith distance of an observed star: add the zenith distance, or the mean of several zenith distances, taken with the face of the instrument to the east, to the zenith distance, or the mean of several zenith distances, taken with the face of the instrument to the west; half this sum is the star's zenith distance: and, half the difference of the above observed zenith distances is the error of the line of collimation.

The rule is the same, if, instead of the zenith distances of stars, we seek to determine their altitudes. We subjoin an instance or two, in which the instrument used, instead of a quadrant, is a circle.

		Altitudes.
6th Sept. Star *Rigel,*	position E.*	30° 21′ 36″.25
	position W.	30 20 22.05
	sum =	60 41 58.30
	true altitude =	30 20 59.15
	difference =	0 1 14.20
	error of collimation =	37.10
Again,	δ *Sagittarii* W. ...	8° 56′ 45″.8
	E. ...	8 58 7.1
	sum. ...	17 54 52.9
	true altitude. ...	8 57 26.45
	difference. ...	1 21.3
	error of collimation. ...	40.65.

If great accuracy be required, the above operations are repeated with several stars, and the *mean* of the whole taken for the error of collimation: thus,

* Position E, position W, denote respectively the graduated side of the circle turned towards the east and west. Rigel, Sirius, α, θ, γ, &c. Capellæ, are the names of certain known stars.

	Error of Collimation.
Rigel.	37″ . 05
Sirius.	40 . 05
δ Sagittarii	40 . 06
X.	42 . 0
α Capellæ.	39 . 45
θ.	37 . 12
γ.	37 . 35
δ	37 . 87
	8)310 . 95
Mean error of collimation	38 . 87

That process, then, of turning the quadrant, or the circle, half way round in azimuth, which finds the altitude and zenith distance, finds also the error of the line of collimation; but it is unimportant to know this latter, if, every time that an altitude is to be determined, the above-mentioned process be resorted to. We may, however, as it is plain, having once determined the quantity of the error of the line of collimation, employ it as a *correction* either additive or subtractive, to the zenith distances of stars determined from one position of the quadrant only, that is, when its face is constantly turned either towards the east, or towards the west.

Thus, suppose that by the mean of twenty observations made at Greenwich, the quadrant facing the east,

the north zenith distance of γ Draconis . . . . . $= 2'\ 21''.76$.

By the mean of 30 observations

the quadrant facing the west, the zenith distance. . $= 2'\ 15''.48$

$0'\ 6''.28$

∴ error of collimation. . . . . . $=$ $3''.14$.

This is the error deduced from one star, γ Draconis, which star is to the north of the zenith of the Greenwich Observatory. When, therefore, the face of the quadrant is to the west, the above correction ($3''.14$) must be added to the north zenith of stars, but subtracted from the distances of those stars which are observed to the south of the zenith*: for, since the instrument, its face

* When the quadrant faces the west, a few stars only, those which are near the zenith, can be observed to the *south* of the zenith (see pp. 64, 65.)

being towards the west, gives the north zenith distance of γ Draconis too little by $3''.14$ (since instead of being $2'\ 15''.48$ it ought to be $2'\ 18''.62$) it must also give the north zenith distances of all stars too small by the same quantity: and if a star were to the north of the zenith by an angular distance equal to $3''.14$, it would, by the instrument, seem to be on the zenith; consequently, a star on the zenith would by the index of the instrument appear to be $3''.14$ to the *south* of the zenith: and a star 1^0 to the south of the zenith would appear to be, by the instrument, $1^0\ 3''.14$ to the south. The contrary will happen if the observations are made with the face of the instrument to the east; for, then, the error of the line of collimation must be subtracted from all north zenith distances, and added to south zenith distances; for instance, if we had the following observations:

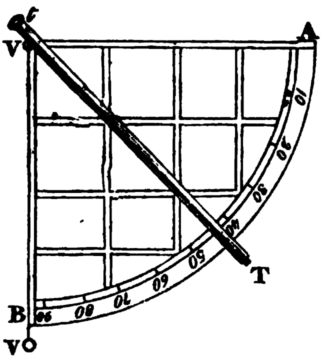
Zenith distance of α Andromedæ	23^0	$24'$	$56''.36$	S.
γ Pegasi....	37	19	32.46	S.
α Ceti......	48	6	55.56,	S.

then the zenith distances, corrected for the error of the line of collimation, (and for that only) would be respectively,

23^0	$24'$	$59''.5$
37	19	35.6,
48	6	58.7.

It appears then, by what has preceded, that, in all quadrants that can be turned round in azimuth, the altitudes and zenith distances of stars can correctly be found as far as the line of collimation is concerned. These, however, must generally be found by applying to their quantities, determined by the quadrant, the error of the line of collimation as a correction of such quantities. They cannot be found, except for stars situated near the zenith, by taking the half sum of zenith distances observed respectively, with the face of the quadrant towards the east and west. The reason is obvious from the inspection of the diagrams (see pp. 59, 60.) If AVB (see the following Figure,) should be in the plane of the meridian, and A should be to the south of VB, the zenith distances of those stars only that are to the north of the zenith

could be determined by such an instrument. If the quadrant were reversed, and the graduated rim now opposite the west were made

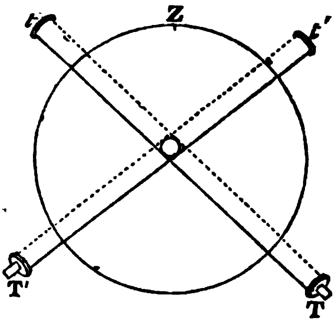


to face the east, the zenith distances of those stars only, that are to the south of the zenith, could be observed. In such a case, the reversion of the instrument would be useless, since, not being able to observe the same star in the two positions of the quadrant, we should be unable to deduce the error of the line of collimation. To remedy this inconvenience, or rather to enable us to avail ourselves of the azimuth motion of the instrument, the arc of the instrument is made to exceed a quadrant, and the graduation, as it is represented in Fig. of p. 59, is extended beyond 90 to 95° or 96°. By this contrivance, the zenith distance of the same star, which is not distant more than 5° or 6° from the zenith, may be observed in the two opposite positions of the instrument, and the error of the line of collimation thence deduced. The star γ Draconis, for instance, which, when it passes the meridian at London, is nearly vertical, would serve the above purpose in every part of England.

But in circular instruments, or declination circles, and endowed with an azimuth motion, any star, either near to, or distant from the zenith, will serve to determine the error of the line of collimation, and with such instruments the method given in pp. 61, 62, &c. may always be practised; that is, we may add the mean of zenith distances observed when the instrument faces the east, to the mean of zenith distances observed in the instrument's reversed position, and then (the error of the line of collimation

being, in fact, compensated for) half the sum will be the zenith distance required.

Thus, suppose the telescope Tt to be directed to a star in the south (so directed, as it must be always understood, that the image



of the star and the middle of the cross-wires are seen, through the eye-tube, in distinct coincidence) the face of the instrument being towards the east: then, if the instrument be turned through 180° of azimuth, so that the face, before opposite to the east, be now opposite to the west, $T't'$ will be the position of the telescope. In order, then, that it may be again directed to the star, and that its position may be parallel to its former one Tt, it must be turned through an angle equal to *twice* its zenith distance: and, consequently, half the difference of the number of degrees indicated by the vernier in its two positions (which difference is no other than the number of degrees intercepted between the two positions of the telescope and vernier) is the star's zenith distance.

It appears then, from what has preceded, that, in all quadrants and circles, used for taking altitudes and endowed with azimuth motions, the altitudes so taken can be freed from the *error of collimation*. But they are instruments of a limited size only * (we are speaking of the practical convenience of the thing) that admit of an azimuth motion; instruments, for instance, of two

* The radii of astronomical quadrants and circles that have an azimuth motion, and are portable, rarely exceed three feet: those of portable zenith sectors may be somewhat larger. The radius of the *stationary* circle of the Dublin Observatory, which has an azimuth motion, is four feet, and the radius of the quadrant at Blenheim, made by Ramsden, and with an azimuth motion, is six feet.

or three feet radii. It would be, almost, an impracticable operation to move, from day to day, such quadrants as the mural quadrants of Greenwich are, of 8 feet radii, and which are very ponderous. Such quadrants when once fixed must so remain, and, consequently, such quadrants are inadequate, from their own properties, to determine the errors of collimation of their telescopes. It is, however, essential to determine those errors. Some subsidiary instrument then must be called in for that purpose. Those circles and quadrants that possess an azimuth motion will not answer that purpose, since, by reason of their small dimensions, they cannot, in the determination of angles, be relied on beyond a certain degree. The error which we seek to investigate in the large instrument (an eight feet mural quadrant for instance) may be within the limits of *inexactness* (if we may so express ourselves) of the smaller. For instance, a quadrant of two feet radius is not to be relied on beyond 8 or 10 seconds: but the sought for error of the line of collimation, of the mural quadrant of 8 feet radius, may not exceed 4 seconds; a quantity of moment in this latter instrument, by which it is purposed to determine angles to within 1 or 2 seconds. It is in vain then we seek for an angle of 4 seconds in an instrument on which we cannot rely to 8 seconds: and, indeed, the error of the line of collimation of a mural quadrant can only be determined by an instrument, of, at least, equal accuracy in the measuring of observed angles, and which, therefore, probably requires, in its essential parts, equal dimensions.

We have already, in explaining the principle of determining the line of collimation, represented the parts or fragments of the Astronomical Quadrant. If we still farther contract the dimensions of the Fig. of p. 60. and suppose the extremities of the graduated arc to be at n and r, the graduation on each side of the lowest point not exceeding 8 or 10 degrees, we shall have, what is, in fact and principle, a *Zenith Sector*, an instrument for measuring small angular distances from the zenith, and, (which is the essential point,) capable of being reversed; which reversion in small instruments is effected by means of an azimuthal movement, and, in large instruments, by removing the *sector* from an eastern to a western wall.

The reason is obvious why these sectors can be moved whilst the quadrants of equal radius cannot. The graduated arc, instead

of containing 90 degrees, contains not more than 10 or 12: sometimes much fewer degrees. The sector, therefore, can be made much less ponderous and unwieldy than the quadrant. The fixed mural quadrants at Greenwich are 8 feet, but the zenith sector's radius exceeds 12 feet.

A sector then of these latter dimensions must, to the extent of what it is able to perform, be more accurate than the mural quadrants. It is capable, for instance, of determining the zenith distance of γ Draconis, more exactly, than the mural quadrant. But it is capable also of determining the zenith distance of that star *truly* by taking the half sum of its zenith distances observed on the eastern and western walls. The difference of that half sum and of the zenith distance of the star, in one of the positions of the sector, is the error of the line of collimation of the *sector*: the difference of that same half sum, and of the zenith distance of γ Draconis observed with the mural quadrant, is the error of the line of collimation of the *mural quadrant*. For instance, by observations of γ Draconis made at Greenwich in 1812 with the zenith sector.

Sector on the eastern wall, mean zenith distance =	2′	14″.61
Sector on the western wall, mean zenith distance =	2	22.63
	2)4	37.24
Mean of eastern and western	2	18.62
Error of line of collimation of the sector	0	4.01

But by observations made the same year, on the same star, with the brass quadrant,

the mean zenith distance =	2′	14″.52
but (see 5th line above) mean of eastern and western =	2	18.62
error of line of collimation of the quadrant	−	4.1

which error (so it happens in this instance) is very nearly the same as the former (see 7th line above,) whereas it might have been different by 2, 3, &c. or more seconds.

By these means, then, the error of collimation of a mural quadrant may be corrected, and, if we use such a quadrant,

we must also be possessed of a zenith sector*. But the uses of this latter instrument are not merely subsidiary and subordinate ones. Its peculiar utility consists in finding, to a great degree of accuracy (we refer to a sector of a large radius, such as Bradley's or the Greenwich one is) the zenith distances of stars situated near the zenith: such, for instance, are, with respect to Greenwich, β and γ Draconis, Capella, α Cygni, α Persei, α Cassiopeæ, η Ursæ Majoris. What are the inferences to be drawn from zenith distances, so circumstanced and so minutely observed, will be hereafter explained.

Having now explained the constructions of the Astronomical Quadrant and of the zenith sector, and shewn the method of freeing them from one error, namely, that of collimation, we ought not to dismiss the subject without explaining, in its principle at least, the method of placing these instruments in the plane of the meridian. We will confine our attention, in the first instance, to a quadrant endowed with an azimuth motion.

A star (see pp. 4, 5,) rising from the horizon, attains its greatest height in the plane of the meridian, and, quitting the meridian, declines, by degrees like those by which it rose, towards the horizon. At equal altitudes to the east and west of the meridian, it is equally distant from its plane. The star so circumstanced, and referred to the plane of the horizon by vertical circles passing through it, is equally distant from the south point of the horizon, or equally distant from the north. In other words, it (see p. 5.) has equal *azimuths*. In the same positions also, namely, those of the star's equal altitudes, the star, with regard to the *time*, is equally distant from the meridian. Draw two *declination circles* (see p. 8,) one passing through the eastern, the other through the western position of the star; then, each circle makes an equal angle with the great circle of the meridian. But such angle, in the terms of sidereal time, expresses how much time will elapse between the star's eastern and meridional

* We are speaking here, as it is plain, of fixed mural quadrants and circles. A quadrant or circle, capable of being reversed, is able to find its own error of collimation. Such is, and perhaps the first of its kind with regard to size and accuracy, the Dublin Circle of 8 feet diameter made by Ramsden and Berge.

altitudes, and also between its meridional and western. Two methods then present themselves, by which the meridian may be found. Half the difference of degrees, &c. on the azimuth circle of the instrument, between any two equal altitudes of a star, is the angular distance of the south or north point, from the eastern or western azimuth of the star: or, half the difference of times elapsed between any two equal altitudes of a star, is the time that the star is on the meridian. In each case, we are able to direct the telescope (to the line of the collimation of which the face of the instrument is parallel) towards the meridian: and as, in the course of a day, we may take several pairs of equal altitudes, we are, by taking the mean of the azimuths, or the mean of the times, able to determine the direction of the plane of the meridian to a considerable accuracy *.

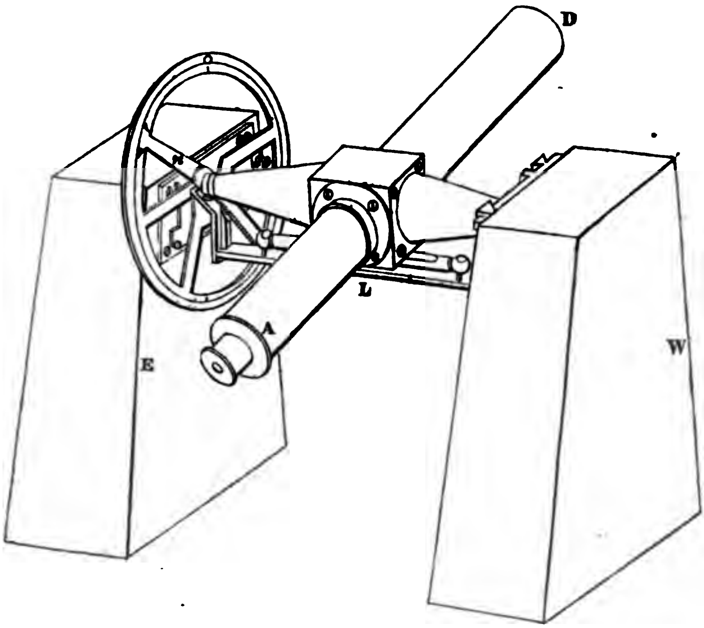
By either of the above methods, or by the aid of both, Astronomical quadrants and circles, such as are furnished with azimuth circles, may be placed, nearly, in the plane of the meridian. By means of such instruments, and by other helps, mural quadrants and mural circles may also be placed in the plane of the meridian. The operation is one of some nicety and is most accurately performed by the aid of the *Transit Instrument*, previously adjusted to move in the plane of the meridian. We will now, then, proceed to explain the *Transit Telescope*, or *Transit Instrument* †.

Let AD represent a telescope fixed, as it is represented in the figure, to an horizontal axis formed of two cones. The two small ends of these cones are ground into two perfectly equal cylinders: which cylindrical ends are called *Pivots*. These pivots rest on two angular bearings, in form like the upper part of a Y, and denominated Y's. The Y's are placed in two dove-tailed brass

* We may, for the above purposes, use the Sun and observe his equal altitudes and azimuths. As we cannot pretend to *bisect* his centre, by a wire of the telescope, we must make our times of observation, those in which the limbs of the Sun are in contact with the wires of the instrument. Since the Sun does not, like a star, describe a parallel of declination, there must be some small correction made, for his changes of declination, during the intervals of observing either equal pairs of azimuths or equal pairs of altitudes.

† *Instrument des Passages.*

grooves fastened in two stone pillars *E* and *W*, so erected as to be perfectly steady. One of the grooves is horizontal, the other



vertical, so that, by means of screws, one end of the axis may be pushed a little forwards or backwards, and the other end may be either slightly depressed or elevated. Which two small* movements are necessary, as it will be soon explained, for two adjustments of the telescope.

Let *E* be called the eastern pillar, *W* the western. On the eastern end of the axis is fixed (so that it revolves with the axis) an index *n*, the upper part of which, when the telescope revolves, nearly slides along the graduated face of a circle, attached, as it is shewn in the figure, to the eastern pillar. The use of this part of the apparatus is to adjust the telescope to the zenith or polar distance (for the one is as easily done as the other) of a star the

* The movements are of small extent since they are only subservient minute adjustments.

transit of which is to be observed. Thus, suppose the index of n to be at o (in the upper part of the circle) when the telescope is horizontal: then, by elevating the telescope, the index of n is moved downwards: suppose the position to be that represented in the figure, then the number of degrees between o and what the index of n marks, is the altitude of the telescope: or, we may so graduate the circle that the index shall mark the telescope's zenith distance: or, if we make the o, the beginning of the graduation, to belong to that position of the telescope in which it is directed to the pole, the number of degrees, &c. between o and any other position of the index, will mark either the telescope's polar distance, or, if we please, may be made to mark the telescope's declination; the telescope, in all these cases, being supposed to move in the plane of the meridian.

There are several other parts and contrivances, belonging to the instrument, not shewn in the Figure: for instance, one of the cones is hollowed, and, opposite the orifice, there is placed, in the pillar, a lamp which, throwing its light on a plane speculum, placed in the axis of the telescope and inclined at an angle of 45^{o}, illuminates the cross-wires. It is usual, also, in large transits to have counterpoises by which the pressure of the pivots of the axis on the Y's is relieved. We will now explain the three principal adjustments of the transit.

1st, To make the axis, on which the telescope moves, horizontal.

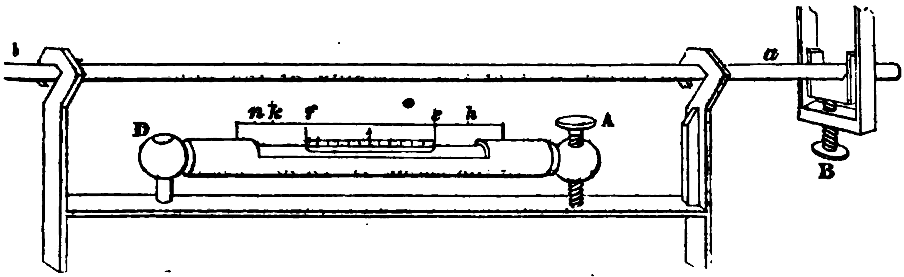
2d, To make the line of collimation move in a great vertical circle, or, which is the same thing, to make it perpendicular to the horizontal axis.

3d, To make it move in that vertical circle which is the meridian.

The first adjustment is effected by means of a level; and in the figure of p. 71, it is intended to represent the level (L) as hanging, by means of its upright arms, (bent, however, in their upper extremities) on the two pivots of the axis. The principle, however, and mode of rendering any axis horizontal, by means of a level, may be best explained by the subjoined Figure.

In this Figure, the spirit-level (including in that term, the brass tube that partly envelopes it, the horizontal bar to which it is affixed, and the two vertical arms by which it is hung on any

cylinder or rod) is represented as hanging on a straight cylinder *a b*, the end towards *a* lying on a crotchet which is capable of



being raised or lowered by a screw *B*. The end *A* of the tube *AD*, which contains the level, is also capable of being lowered or raised by means of a screw at *A*, as it is shewn in the Figure.

If *a b* were horizontal, and the tube of the spirit-level were parallel to *a b*, then the bubble would occupy the middle, or, the two extremities of the bubble would be equidistant from the centre, and would be, for instance, at *f* and *e*. The same thing would happen if the level were reversed, that is, if it were taken off the rod, turned round, and again hung on, so that *D* in the second position, should occupy the place that *A* did in the first, or should be to the right hand. But, if *a b* should not be horizontal, the above circumstances cannot take place. Suppose the end *a* to be *lower* than the end *b*, then if the level should not be parallel to *a b*, the bubble might still stand in the middle, by the end at *A* being, by a certain quantity, higher than the end at *B*. But on *reversing* the level, the bubble cannot occupy its middle, since then the lower part of the rod *a b* and the lower part of the level would both be situated to the right hand. The bubble, however, may not stand in the middle from two causes, the want of horizontality in *a b*, and the want of parallelism to it in the tube contained between *AD*.

If the level were parallel to *a b*: and the extremity of the bubble, instead of being at *e*, should be at *h*, on reversing the level, the other extremity of the bubble (which by the reversion would be towards *a*) would be at *k*, *f k* being equal to *e h*. But suppose this is found not to be the case, and that the extremity of the bubble, on reversing the level, is at *n*, then the circumstance of the bubble not standing at the two points *e* and *f*,

cannot arise solely from the end a being higher than b, but the level cannot be parallel to $a\,b$, and, in the case we have put, the end at A must be lower than the end at D: when the level then is in the second or the reversed position, so elevate the end at A, by means of the screw A, that the extremity of the bubble shall descend from n and occupy a place intermediate to n and k, and then the level is made parallel to $a\,b$; this is the first adjustment. Next, by means of the screw B, so depress the end a *, that the extremities of the bubble shall be, (as they ought to be, $e\,f$ being the length of the bubble) at e and f; then is $a\,b$ adjusted or made horizontal: this second adjustment completes the operation.

In the preceding reasonings, $a\,b$ has been considered, (the whole of it,) as cylindrical. But this is not necessary: it is sufficient if its extremities at a and b (the pivots), on which the level is hung, be equal cylinders, the axes of which lie in the same straight line. The intermediate parts of the axis of the transit between the pivots, may be of any form: they may be formed, as they generally are, of two cones. The preceding process, then, will render the axis of the *transit* horizontal; the level, whether in its primary or in its reversed position, being supposed to be hung on the equally cylindrical pivots.

The axis being now horizontal, the next operation is to make the line of collimation describe a great vertical circle, or, which is now the same thing, to make the line of collimation perpendicular to the axis of the transit.

The telescope AD (p. 71.) is furnished, like the telescope of the quadrant, with a system of cross-wires placed in the principal focus of the object-glass. Suppose the wires so placed that the line of collimation (see p. 61,) is perpendicular to the axis of the transit. If then a small and well-defined object be bisected by the centre of the cross-wires, it will still be bisected when the transit is lifted off its angular bearings, reversed and directed to the object; that is, illustrating our meaning by the Figure of

* The end of the cylinder at a rests upon an angular bearing (it might have been a Y), placed in a groove, and capable of being moved vertically by the screw at B. This part is, in fact, the same as that which is mentioned in the brief description of page 71.

p. 71, if the end of the axis carrying the index *n* which is placed on the eastern *Y* should be placed on the western. Let now the wires be deranged, so that their intersection is moved, not, as in the former case, in the plane of the meridian*, but in a direction perpendicular to that plane, and suppose it moved a little towards the east. In this case, the object before *bisected* is no longer so, but will be seen in the field of view a little to the west of the present centre of the cross-wires. Reverse the telescope, then the centre will be towards the west and the original object will be seen a little to the east of the centre: as much towards the east as it was before towards the west. If therefore there should be two objects or marks (on the horizon, for instance,) bisected by the centre of the wires in the two positions of the transit, the correction or adjustment of the line of collimation would consist in moving the centre of the cross-wires half way towards that object which is not on the centre.

But the moving the centre of the cross-wires, half way towards an object, is a matter of guess and not of certainty. In order to ascertain whether, in moving the centre, we have adjusted it rightly, we may avail ourselves of that angular bearing, or *Y*, which, (see p. 71,) by means of an horizontal groove and screw, we can move, together with the pivot of the axis, in azimuth. So move these then, that the object, to which we have already made the centre to approach half way, may be exactly bisected by that centre. Reverse the transit, and the object and centre are either coincident, or very nearly so. If the latter be the case, again, by their proper motion, move the centre of the wires half way towards

* We have supposed, in the quadrant, the derangement of the centre of the cross-wires to be made in the plane of a vertical circle; or, in the plane of the meridian, if meridional altitudes are to be taken: for such derangement is the essential one: a small deviation or derangement to the *east* or *west* would very slightly affect the determination of the altitude. But in the transit instrument the reverse is the case: the essential derangement is that which moves the centre of the cross-wires to the east or west of the meridian, and which makes the star to appear to pass the meridian too late or too soon. A small derangement of the cross-wires in the direction of the meridian, is of no consequence, since such derangement will neither accelerate nor retard the star's transit.

the object and move it the other half way by the screw that acts on the axis *. Reverse the instrument, and again, if it be necessary, repeat the above operations.

By these means, after a few trials, we are sure of making the line of collimation, or axis of vision, perpendicular to the axis of the transit; and, when that is effected, the cross-wires are no longer to be meddled with, although we must continue to use the above horizontal movement of the axis (see pp. 71, &c.) for the purpose of placing the line of collimation in the plane of the meridian. That line now moves in a vertical circle, and produced passes through the zenith: it is farther necessary to make it pass through the pole.

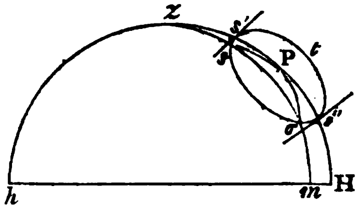
The transit instrument, (that which in the preceding pages we have spoken of) is supported between two fixed pillars. It must be supposed to be nearly in the meridian (the direction of the meridian being known, to a tolerable degree of accuracy, by some of the methods described in pp. 69, &c.) and to need only some slight adjustments to place it there exactly. It would be easy to effect this were the pole star exactly in the pole; for, then, it would be only requisite to bisect that star by the middle vertical cross-wire. But the pole star being, in fact, a circumpolar one, we must compute, by means of existing Tables and observations, (for the question is not now concerning the independent derivation of all Astronomical Elements from first principles) the time of its transit, and, at that computed time, *bisect* the star by the middle vertical wire. By these methods we may place the transit very nearly in the plane of the meridian.

We will now shew how to place it there more exactly by means either of the polar, or of any other circumpolar star.

The axis being horizontal, the optical axis perpendicular to it passes through the zenith: let ZPH be the true meridian and

* It is plain that the horizontal or azimuthal motion given to the Y and pivot, has nothing to do in the adjustment of the line of collimation. The adjustment is solely effected by the screw (or other contrivance) that gives motion to the cross-wires. The motion we can give to the axis only enables us to ascertain whether the last adjustment we have made with the cross-wires be sufficiently exact, or whether a farther one be necessary.

Zsm the vertical circle described by the optical axis or line of



collimation : then Hm, which is the measure of the angle at Z, is the deviation of the Transit from the meridian.

Let $ss's''\sigma$ represent the circle described by a circumpolar star, which is seen, through the transit telescope, at σ its inferior passage, and at s its superior. Now, when the Transit is not in the meridian, the time from σ to s cannot equal the time from s through s' and s'' to σ : for, P being the pole, the former time is p. 9,) proportional to the angle $\sigma P s$, or

$$180^\circ - \angle sPs' - \angle \sigma Ps'',$$

the latter to

$$180^0 + \angle sPs' + \angle \sigma Ps''.$$

Hence, if the interval between the inferior and superior passage should be less than the interval between the superior and inferior, the plane in which the Transit moves from the zenith to the north of the horizon (P being the north pole) is to the eastward of the true meridian.

But, in order to estimate the quantity of deviation from the observed difference of intervals between the passages, we must compute the angles

$$sPs' \text{ or } sPZ, \text{ and } \sigma PH,$$

now,

$$\sin. sPZ = \sin. sZP \times \frac{\sin. Zs}{\sin. Ps},$$

$$\sin. \sigma PH = \sin. \sigma PZ = \sin. sZP \times \frac{\sin. Z\sigma}{\sin. P\sigma}.$$

Let $\angle sZP$ (measured by Hm) $= Z$,

$Ps = P\sigma = \pi$,

the latitude of the place ($= HP$) $= L$,

then since Z, or the deviation from the meridian, is, by the conditions, very small, we have, nearly,

$$\sin. Z = Z,$$
$$Zs = ZP - Ps = 90^0 - (L+\pi),$$
$$Z\sigma = ZP + Ps = 90^0 - (L-\pi),$$

consequently, sPZ (which is, nearly, $=$ its sine)

$$= Z \cdot \frac{\cos. (L+\pi)}{\sin. \pi} = Z (\cos. L \cot. \pi - \sin. L),$$

and $\sigma PH = Z \cdot \dfrac{\cos. (L-\pi)}{\sin. \pi} = Z (\cos. L \cot. \pi + \sin. L).$

Hence, the time from σ to $s = 180^0 - 2Z \cos. L \cot. \pi$,

and from s to $\sigma = 180^0 + 2Z \cos. L \cot. \pi$.

Let the former time $= 12^h - \Delta$,

the latter $= 12^h + \Delta$;

then, since 180^0 (see pp. 9, 10.) is the angular measure, or exponent of 12 hours of sidereal time,

$$12^h - \Delta = 12^h - 2Z . \cos. L . \cot. \pi,$$
$$12^h + \Delta = 12^h + 2Z . \cos. L . \cot. \pi,$$

whence

$$Z = \frac{\Delta}{2 \cos. L . \cot. \pi},$$

or, (see *Trig*. p. 18.)

$$= \frac{\Delta}{2} . \sec. L \tan. \pi,$$

and, the logarithmic formula will be (see *Trig*. p. 19.)

$$\log. Z = \log. \frac{\Delta}{2} + \log. \sec. L + \log. \tan. \pi - 20,$$

which is the substance of the Rule that is given by Wollaston at p. 74, of the Appendix to the Fasciculus Astronomicus.

As an example to this formula, let the observed star be the pole star, with a north polar distance equal to $1^0\ 39'\ 25''.05$, and, the place of observation, Cambridge, assuming its latitude

to be $52^{0}\ 12'\ 36''$: and let Δ, the difference of the intervals of the transits, equal $7^{m}\ 22^{s}$ ($= 442^{s}$): we have then

$$\begin{aligned} \log.\ 221 \ldots\ldots\ldots\ldots &= 2.3443923 \\ \log.\ \sec.\ 52^{0}\ 12'\ 36'' \ldots &= 10.2127030 \\ \log.\ \tan.\ 1^{0}\ 39'\ 25''.05 &= \underline{8.4513064} \\ & \quad 21.0084017 \end{aligned}$$

Hence, $\log.\ Z = 1.0084017$,

and $Z = 10^{s}.195$.

The result is here expressed in *time*, as it must needs be from the expression of p. 78, l. 18, if Δ be so expressed. It may, however, (should it be thought necessary) be expressed in measures of space or of angular distance: for, since 24 hours of sidereal time is held to be equivalent to be 360^{0}, 1^{h} will equal 15^{0}, 1^{m} will equal $15'$, and 1^{s} will equal $15''$: and, consequently,

$$10^{s}.195 \text{ must equal } 101.95'' + \frac{1}{2}(101.95''),$$

or $2'\ 32''.925$, which will be the value of the deviation of the line of collimation from the plane of the meridian.

Nothing, however, is gained (if we look, in the present case, to the practical convenience of the thing) by this conversion of a measure in time into an angular measure: for the approach of the plane, in which the line of collimation is, to the plane of the meridian is effected (see p. 71.) by means of a screw: suppose, for the sake of illustration, the head of this screw to be graduated like that in the figure of p. 55. Let the time of the transit of an equatoreal star over the middle vertical wire be noted on a particular day. Alter the inclination of the plane, in which the line of collimation moves, to the plane of the meridian, by turning the screw once round, and observe, the next day, the time of the star's transit: suppose the difference of the times of transit, on the two successive days, to amount to two seconds, then will one revolution of the adjusting screw be equal to two seconds, half a revolution to one second, one eighth of a revolution to one quarter of a second, and so on: so that, having thus once obtained the value of the motion of the adjusting screw we may immediately

apply it to the result of Z, expressed in time, (see p. 79.), and correct, accordingly, the Transit's deviation.

It appears then, from the preceding computation, that a deviation of about 10 seconds of time, in the transit telescope from the plane of the meridian causes the time, between the inferior and superior transit of the pole star, to differ, from the time between the superior and inferior transit, by about 7 minutes. The difference, it is probable, will not be the same in another circumpolar star. Let us examine what it will be in *Capella*, the north polar distance of which in January 1, 1819, was $44^0\ 19'\ 53''$, and which, consequently, passes the meridians of Greenwich and Cambridge to the south of their zeniths. In this case (estimating separately the angles sPZ, σPH), we have

$$sPZ = -Z \cdot \frac{\cos.\,(L+\pi)}{\sin.\,\pi} = -Z \cdot \cos.\,(L+\pi)\ \text{co-sec.}\ \pi,$$

$$L = 52^0\ 12'\ 36''$$

$$\pi = 44\ \ 19\ \ 53 \ldots\ \text{log. co-sec.} = 10.1556425$$

$$96\ \ 32\ \ 29 \ldots\ \text{log. cos.}\ldots = 9.0566035$$

$$\Delta = \quad 10.195 \ldots\ \text{log.}\ldots\ldots = 1.0084017$$

$$20.2206477$$

$$\therefore\ \text{log.}\ sPZ = .2206477,$$

$$\text{and } sPZ = 1''.662:$$

for the inferior passage of Capella,

$$\sigma PH = 14''.452.$$

It appears then from the above results that although the plane, in which the line of collimation of the transit telescope moves, deviates more than 10 seconds from the plane of the meridian, yet the time of passing the middle vertical wire, at the superior passage of Capella, differs but very little ($1^s.662$) from the time of passing the meridian; and the reason is obvious: Capella in its upper passage, passes near the zenith, and the line of collimation, by means of previous adjustments, describes a great vertical circle, and, consequently, passes through the zenith. But the case is different with the inferior passage; at that, the

time of passing the middle vertical wire differs from the time of passing the meridian by $14^s.452$.

If we wish to determine the difference between the intervals of the successive transits, we have

$$\text{the time from } \sigma \text{ to } s = 12^h - 14^s.452 + 1^s.662,$$
$$\text{from } s \text{ to } \sigma = 12^h + 14^s.452 - 1^s.662,$$

and, consequently, the difference of times equals

$$28^s.904 - 3^s.324, \text{ or } 25^s.58.$$

But with the pole star the difference arising from the same deviation of the transit telescope $(10''.195)$ amounted to 442^s. This latter star then, *if all other things were equal,* is much better adapted than Capella, or than any other circumpolar star (provided its north polar distance exceeds that of the pole star) to adjust, by the preceding method, the transit telescope to the plane of the meridian.

But there are circumstances attending the pole star that detract from this superiority. The slowness of its motion is such that it is difficult to note the exact time of its *bisection* by the middle vertical wire of the telescope. There must always be some uncertainty on this head: more or less, according to the magnifying power of the telescope and the fineness of the wires that are placed in the common focus of the object and eye-glasses. In small Transits the star is hid for some seconds behind the wire. In the late transit instrument* of Greenwich, the *uncertainty* of the time was esteemed at about 2 seconds: in

* The transit instrument used by Bradley and Maskelyne was made in 1750 by Bird, was eight feet in length, had an aperture limited to an inch and half, and magnified 50 times. After Dollond's discovery of the different relations which rays of light bear to different kinds of glass, but possessing the same mean power of refraction, an *achromatic* object-glass, of $2\frac{7}{10}$ inches diameter, was substituted instead of the original one, the eye-glasses were changed, and the magnifying power of the telescope increased to eighty times. The present transit telescope put up July 16, 1816, was made by Troughton, is ten feet in length, has an object-glass of five inches diameter, and will magnify distinctly with a power of 300.

the present transit instrument, it is reduced to about 1 second *. But this uncertainty will, it is plain, be reduced within narrower limits, by observing with stars that have greater north polar distances. The time which an *equatoreal* star takes in passing over a given interval, is to the time which *Polaris* takes in passing over the same interval, nearly, as 183 is to 6000, or as 1 is to 33. And in such proportion will the uncertainty, respecting the precise time of a star's transit, be reduced.

But the above circumstance, the slowness of the motion of the pole star, only renders that star less convenient than it otherwise would be, for adjusting the plane in which the line of collimation moves to the plane of the meridian. It is still, on the whole, the most convenient star to be made use of.

On principles, like the preceding, is founded a method for bringing the Transit into the plane of the meridian by means of the pole star, and of another star which passes the meridian near the zenith of the place of observation. Capella, for instance, as we have seen, is, in our latitudes, under such predicaments. Now in its superior passage, such a star, should the Transit deviate, only slightly, from the meridian, would pass the meridian *very nearly* (see p. 80,) at the time of its passing the middle vertical wire of the telescope. Assume it to pass *exactly*, and then (that is, when the star is on the middle wire) make the clock denote the right ascension of Capella, known from Catalogues and Astronomical Tables: or, which is the same thing in practice, note how much the clock differs from the registered right ascension. Next observe the clock when the pole star is on the vertical wire. The time shewn by the clock cannot be the right ascension of the pole star, or the interval of time between Capella and the pole star being on the vertical wire, cannot be the right ascension of the latter star, or the difference of the catalogued right ascensions of the two stars, because the transit instrument is not in the plane of the meridian. Compute

* These assertions are not to be taken absolutely and according to the letter. The estimation of the time which a star *hangs* on the wire, or takes in passing the wire, will vary with circumstances; the state of the air, the time of day, the brightness and magnitude of the star, &c.

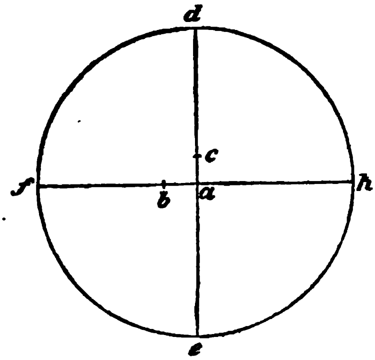
according to the difference of the right ascensions of the pole star as shewn by the clock, and as expressed in catalogues, the deviation of the transit instrument (see pp. 77, &c.) and adjust it accordingly. The instrument so adjusted will be very nearly, but not exactly, in the plane of the meridian.

It will be *not exactly* adjusted, because Capella, although very nearly on the meridian, when on the vertical wire, was not there exactly. If, as in the Figure of p. 77, the telescope directed towards the pole, moves in a plane to the east of the meridian, then Capella, in its superior passage, will be on the vertical wire of the telescope, after it has passed the meridian. Suppose the error of time, as computed in p. 80, to be $1^s.66$, and the right ascension of Capella to be $5^h\ 3^m\ 11^s$: then the clock, when Capella is on the middle wire, ought not to denote $5^h\ 3^m\ 11^s$, but $5^h\ 3^m\ 12^s.66$. The clock, therefore, by the rule (see p. 82,) is made too slow: suppose then the clock, Polaris being on the vertical wire of the Transit*, to denote $50^m\ 0^s$, the catalogued right-ascension being $56^m\ 18^s$. $6^m\ 18^s$ would, by the rule (see p. 82,) be the error of time from which the deviation of the transit is to be computed, whereas $6^m\ 18^s - 1^s.66$, or $6^m\ 16^s.44$ ought to be the error, which, so taken, would cause a slight difference, and a very slight one, in the resulting quantity of the Transit's deviation. This slight difference must be got rid of by a renewed process of computation and adjustment.

The line of collimation being now supposed, by means of the previous adjustments, to describe a great circle passing both through the pole of the Heavens, and the zenith of the observer, the transit instrument is in a fit state to note the passages of stars cross the meridian. A star *passes* the meridian at the instant it is coincident with *a* the centre of the cross wires: but if *de* were truly vertical, a star on any point of *de* would be on the meridian. It is desirable then to make *de* vertical, since then we should have the power of observing the star's transit on any part of *de*. This may be thus effected. Direct the transit

* Transit, transit instrument, transit telescope, are used in these pages to denote the same thing.

telescope to some well-defined small object, so that it is *bisected*

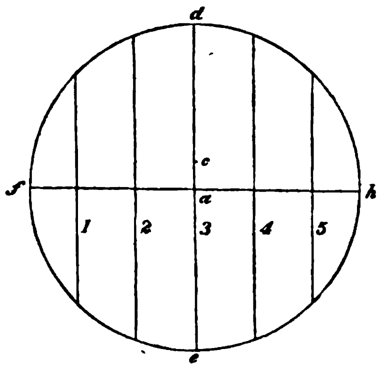


by some point of *de*. Move the telescope round its horizontal axis and observe whether the same object is bisected by every part of *de*, or, in other words, whether it *runs* along the wire *de*. If it does, the wire is vertical, or the middle wire is also a *meridional* wire. If it happen otherwise, the wire must be adjusted till the above test of its *verticality* be obtained.

When the transit instruments are large, the various adjustments, that have been described, are not made without trouble and difficulty. But the results now exacted of large transits are of such nicety that we cannot rely on observations except we are assured that, at the times of making them, the instruments were properly adjusted. The transit instrument, then, requires a daily and continued examination. But, in order to avoid the repetition of troublesome verifications, two marks are set up, one to the north, the other to the south, and their places determined by means of the middle and meridional wire. The *marks* used at Greenwich are vertical stripes of white paint on a black ground, on buildings about two miles distant from the Observatory. They are first placed by means of the instrument adjusted to the meridian, and then are subsequently used to bring the instrument into the meridian, should it become deranged.

But, besides the middle or meridional wire, it is usual to place on each side of it and at equal distances from it, parallel side wires. Their use is to check the observation at the middle wire, and to supply its place, should it become defective by inter-

vening clouds or other accidents. The old Transit at Greenwich (see p. 81,) had four side wires, and, therefore, in all, five wires. The present Transit has 7. There are five wires represented in the subjoined Figure, and numbered 1, 2, 3, 4, 5. If



1 and 5 are equidistant from 3 the middle wire, half the sum of the times at 1 and 5 will be the time at 3: and adding together the times at 1, 3 and 5, one third of their sum will the *mean* time of transit cross the middle wire. The like will take place with the wires 2, 4, if these be at equal distances from 3. And if we add together the five times of the star's passage cross the wires 1, 2, 3, 4, 5, and take one-fifth of the sum, the result will be the mean time of the star's passage over the meridional wire.

Let t be the time at the middle wire; $t-20^s$, $t-40^s$, the respective times at the wires 2 and 1, $t+20^s$, $t+40^s$, at the wires 4 and 5: then the sum is $5t$ and one-fifth is t, the time at the middle wire: and if the cases in practice were like this, nothing would be gained by the side wires. But the fact is that we are not able to note absolutely the times at the several wires. It is probable no beat of the pendulum will happen exactly when the star is on a wire. The beat of the pendulum may happen just before the star reaches a wire, and the next beat after the star has quitted the wire. The observer then is obliged, in default of other means, to estimate, according to the best of his judgment, the fraction of a second at which the star was on the wire: which estimation must needs be somewhat uncertain and erroneous. A tenth of a second may be put down too much at one wire, and too little at another: but it is probable

that the errors will, in degree at least, compensate one another, and that the mean result will be entitled to more confidence than a single observation at the middle wire.

Thus by an observation made in 1816 on α Ceti, the observer saw the star a little to the left of wire 1 * at $2^h\ 51^m\ 12^s$; at the next beat, that is, at the 13 seconds, it was to the right of the wire, and judging the star's distances, to the left and right at the times of the two beats, to be as 7 to 3, he put down the time at the wire 1 at

$$2^h\ 51^m\ 12^s.7.$$

The star took more than 18 seconds in passing to the second wire. At the beat of the thirty-first second, the star was to the left of the wire 2, at the thirty-second, to the right, and, the distances being apportioned as before, the time at the second wire was put down at

$$2^h\ 51^m\ 31^s.1:$$

in like manner

at the third wire at	2^h	51^m	$49^s.4$,
at the fourth	2	52	7.6,
at the fifth	2	52	25.9.

Here the intervals of time between the wires are 18.4, 18.3, 18.2, 18.3, a little different the one from the other, not necessarily different from real inequalities in the respective spaces between the wires, but, probably, from the cause assigned above, namely, the uncertainty of the observer when he *guesses* at the tenth of a second. If we add the above five times together, their sum amounts to

$$5 \times (2^h\ 51^m) + 246^s.7,$$

the fifth of which is

$$2^h\ 51^m\ 49^s.34.$$

* Since objects appear inverted through the telescopes of Astronomical instruments, a star will appear to enter the field of view to the right of the extreme wire to the right, which, in the preceding figure, would correspond to the wire 5. The principle, however, of the explanation is precisely the same whether the object is seen inverted, or in its natural position.

The time at the middle wire * was

$$2^h\ 51^m\ 49^s.4.$$

The former time, the mean time, is *probably* the truer time, although it is plain that nothing positive can be affirmed on this head.

The intervals between the wires are made very nearly equal by the instrument maker. But the power and accuracy of modern transit instruments is such that a good observer will, from his observations, be able to discover inequalities in the intervals not otherwise, or mechanically, ascertainable. The intervals are examined, and their values in seconds of time found by taking, from a great number of observations, the means of the times a certain star takes in passing respectively from the first to the second wire, from the second to the third, &c. If, as is frequently the case, the intervals are unequal, then, in estimating the

* It can very rarely happen that the minutes of the time at the middle wire differ from the minutes of the deduced mean time. For that reason, in registering the several times, the hours and minutes are only once expressed for the middle wire, it being sufficient to note the seconds alone at the side wires. Thus, the above results are thus registered.

I.	II.	Middle Wire.	IV.	V.	Reduction of Wires.
12.7	31.1	$2^h\ 51^m\ 49^s.4$	7.6	25.9	- 49.34

The seconds added together make 126.7: now, if we divide by 5, the first figure of the seconds would be 2, which must be wrong, since the number of seconds must be, what it is in the middle wire, nearly 49: in order to make the first figure 4, we must add 120 (two minutes) to 126.7: the sum 246.7 divided by 5 gives 49.34: the two minutes (120^s) added come in fact, from the fourth and fifth wire; where the minutes instead of 51, are 52. But, as it is plain, we need not concern ourselves about the minutes. If the sum of the seconds added together and divided by 5 do not give the first figure, the same as the first figure of the seconds at the middle wire, we must add either 60, or 120, to the number of seconds till that fact takes place, and the result cannot fail to be right. In the sixth column entitled the *Reduction of the Wires*, the mean result of the seconds is put down.

time of a star's transit from the mean of the times at the several wires, some allowance must be made for the inequalities of the intervals *.

We subjoin an instance or two from the Greenwich Observations of 1816 to illustrate the preceding matter.

	I.	II.	Middle Wire. III.	IV.	V.	Reduct. of Wires.	Stars.
Nov. 3.	1.4	20.0	$21^h\ 55^m\ 38^s.4$	56.5	15.2	38.30	α Aquarii.
	22.6	55.2	0 29 27.5	0.0	32.5	27.56	α Cassiopeæ.
Nov. 4.	0.4	18.4	21 55 37.2	55.7	14.1	37.16	α Aquarii.

The sum of the seconds at the five wires in the first horizontal line is 131.5: but the first figure of the seconds (see Note of p. 87.) must be 3, 38^s being the seconds at the middle wire. We must, therefore, add 60 to 131.5, in order that the first Figure of the quotient may become 3, and, accordingly, $\frac{191.5}{5} = 38.30$ *the reduction of the wires*: or, the mean time of the star's transit is

$$21^h\ 55^m\ 38^s.30.$$

Again, the sum of seconds in the second horizontal line is 137.8: and dividing by 5 the first figure of the quotient is 2, which is right, (27 being the number of seconds at the middle wire) or, it

* Delambre and other authors give rules for estimating the thickness of the wires, and for allowing, in registering the observations, for such thickness. But it is the practice at the Greenwich Observatory not to make any allowance. The thickness of the wire used in the new transit is $\frac{1}{934}$ths of an inch. Which is a thickness greater, if we rely implicitly on Dr. Maskelyne's statement (see vol. III. Greenwich Observations, p. 339.) than that of the wires in the old transit; which is, in the observations just alluded to, stated at $\frac{1}{1000}$th of an inch.

is not necessary to add either 60, or 120, to 137.8. Accordingly,

$$\frac{137.8}{5} = 27.\overset{5}{\cancel{1}}6,$$

the reduction of the wires, or the mean transit at the meridional wire is

$$0^h \ 29^m \ 27^s.\overset{5}{\cancel{1}}6.$$

In the third column the sum of seconds is 125.8: divide by 5, and the first figure in the quotient is 2, but it ought to be 3, since 37 is the number of seconds at the middle wire, add therefore 60, and then

$$\frac{1}{5}(185.8) = 37.16,$$

the reduction of the wires, and the mean time of transit is

$$21^h \ 55^m \ 37^s.16.$$

The mean time is expressed to the hundredths of a second. But this is an exactness altogether arithmetical, or which results from arithmetical operations, and is, in no wise, connected with any presumption on the part of the observer to distinguish such small portions of time *.

The intervals between the several wires, as estimated from the same star (α Aquarii), are from the first and third rows,

18.6, 18.4, 18.1, 18.7,
18.0, 18.8, 18.5, 18.4,

so that, if we were limited to these two observations, we should find it difficult to say whether the intervals between the wires were equal or unequal.

The intervals between the wires from the observations of α Cassiopeæ are

32.6, 32.3, 32,5, 32.5,

in which, the intervals appear to be much more nearly equal than they were in the former instances.

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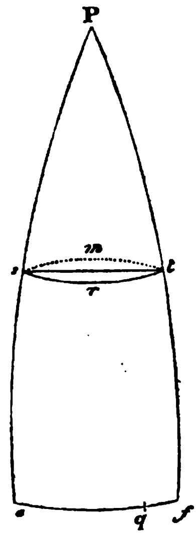
* "Tam exigua et evanescentia temporis momenta."

It appears from the above examples that the star α Cassiopeæ is almost twice as long in passing from wire to wire as the star α Aquarii. The latter star is near the equator, its north polar distance being (in 1816) about $91^0\ 12'\ 30''$, whereas the north polar of α Cassiopeæ was, at the same period, $34^0\ 28'\ 23''$. Now it is easy to prove that the time of a star's describing small spaces perpendicular to the meridian (such as the intervals of the cross-wires would be) varies inversely as the cosine of its declination. For let P represent the pole, Pe, Pf two arcs of 90^0



each. Let st represent the interval of the wires, nearly, by reason of its smallness, coincident with srt. Take $eq = st$; then (see pp. 9, 10.) a star apparently moves from s to t in the same time as another star moves from e to f in.

But the time through st (= the time through ef) = time through $eq \times \frac{ef}{eq}$ = time through $eq \times \frac{ef}{st}$ =, nearly, time through $eq \times \frac{\text{radius}}{\text{co-sin. } se}$. Hence, if the time through eq, that is, if the time of an equatoreal star moving across the interval eq be given,

the time of moving across an equal interval (st) varies as $\frac{1}{\text{co-sin. } se}$; or, directly as the secant of the star's declination.

But there are no stars exactly in the equator, and consequently, the *equatoreal* interval of time, through a space equal to *st*, cannot be determined by direct observation. It may, however, be easily determined by observing the time that any known star (α Aquarii, for instance,) takes in passing that interval, and then by lessening that time in the ratio of the cosine of the star's declination to radius. Thus the mean time of α Aquarii passing an interval of the cross-wires being $18^s.4$, the time of an imaginary equatoreal star passing the same interval, equals

$$18^s.4 \times \cos. (1^\circ\ 12'\ 30'') = 18^s.395.$$

This is the quantity from one star, and, if we employ several stars, we shall obtain, from a mean of the results, a result of greater exactness. For instance, the north polar distance of α Cygni is $45^\circ\ 22'\ 5''$, that of α Aquilæ is $81^\circ\ 36'\ 42''$, and the mean times which these stars took in passing the interval between two successive cross-wires, were, respectively, $25^s.8$, and $18^s.55$. Hence, since the cos. star's declination = sin. star's N. P. D. we have

For α Cygni.

$$\begin{array}{rl} \log.\ 25.8 \ldots & = 1.4116 \\ \log.\ \sin.\ 45^\circ\ 22' \ldots & = 9.8522 \\ \hline & 1.2638 = \log.\ 18.35. \end{array}$$

For α Aquilæ.

$$\begin{array}{rl} \log.\ 18.55 \ldots & = 1.2683 \\ \log.\ \sin.\ 81^\circ\ 36' \ldots & = 9.9953 \\ \hline & 1.2636 = \log.\ 18.35. \end{array}$$

The time of an equatoreal star's passing an interval between the cross-wires, being thus determined by computation, from the observed times of known stars, but not in the equator, the times which other stars will take in passing the intervals of the wires may be determined by *increasing* the *equatoreal* time in the ratio of radius to the cosine of declination, or, in the ratio of radius to

the sine of north polar distance. Thus, the *equatoreal* time of passing the interval being assumed equal to $18^s.3$, the times which the stars β Draconis, μ Ursæ Majoris, the north polar distances of which are (in 1816), respectively, $37^0\ 33'\ 26''$, $47^0\ 34'\ 44''$, will take in passing the same interval, will be

$$18^s.3 \times \text{sec.}\,(52^\circ\ 26'\ 34''), \text{ and } 18^s.3 \text{ sec.}\,(42^0\ 25'\ 16'').$$

Hence,

log. 18.3	1.2624	log. 18.3	1.2624
log. sec. 52° 26′ .	10.2149	log. sec. 42° 25′ . .	10.1318
	11.4773		11.3942

Hence, deducting 10, the logarithms of the times are 1.4773, and 1.3942, and the numbers 30.01, 24.786: which times agree, very nearly, with the following observations made in Sept. 1816:

I.	II.	III.	IV.	V.	Stars.
41.5	11.5	$17^h\ 25^m\ 41^s.6$	11.6	41.7	β Draconis.
54.6	19.5	10 10 44.1	9.0	33.8	μ Ursæ Majoris.

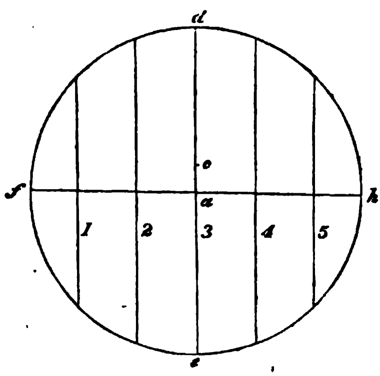
The mean interval, in time, of the first row is $30^s.05$ and of the second $24^s.775$.

The pole star is about ten minutes in passing over the above intervals *.

* Dr. Maskelyne in vol. III, of his Observations, observes that the stars generally observed (some of the thirty-six stars of his catalogue) and which are near the equator, move over the vertical wire ($\frac{1}{1000}$th of an inch in thickness) in about $\frac{2}{15}$ths of a second. Consequently, the pole star, under ordinary circumstances, would be about $\frac{10 \times 60}{18.3} \times \frac{2}{15}$, or about $4\frac{1}{3}$ seconds in passing the vertical wire, or would appear to hang on the wire for about that time.

By the preceding methods and computations the upright wires of the transit telescope may be adjusted vertically, and the intervals between the wires found in parts of sidereal time. For the purpose of knowing whether the wires, which ought to be at right angles to the former, are strictly horizontal, direct the telescope towards a star near the equator, and if the star entering at *h* (the telescope is supposed to reverse its objects) *runs* along the *hf*, then *hf* is horizontal.

This test of the *horizontality* of the cross-wire, is literally true only with respect to a star situated in the equator. If the star be out of the equator it cannot be bisected during its



passage through the field of view by every point of the wire *fh*, whatever be *fh*'s position. The reason is easily arrived at. When the telescope is directed to the equator, the cross-wire *fh* is the chord of an arc of the equator, in the centre of which great circle the eye is situated. The eye, therefore, being in the same plane with the subtense *fh* and the arc which the star describes, sees the star moving along the subtense (which in this case is the cross wire *fh*) whilst it describes the arc. The same would be true of the arc of every other *great* circle and its subtense or chord. But if the star be out of the equator it does not describe a great circle but a small circle. In the Figure, p. 90, let *smt* be an arc of a great circle: then a star describing *smt* would seem, to an eye situated in a plane passing through *smt* and *st*, to describe *st*: but *srt*, part of a small circle parallel to *ef* is the star's apparent path, which, coinciding

with the chord st at its two extremities s and t, would (the telescope reversing) appear to describe a curve below the cross horizontal wire, the apparent path of the star through the field of view being the more curved, the less the star's north polar distance.

The method given in p. 90, for determining the time of an equatoreal star's passing the interval between two successive wires, is, strictly examined, an approximate method. If we wish for an exact one, we may obtain such by means of the Figure of p. 90. Suppose st to represent the interval of the cross wires, then

$$st = \text{chord } srt = 2 \sin. \frac{srt}{2} \quad (\text{radius} = \sin. Ps)$$

$$= 2 \sin. \frac{ef}{2} \times \sin. Ps \ (\text{radius being} = \sin. 90^\circ);$$

$$\text{but } ef = \frac{t}{24^h} \times 360^\circ \ (t \text{ being the time of describing } srt)$$

$$= \frac{t \times 360 \times 60 \times 60''}{24 \times 60 \times 60} = 15''t,$$

t being now the number of seconds of time.

Hence $st = 2 \sin. \dfrac{15''t}{2} \times \sin. Ps$, which is a general expression, whatever Ps is, st the interval of the cross-wires being supposed the same. Hence at the equator, t' being what t becomes

$$st = 2 . \sin. \frac{15''.t'}{2};$$

$$\therefore \sin. \frac{15''t}{2} \times \sin. Ps = \sin. \frac{15'' \times t'}{2};$$

$$\text{or, } \sin. \frac{15''t}{2} \times \sin. \text{star's north polar distance} = \sin. \frac{15''t'}{2}.$$

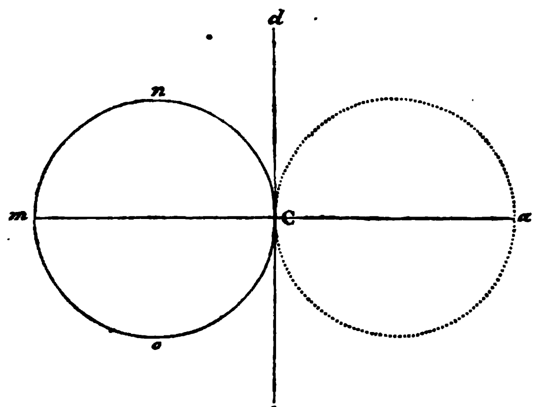
But the sines of small arcs are nearly equal to the arcs themselves. Consequently, since $\dfrac{15''t}{2}$, $\dfrac{15''t'}{2}$ are small arcs, we have, *nearly*,

t . sin. star's north polar distance $= t'$,

which agrees with the formula of p. 90.

What has preceded relates to the transits of stars that are but as points and without disks. We must now find out the means of determining the transits of heavenly bodies, such as the Sun and Moon, which have disks but no distinct or marked centres. The transit, however, of a heavenly body means the transit of its centre. In this case then, we cannot avail ourselves of direct observation. But we may compute the time when the centre (the Sun's centre, for instance) is on the middle wire, from having noted the two times of contact of its western and eastern limb with that wire. For, as it is plain, half of those observed times is the time required.

Let *mno* represent the Sun's disk, in contact with *de* a vertical wire. If the Sun's centre be crossing the meridian in the direction *ma*, *m* cannot pass on to *C*, or the eastern limb



cannot come into contact with the middle wire, except by *m's* moving through a space equal to *mC*, and in a time equal to that in which a star, having the same declination with the Sun, would describe a space equal the Sun's diameter. In half that time then the middle point between *m* and *C*, or the Sun's centre, will be at *C*, or on the middle wire.

But, as in the case of stars, so here we may avail ourselves of the side-wires. Thus, the linear distances of the wires from the

centre being supposed equal, half of the interval of the times between a star being on the first and fifth wire, is the time that a star is on the meridian; so, half the time between the *contacts* of the Sun's limb (whether it be the eastern or western limb) with the first and fifth wire, is the time of the contact of the same limb with the middle or meridional wire. But half the time between the contacts of the Sun's western and eastern limb with any given wire, is the time of the *bisection* of the Sun's centre by the same wire. Add, therefore, the times of contact of the western or first limb *, with the several wires, to the times of contact of the eastern or second limb, with the same wires, and the sum divided by the *whole* number of contacts, will be the mean time of the Sun's passage cross the meridian.

Thus, by the Greenwich Observations of 1815, Nov. 6,

I.	II.	III.	IV.	V.	
$40^m\ 45^s.5$	$41^m\ 4^s.6$	$14^h\ 41^m\ 23^s.6$	$41^m\ 43^s$	$42^m\ 1^s.8$	$\odot\ 1L$
43 0.6	43 19.7	14 43 39	43 57.8	$44\ 17^s.2$	$\odot\ 2L$

The sum of the times of contact is $10 \times 14^h + 425^m + 12^s.8$; the number of contacts is 10. The mean time, therefore, of the Sun's transit is

$$14^h\ 42^m\ 31^s.28,$$

in which, as before, (see pp. 87, 88, &c.) $31^s.28$ is the *reduction of the wires*. The time of the Sun's transit estimated from half the sum of the times at the middle wire, is

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* The telescope inverting objects, the Sun's western limb appears to the east, in the field of view, and the eastern limb to the west, and the Sun's motion is apparently from the right to the left. The Sun's limb that first comes into contact with a vertical wire is symbolically denoted thus $\odot\ 1L$, and the other limb thus $\odot\ 2L$: and the corresponding symbols for the Moon are ☽ $1L$, ☽ $2L$.

$$\frac{1}{2}(28^h\ 85^m\ 2^s.6), \text{ or, } 14^h\ 42^m\ 31^s.3,$$

which differs from the mean time only by $0^s.02$.

Again, by observations made at Greenwich, Nov. 8, 1816, with the new transit and with five of its seven wires,

II.	III.	Meridional Wire. IV.	V.	VI.	
$51^m\ 29^s.4$	$51^m\ 48^s.5$	$14^h\ 52^m\ 7^s.6$	$52^m\ 26^s.7$	$52^m\ 46^s$	☉ 1 *L*
53 45	54 4.3	14 54 23.4	54 42.5	55 1.6	☉ 2 *L*

The sum of all the times is

$$10 \times 14^h + 532^m + 35^s,$$

the number of contacts is 10; therefore, the mean transit of the Sun's centre is

$$14^h\ 53^m\ 15^s.5,$$

which is the same result as the half of the times at the middle wire.

We cannot use exactly the same method in finding the transit of the Moon's centre; because the Moon shines only once with a full orb during her revolution round the Earth. At all other times, amongst which will almost always be found the times of observation, either her western or eastern limb is more or less deficient: so that, on the deficient side, either no contact, or an imperfect one, takes place. On this account the contact of one limb only, that which is turned towards the Sun, is observed. Thus amongst the Greenwich Observations, Jan. 15, 1816, we find the following:

II.	III.	Meridional Wire. IV.	V.	VI.	
$49^m\ 38^s.6$	$49^m\ 59^s$	$8^h\ 50^m\ 18^s.8$	$50^m\ 39^s$	$51^m\ 0^s$	☽ 2 *L*

The sum of these times is

$$5 \times (8^h\ 50^m) + 95^s.4.$$

The number of contacts is 5, and consequently, the mean time of contact, of the Moon's second limb with the meridional wire, is

$$8^h\ 50^m\ 19^s.08,$$

from which, deducting the time which the Moon takes in passing over a space equal to her semi-diameter, we shall have the transit of the Moon's centre over the meridian.

We must proceed in a like manner, when we wish to determine the altitude of the Sun's, or of the Moon's centre by the quadrant or circle. The altitude of the upper (⊙ U. L.), or lower limb (⊙ L. L.) must be found by bringing it into contact with the horizontal wire. The Sun's semi-diameter deducted or added will give a result equal to the altitude of the Sun's centre. Or, half the sum of the altitudes of the upper and lower limbs will give the altitude of the centre.

Thus, by observations made in 1816, with the Greenwich mural circle,

	Barometer.	Thermometer. In.	Out.		N. P. D.
June 3.	29.89	59	64	⊙ U. L.	$67^0\ 23'\ 8''$
June 4.	29.86	58	64	⊙ L. L.	67 47 34.1

and by observations made in 1787, with the south mural quadrant of Greenwich,

					Z. D.
June 4.	29.83	·55	58	⊙ L. L.	$29^0\ 15'\ 50''.3$
				⊙ U. L.	28 44 13.5

From the last of these observations the Sun's diameter, as it simply results from the difference of the two zenith distances, is $31'\ 36''.8$.

The zenith distance of an heavenly body means the zenith distance of its centre. Now the planets possess disks of sensible

magnitude. Dr. Maskelyne appears to have taken their zenith distances with the mural quadrant by making the middle horizontal wire of its telescope bisect the planet's disk. Thus we find in the Greenwich Observations of 1775,

Oct. 17	♃	*centrum*	72° 36′ 24″
Dec. 3	♄	*centrum*	31 16 11.6.

In the observations made with the present mural circle of Greenwich, the practice seems to be, to bring the upper or lower limb into contact with the middle horizontal wire, and, by means of a screw, with a graduated head, to move another wire (which always keeps a direction parallel the horizontal wire) till it comes into contact with the lower or upper illuminated part of the planet.

Thus, by the Greenwich Observations of 1813,

			N. P. D.	Diff.
July 25,	♂	L. L.	114° 17′ 6″.2	28″.6
	♂	U. L.	114 16′ 37.6	
July 29,	♀	L. L.	74 56 29.8	9.5
			74 56 20.3	
Mar. 10,	♃	L. L.	68 58 29.8	43.7
			68 57 46.1	

The construction and uses of, and the means of correcting, the Astronomical Quadrant and Transit Instrument, being now gone through, it remains to notice, briefly at least, the *Astronomical Clock*, which, in p. 47, was mentioned as one of the *Capital Instruments* of an Observatory; which, indeed, is as essential to the finding of the right ascensions of bodies as the transit instrument.

The declination of a star can be found, and in angular measure, by one instrument. The right ascension of a star, (see p. 47,) the other condition for determining its place, cannot be conveniently or correctly found in angular distance by one instrument. It is, according to the practice of modern science, conveniently found by two instruments. The transit instrument which observes the star when on the meridian, and the Astronomical Clock, which marks the time of that observation.

If the stars which appear on the concave Heavens accede to, or recede from, the meridian of a place, in consequence of the

Earth's *uniform* rotation; a clock, which is to measure such approach and recess, ought to *go* equably. A clock, then, ought to preserve its equable motion during any change in the state of the atmosphere, and during the vicissitudes of heat and cold. It is not within the plan of the present Treatise to describe the several contrivances by which ingenious artists have endeavoured to make a clock possess the above requisites. We shall confine ourselves to more simple views. We will first state the method now practised of ascertaining the equable motion of a clock, and next we will examine the reason and principle of such method.

The first point is to examine whether the clock is *adjusted to sidereal time*. The hour-hand moves through a circle of twenty-four hours. The minute and second hands mark the minutes and seconds. The second-hand moves over one of the divisions of its circle between two successive beats of the pendulum. In twenty-four hours then the pendulum makes 86400 vibrations, and the second-hand moves over as many divisions. Set the several hands to zero, or let them begin from 0^h, when a given star is bisected by the centre of the cross-wires, and if, when the star is next bisected, the hour-hand shall have made a complete circuit of twenty-four hours, and neither more nor less than a circuit, then is the clock *adjusted to sidereal time*.

But this, should it take place, is no proof of the clock's equable motion. During the twenty-four hours, the clock, from the vicissitudes of heat and cold, may have been both retarded and accelerated, whilst such circumstance would not be discovered by the above test. In the second place, the clock may go equably, although it is not adjusted to sidereal time. For instance, suppose, on the first return of the preceding star to the meridional wire of the telescope, the hour-hand to have made a complete circuit, and besides, the second-hand to have moved through three of its divisions, or that the pendulum has made 86403 vibrations. On the second return, and between the first and second, of the star, suppose the pendulum to have again made 86403 vibrations, then the index-hand of the clock, which, on the first return of the star, noted

$$0^h\ 0^m\ 3^s;$$

would, on the second, note

$$0^h\ 0^m\ 6^s;$$

and, if the like circumstance took place at the end of the third sidereal day, the clock would note

$$0^h\ 0^m\ 9^s.$$

And in this case, it is plain, the *mean gain* of the clock in a sidereal day (which gain is called its *rate*) would be three seconds. It would not, indeed, be adjusted to sidereal time, but it may, for all that appears to the contrary, have gone throughout its circuit equably. We cannot, however, presume that it has so gone; indeed, whether or not the clock be adjusted to sidereal time, we are unable, from the observations of a single star, to determine any thing relatively to the *equability* of its motion.

And indeed we should remain in the same uncertainty whatever number of stars were observed, if we merely examined whether their returns to the meridional wire were contemporaneous with the returns of the index of the clock to the same divisions of the dial-plate that marked their original departures, or happened after the same number of beats of the pendulum. It is necessary to examine the *differences* of the transits of different stars at *different* times. And if these differences should not be the same, then we must conclude the clock, at one period or another, not to have moved equably. Suppose, for instance, the clock being adjusted to sidereal time in the way above described, (namely, that its second-hand has moved through 86400 ($= 24 \times 60 \times 60$) of its divisions during two successive transits of the same star) and that we observe a star on the meridian at midnight. Suppose moreover, the clock to be then at its greatest acceleration. Another star, by the clock, passes the meridian an hour after the first; but 1^h or 15^0 cannot be the just difference of the right ascensions of the two stars; since, by the hypothesis, the clock, at the time of the star's transit, was *going* beyond its mean rate. But a star, which on a certain day is on the meridian at midnight, will, on each succeeding night, pass the meridian at a more early hour. If the cause, therefore, of the acceleration of the pendulum, should happen to depend on the hour of observation, the clock, on some night after the first, may be returning towards its mean rate; in which case, there will be fewer *beats* between the

transits of the two stars than before: in other words, the difference of their right ascensions, will not, as before, be noted by 1^h, but by some quantity less than 1^h. For instance, if the number of beats of the pendulum between the two transits should be 3599, the difference of the right ascensions as shewn by the clock, would be $0^h\ 59^m\ 59^s$. But the difference of the right ascensions of the two stars being constant, cannot be expressed both by 1^h and by $0^h\ 59^m\ 59^s$: one or other of these quantities must be wrong: or, should the clock, in the interval of the transits, not happen to be going at its mean rate, neither may be right.

From the preceding instance then, which has been imagined, we may perceive the possibility of ascertaining the equability of a clock's motion, should an observer possess no other means than his own observations. But Astronomical Science has provided, in its Catalogues of stars and its Tables, means much more simple and expeditious. A clock adjusted to sidereal time, and going equably, ought to shew between the transits of two stars an interval of time equal to that difference of their right ascensions, which Catalogues of Stars and the auxiliary Tables afford. If not adjusted to sidereal time, but going equably, it ought to note, between the transits of different stars, intervals of time proportional to the differences of their right ascensions: such right ascensions being computed from Catalogues and Tables. For instance,

	Right Ascension.	Differences.
α Serpentis. . . .	$15^h\ 35^m\ 18^s.46$	
		$8^h\ 58^m\ 11^s.14$,
Sirius	6 37 7.32	
α Arietis	1 56 55.96	4 40 11.36.

If the clock, therefore, should, between the transits of α Serpentis and of Sirius, note an interval of time equal to $8^h\ 58^m\ 8^s$, instead of $8^h\ 58^m\ 11^s.14$, it ought, on the supposition of an equable motion, to note between the transits of Sirius and of α Arietis a time equal to $4^h\ 40^m\ 11^s.36 \times \dfrac{8^h\ 58^m\ 8^s}{8^h\ 58^m\ 11^s.14}$.

The practical method of determining the clock's *daily rate*, that is, its gain or loss during two successive transits of a star, is

to subtract the mean meridional passages of certain stars on one day (as shewn by the clock) from the passages of the same stars on the next, or on some following day. The sum of the differences divided by the number of days intervening between the observations, and by the number of stars, is the clock's *mean* daily rate; to which quotient, or result, should the clock *gain*, the sign + is affixed; should it *lose*, the sign −.

Thus, by the Greenwich Observations of 1798, the mean transits of the following stars were

			Stars.
Jan. 23.	$1^h\ 56^m\ 53^s.32$	Jan. 25, $1^h\ 56^m\ 55^s.10$	α Arietis.
	5 5 55.70	5 5 57.46	Rigel.
	5 14 37.57	5 14 39.32	β Tauri.

Here the several differences are 1.78, 1.76, 1.75, their sum 5.29 divided by 2, the number of intervening days, is 2.645, and again divided by 3, the number of stars, is .881; and, since the clock gains, the mean daily rate is thus to be expressed, $+0^s.88$.

In practice, a clock is adjusted very nearly to sidereal time. Its daily gain or loss seldom exceeds three or four seconds. In computing its rate then, we need not concern ourselves with the degrees and minutes of the star's right ascension; it is sufficient to attend solely to the seconds, and to those, which, in the Registers of Observations, are inserted in a column entitled the *Reduction of the Wires*, (see p, 88.)

Thus, in the Greenwich Observations of 1816, we find

	Reduction of Wires.	Number of Days.	Daily Rate of Clock.	Names of Stars.
Aug. 6	25.58			α Orionis.
Aug. 7	56.32			α Lyræ.
Aug. 8	55.28	1	− 1.04	α Lyræ.
	23.18	2	− 1.2	α Oreonis.

The difference between the *reductions of the wires*, in the interval of two days, for *α Orionis* is 2.4, and half, that is, 1.2, is to be written − 1.2, since the clock *loses*, or its pendulum made between the transits of α Orionis, on the sixth and eighth day, only 172797.6 (=2 × 86400 − 2.4) beats.

The daily rate of the clock from two successive transits of α Lyræ is -1.04, and therefore, the mean daily rate from the two stars is $-\frac{1}{2}(2.24)$, or -1.12. It is part of the regular daily business of an Observatory to determine the rate of the clock. But the weather may prevent this practice, so that an observer, in order to determine the rate of his clock, may be obliged to compare observations distant from each other by intervals of four or five days. The greater, however, the number of intervening days, the less accurate is the method (that which has been explained and exemplified) of determining the clock's rate. Indeed, if the number of days be considerable, the method, as we will hereafter shew, is erroneous.

The *rate* of the clock being determined, there remains another point to be settled, which is the *error* of the clock dependent partly on the rate and, under certain considerations, caused entirely by it.

There are certain circumstances (circumstances of convention) that require previously to be explained, in order that we may know what the error of the clock is, or what it consists in. The position of a star (as it has been explained in p. 46.) depends, or is made to depend, on the arcs of two great circles, one measured from the pole, the other along the equator and from some point in it. The pole is not marked by any star, but is a point variable with respect to the stars, ascertainable, however, at any given period, by observation and computation. The point from which Astronomers have agreed to measure the right ascension is, like the former, variable from time to time, but capable of being ascertained at any assigned time. This point (a point of convention) is the intersection of the equator and ecliptic: it is not, and cannot be, permanently marked by any star, but still it is a determinable point. All right ascensions are to be measured from it. When such point is on the meridian, the clock, which is adjusted to sidereal time, ought to mark 0^h. The right ascension of a star passing the meridian an hour after would be 1^h; of a second star, passing $2\frac{1}{2}$ hours, $2^h\ 30^m\ 0^s$; and so on. Suppose then on Feb. 3, that the clock rightly noted, the right ascension of α Arietis, and that it was

$$1^h\ 56^m\ 55^s,$$

ten days after, on Feb. 13, if the daily rate of the clock were $+.88$, the gain of the clock would be $8^s.8$: consequently, at the passage of α Arietis over the meridian, the clock would denote

$$1^h\ 57^m\ 3^s.8,$$

and, if the right ascension of the star remained the same, the clock's *error* would be $8^s.8$. In twelve days the *rate* having increased to $1^s.02$, the clock's error would be

$$8^s.8 + 2.04, \text{ or } 10^s.84.$$

In what manner the right ascension of a star is computed will be hereafter explained. But admitting, for the present, that we are able to find it, from Catalogues and subsidiary Tables, it is easy to shew that the *error* of the clock, and the *rate* of the clock may both be found by the same process. Thus, suppose, on March 11, the *catalogued* apparent right ascension of Sirius

to be......................	6^h	37^m	$19^s.4$
whilst the clock denoted....	6	37	10.3
			9.1

The clock then would, on March 10, be absolutely too slow by $9^s.1$, or its *error* would be $9^s.1$.

Again, on March 16, let the star's apparent R. A.	6^h	37^m	$19^s.3$
the clock denoting..................	6	37	14.1
			5.2

On March 16th, then, the clock's error is $5^s.2$, too slow.

The clock's gain in five days is $9^s.1 - 5.2 = 3.9$, and consequently, (see p. 103.) its *mean daily rate*, so estimated, is

$$+\frac{1}{5}(3.9) = +.78.$$

This latter result is the true daily rate: the daily rate, estimated from the difference of the transits as shewn by the clock, would be

$$+\frac{1}{5}(3.8) = +.76.$$

Now the results for the daily rate do not agree. The question, then, is which is the right result; and this immediately leads us to the point to which, in p. 100, we promised to advert, namely, the principle and ground of the practical method of determining the *rate* of a clock.

Let the telescope be directed to a star (*α* Aquilæ for instance,) on some day, the 7th of March, and note the index of the clock when the star is bisected by the centre of the cross-wires. If the two events, the index at the same division, and the *bisection* of the star, are contemporaneous on the 8th of March, on the 9th of March, &c. the clock is said to be duly adjusted to sidereal time, and its mean motion in twenty-four hours is said to be uniform. Now this depends on the supposition, that the same absolute time is always absolved between each successive transit of a star over the meridian. And this latter supposition, the equality of time between successive transits, is founded on another, which is the uniformity of the Earth's rotation round its axis. This supposition, then, is completely compatible with the above rule. It remains now to examine, whether the time between two successive transits of the same star, depends solely on the time of the Earth's rotation, and, if it should not solely depend, whether the impeding circumstances are of magnitude sufficient to vitiate the practical rule.

If the Earth's rotation were uniform, and its axis produced were always directed to the same point of the Heavens, and if, besides, no cause, dependent on the relative position of the Earth and a star, made the latter, at one time, appear on the meridian before its real passage, at another time, after it, then would all the several intervals between the successive transits be equal. And this would also take place, if the *deranging* causes to which we have alluded, altered equably, and the same way, the star's right ascension. But, as it will be shewn in the succeeding Chapters, the deranging causes not only exist, but are variable, both as to degree and direction, in their effects. It is true their effects are very small: so small as not to be ascertainable, in the short intervals of two or three days, by our measures and reckonings. But still they exist, and become perceptible in their accumulations.

Although the Earth, then, should complete her diurnal rotations in equal portions of absolute time, it does not thence follow that a star will always return to the same point (the wire, for instance, of a fixed telescope) after equal intervals of absolute time. It may seem to do so when we compare one interval with another that succeeds it: but it may seem to do so, only because we have no means, either by our eye or our ear, of distinguishing the hundredths of a second of time.

What then shall we define a sidereal day to be? We may define it to be the portion of time between two successive transits of a star over the meridian: but then, if the preceding statements be admitted to be true, all sidereal days would not be equal. The definition, then, would not be a good one. If we define a sidereal day to be the portion of time absolved whilst the Earth makes a complete rotation round its axis, then, on the hypothesis of an uniform rotation, all sidereal days would be equal. It is no valid objection against this definition, that a sidereal day, not being identical with the interval between two successive transits of a star, and, therefore, not immediately ascertainable by observation, would thus become a quantity to be determined by calculation. A sidereal year must be so determined.

This is not the place to state the physical causes that prevent the time of the recurrence of a star to the meridional wire of a Transit from being solely dependent on the Earth's rotation: but, if we wanted a practical proof of the fact, we could easily find one in the instance of the pole star. That star is about $1^{\circ}\ 40'$ distant from the pole: but, if the times of the transits of stars over the meridian arose solely from the Earth's uniform rotation round a fixed axis, the several intervals between the successive transits of the same star would all be exactly equal, wherever that star were situated, whether near the equator or near the pole. In such case, if on the first of next January, (1822), *Polaris* should be (as he will be) on the meridian at $0^{h}\ 57^{m}\ 20^{s}.3$ of sidereal time, he ought to be again there on January 2, at the same sidereal time; whereas, on this latter day, the time of the transit will be, nearly,

$$0^{h}\ 57^{m}\ 19^{s}.7,$$

and the succeeding day, January 3, at

$$0^h\ 57^m\ 19^s,$$

and the apparent motion of Polaris will so increase that, after ten days, he will be on the meridian at

$$0^h\ 57^m\ 13^s.3,$$

and on January 20th, at

$$0^h\ 57^m\ 6^s.3,$$

the apparent mean daily acceleration of the star being, during the above period, about $\frac{7}{10}$ths of a second.

In the above case, the real differences of the intervals of successive transits become discernible from the peculiar situation of the star. But, with other stars, the case is different. The star *Procyon* (the lesser Dog Star), for instance, which is near to the equator, will be on the meridian, at the latter period, (January 20, 1822), at

$$7^h\ 50^m\ 0^s.9,$$

and the real differences, between the intervals of its transits for the next twenty days, are so minute as completely to baffle detection, with whatever instrument the eye and ear be assisted. The same circumstance takes place, very nearly, with other stars that are not near the pole. It takes place with all those stars which are used in determining the clock's daily rate. With stars, then, such as the last, the rule for finding the clock's daily rate, from the difference of two successive transits, is sufficiently exact for all practical purposes. It can never, so applied, lead into error; which it would do, were Polaris the star. The latter star may indeed be used for finding either the clock's error, or the clock's rate, but then we must have recourse to operations less simple than those of merely noting the times of its transits *.

* We have, on the preceding subjects, somewhat dilated, and been digressive. But the subjects are those on which students (we are speaking in general terms) have no precise notions, nor, through books in ordinary use, any means of acquiring such.

It

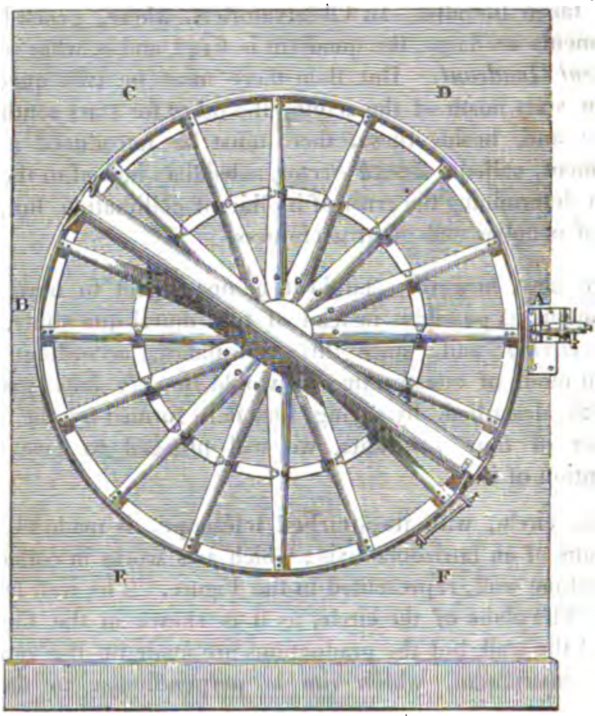
The three *capital* instruments of an Observatory, it has been said, are the quadrant, the clock, and the Transit. But this is not to be taken literally. In Observatories, where, generally, the instruments are large, the quadrant is fixed, and is, what is called, a *Mural Quadrant*. But then there must be two quadrants; one for stars north of the zenith, the other for stars south of the zenith; and, beside these, there must be introduced a fourth instrument, called a *Zenith Sector*, subsidiary indeed to the quadrant in determining the error of its line of collimation, but, moreover, of peculiar and great usefulness.

We may, however, should it be our object to have as few instruments as possible, instead of two mural quadrants, use a *mural circle*; and, since this instrument, according to the present mode of constructing it, would be very loosely and imperfectly described, by saying, that it is formed by the putting together of four quadrants, we will proceed to give a brief description of it.

The circle, with its attached telescope, is made to revolve by means of an horizontal axis; which axis works in collars fixed in the stone wall, represented in the Figure. The wall faces the east. The plane of the circle, as it is shewn in the Figure, is parallel the wall, but the graduations are made on the outer rim of the instrument, which rim is perpendicular to the wall.

It has been said, that art and science render each other mutual assistance, and are contemporaneously progressive. In the subject which has been under discussion, namely, that of the instrumental means of measuring time, a refined state of science is absolutely necessary to enable us to pronounce on the quality of such means. If the antients had invented exact time-keepers, could they have verified their exactness? Suppose, for instance, a Watchmaker of Alexandria had constructed a perfect clock, the Astronomer of Alexandria would have found it faulty, since the clock would have indicated an inequality in the revolution of the *primum mobile*. There seem to be no other means than Astronomical ones of verifying time-keepers; and these means, if they are to be exact, cannot be made so, except with great difficulty, nor without the results and formulæ of refined science.

These graduations are viewed and *read off*, by six microscopes fixed to the wall, one of which microscopes is represented at *A*,



and the places of the five others (precisely similar to the former) are marked by the letters *B, C, D, E, F*. The microscopes are distant from each other sixty degrees, or so placed, as nearly as can be, by the instrument-maker.

The circle's diameter is six feet. Its rim is divided into equal parts of five minutes each, and the *readings off* to a less number of minutes and to single seconds, are effected by the *Micrometer Microscopes, A, B*, &c. The construction of which is as follows. The microscope *A*, or micrometer microscope *A* is directed, as it is shewn in the Figure, to the rim on which the graduations are made. Consider the *object* to the microscope to be one graduation of the instrument, or the space occupied by five minutes. The image of this space will be formed in the conjugate focus of the object-glass, and will be seen distinctly

through the eye-glass of the microscope, when the above-mentioned image is in its focus. In this latter focus (the focus of the eye-glass) are placed, a thin indented slip of metal and a wire * capable of being moved, in a parallel direction, from one mark of division to another by means of a screw. The revolutions of the screw, and parts of its revolution, are noted by means of a screw-head aud graduated plate, similar in the principle of its construction to the one of p. 55. Now it is desirable, for the more convenient noting of the results of observations, that, by five revolutions of the screw, the wire should be translated through the space occupied by five minutes: in which case, one revolution would answer to one minute, and one-sixtieth to one second. The mode of effecting this may be thus explained. Suppose, the object-glass of the microscope being at a certain distance from the graduated rim, and there being distinct vision, that the moveable wire appears to be translated through the five minutes, by $5\frac{1}{2}$ revolutions of the screw. In such case, the image of the five minutes is too small. It will be increased by moving the object-glass towards the graduated rim. But, if the whole microscope be moved, there will no longer be distinct vision, since the object being nearer to the object-glass, its image will be formed at a greater distance from the object-glass, and beyond the focus of the eye-glass. The eye-glass, therefore, with its wire, &c. must, by a separate movement, be withdrawn from the object-glass till distinct vision ensues. In this second position, a second trial must be made to ascertain whether five revolutions of the screw are equal, or not, to the translation of the wire over the image of that portion of the divided limb which contains five minutes. Should there be no equality, the adjustments must be made both of the object-glass and of the eye-glass, by their peculiar movements, till five revolutions of the screw shall correspond to the translation of the wire over five minutes.

* Instead of one wire moveable, in a direction parallel to the marks of graduation, two wires crossing each other, at an acute angle, are substituted. These wires, in measuring the distance from the index to a graduation, are to be stopped when the mark of the graduation bisects the angle of their intersection.

The adjustment, which we have described, is merely a matter of convenience: it saves the observer the trouble of reducing the graduations of the screw-head to their values in minutes and seconds. If the microscope micrometer were suffered to remain in its first state, then, since 5.5 revolutions $=5'$, one revolution would equal $50''.454$, &c.

But, whatever be the value of a revolution, the uses of the moveable wire and the indented slip of brass are the same. A star is observed on the centre of the cross-wires of the telescope. On looking through the microscope, the index, or what serves as one in the slip of brass, occupies a place between two graduations. The wire moved from the index, either to the upper or lower graduation, measures by the revolutions of the screw-head, the distance from the mark of graduation: and, for convenience, each tooth of the indented brass answers (one revolution of the screw being equal to one minute) to one minute: so that, if the wire is moved from the index past two teeth, and the index of the screw-head points to 55, then $2'\ 55''$ are to be added to or subtracted * from the degrees and minutes which are read off by the naked eye, or without the aid of the micrometer microscope.

In every observation all the six microscopes are to be used for the purpose of diminishing the errors of division, and the effects of partial expansion.

In *reading off* the angles at the several microscopes, we need only to attend to the seconds; which may be thus explained. Suppose a star to be in the pole and that the telescope is to be directed to it. The whole circle then must be turned round in the direction from *B* towards *C*, *D*, &c. and the end of the telescope containing the object-glass, instead of being directed as it is in the Figure, to a point in the south, between *B* and *C*, will be directed to a point between *D* and *A*. If, the telescope being directed to the pole, the *reading off* at the micrometer at *A* were $0^0\ 0'\ 0''$, the *Index error*, as it is called, would be 0. The *readings off* at the other microscopes *F*, *E*, *B*, *C*, *D*, (were those microscopes placed at exactly equal distances from each

---

* Accordingly, as the distance of the index from the upper or lower graduation is measured.

other) would be 60°, 120^{0}, 180^{0}, 240^{0}, 300^{0}. But these circumstances are not likely to take place. The *index error* will probably be of some magnitude: a few seconds, for instance: that is, when the telescope is directed to the pole, the *reading off* at the microscope *A*, instead of being $0^{\circ}\ 0'\ 0''$, may be $\pm 5''$, or $\pm 7''$, or $\pm 8''$, &c. In like manner, the *readings off* at *F, E, B, C, D*, may be, from their not being placed at exactly equal distances, or from inequality of graduation, or from partial expansion, or conjointly from all these causes (for in practice they may all operate) either

$$
\begin{array}{ll}
60^{0}\ 0'\ 7'', \text{ or } & 60^{0}\ 0'\ 10'', \text{ or \&c.} \\
120^{\circ}\ 0'\ 8'', \text{ or } & 120^{\circ}\ 0'\ 12'', \text{ or \&c.} \\
\text{\&c.} & \text{\&c.}
\end{array}
$$

Suppose, independently of the degrees and minutes, the seconds at the six microscopes to be respectively,

$$+5'',\ +7'',\ +4'',\ +12'',\ +8'',\ +9'';$$

then these are the several *index errors:* and, if the polar distance of an observed star were *read off* only at one microscope, the index error belonging to such microscope must be added to, or subtracted from, the distance so read off. Thus, if the microscope *B* were only used, the index error of which is $+12''$, and the north polar distance of *β Ursæ Minoris read off* were $195^{0}\ 4'\ 46''$, then, deducting 180^{0} for the position of the microscope, and $12''$ for the index error, we should have

$$\text{the north polar of } \beta \text{ Ursæ Minoris} \ldots\ldots = 15^{0}\ 4'\ 34''.$$

But, all the six microscopes being used, it is convenient to consider a *mean* index error, which will be one-sixth of the several index errors, and, which, in the preceding instance (see l. 16.) will be $\frac{45''}{6}$, or $7''.5$.

We have in the preceding illustration, for the sake of simplicity, supposed the telescope to be directed to the pole, which, as it has been several times stated, is not marked by any star, but is a point to be assigned by calculation and angular measurement. But the illustration will be, in substance, the same if we suppose the telescope directed to a known star, *Polaris*, for

instance. If, by previous Catalogues and Tables, we should know the north polar distance of this star to be $1^\circ\ 41'\ 41''.3$, the micrometer microscope A marking $1^\circ\ 41'\ 48''.5$; then the index error would be $+7''.2$*, and, in like manner, we should know, by the same star, the index errors for the other microscopes, and thence the *mean index error*.

We shall, in another part of the Work, explain the use of the observations made with this instrument, and of the index error, in correcting the catalogues of polar distances. At present we shall be content in shewing, by a kind of exemplification, that the uses of the instrument do not depend on the accurate positions of the several microscopes.

Suppose, the telescope being directed to the pole, the number of seconds indicated by the micrometer microscope A to be 7.

Let B indicate $b+23''$ (b, c, d, &c. denoting degrees and minutes)
C .... $c+\ 4$
D .... $d+\ 5$
E .... $e+\ 9$
F .... $f+15$

Let X be the north polar distance of any star, (of Capella, for instance, X being $=44^\circ\ 12'\ 16''$), and let the number of seconds in X be 16, so that, Y being the degrees and minutes, $X=Y+16''$; then, the instrument being directed to Capella, (and, consequently, turned round through an angle X) and the errors of division, expansion, and the uncertainty of the reading off not being considered, the number of seconds in A, will be 23,

in B 39,
in C 20,
in D 21,
in E 25,
in F 31,

the sum of these is 159, and one sixth is $26''.5$; the north polar distance, therefore, of Capella by the instrument, and, by the above method of taking the mean of the seconds, is

$$Y+26''.5\quad(=44^\circ\ 12'\ 26''.5),$$

* Therefore, the *equation* for the north polar distance is $-7''.2$.

and, consequently, the mean index error

$$Y + 26''.5 - X, \text{ or } Y + 26''.5 - (Y + 16''), \text{ or } 10''.5.$$

This is the index error from one star; but the process is the same with any other star, since X may be any angle. If the catalogues were exact, and there existed no source of error from inequality of graduation, &c. the same index error would result whatever star were observed. Thus, suppose the number of seconds in X, instead of 16, to be 36, then the number of seconds from the six microscopes instead of being 159 would be $159+6 \times 20$, and consequently, the mean number would be

$$26''.5 + 20 = 46''.5,$$

and in this case the index error would be

$$Y + 46''.5 - (P + 36) = 10''.5.$$

But neither are the catalogues of stars perfect, nor is the instrument altogether exempt from the errors of graduation, and of partial expansion. It will, therefore, happen in practice, that the index error is different with different stars. If the index error resulting from the observations of twelve stars, should be respectively,

$$\begin{array}{cccccc} 10''.5, & 9''.3, & 6''.8, & 13''.1, & 11''.2, & 9''.1, \\ 8.4, & 13.2, & 8.5, & 10.2, & 7.9, & 8.7, \end{array}$$

the sum being $116''.9$, the mean would be $\frac{116.9}{12} = 9''.74$.

This is not the place to enter more fully into the special uses of the instrument. We will, however, give a specimen of the method of registering the *readings off* by the six microscopes.

Oct. 15, 1812. Position of the telescope 0°.

Bar.	Therm. In.	Therm. Out.	Names of Stars.	Deg. & Min.	Microscopes. A.	B.	C.	D.	E.	F.	Mean.	N. P. D.
29.12	51	53	γ Drac.	38° 28′	40″.2	44″.5	46″.0	41″.2	42″.5	41″.4	42″.6	38° 28′ 42″.6
		49	α Lyræ.	51 22	28.1	30.0	33.5	28.0	27.2	29.8	29.4	51 22 29.4
29.13	50	47	α Aquilæ.	81 35	56.5	58.0	1.4	57.2	57.4	58.6	58.2	81 35 58.2

The sum of the seconds, belonging to the six microscopes, is, in the first row, equal to 255.8; one-sixth of which is 42.6, the *mean*. The sum in the second row, is 176.6: one-sixth of which (as far as one decimal place) is 29.4 the *mean*. The sum in the third row is 289.1; divide by 6, and the quotient is nearly 48.19; but, it is clear, it ought to be 58.19. Now if we look to the number of seconds under C, which are 1.4, it is obvious that if we attended solely to that microscope, the number of minutes instead of being 35, would be 36, or the north polar distance of α Aquilæ would be $81^0\ 36'\ 1''.4$; but, as it is clear, from the number of seconds belonging to the other microscopes, that the mean number of minutes cannot exceed 35, we must, in taking the mean of the seconds, consider $81^0\ 36'\ 1''.4$, as $81^0\ 35'\ 61''.4$, or we must add 60 to the seconds added together in the usual way, or, which is the more simple way, we must add 10 ($=\frac{1}{6}60$) to one-sixth of the former result; in which case, the *mean* becomes 58.19, or nearly 58.2. In like manner, we must treat other like cases, should they occur: which, it is plain, can be but seldom. In some cases it may be necessary to add 120 to the sum of the seconds: for instance, if the several seconds were

57.1, 59,5, 1.9, 57.8, 57.8, 57.9, 1.1,

their sum is 235.3, add 120, and the mean is $\dfrac{355.3}{6} = 59.2$, or, by the former rule, (see l. 15,)

$$\frac{1}{6}(235.3) + 20 = 59.2.$$

At the head of the preceding Table of results, (see p. 116,) is written, 'Position of the telescope 0^0.' For the purpose of still farther lessening the errors of division, the telescope can be placed in several positions. When it is at the position 0^0, the telescope is directed to the pole, and the microscope A, which is the reading microscope, marks 0^0: and it is at the positions 10^0, 20^0, 30^0, when, the telescope, in each case, being pointed to the pole, the microscope A marks 10^0, 20^0, 30^0, respectively.

The mural circle, like the transit instrument, requires three adjustments. 1st, Its axis must be made horizontal. 2dly, Its

line of collimation must be made perpendicular to the horizontal axis. 3dly, The line of collimation must be made to move in the plane of the meridian.

A simple mechanical contrivance exists for carrying the first of the adjustments into complete effect. When the axis is made horizontal, the line of collimation describes a vertical circle: but it may describe a *small* vertical circle. To make it necessarily describe a great vertical circle, and a meridional circle, there are no mechanical means. Astronomical ones must be resorted to: and even with those, the two latter corrections are not accomplished without great difficulty. We may, on this occasion, use (as it was stated in p. 70,) the transit instrument. When a star is on the meridional wire of the transit instrument, so move the mural circle that the star may be on its middle wire. Next, observe by the transit instrument when a star, on, or very near to, the zenith, crosses the meridian: if, at that time, the star is on the middle vertical wire of the telescope of the mural circle, then its line of collimation is rightly adjusted. If the star is on the middle wires of the two telescopes at different times, note their difference and adjust accordingly*.

The great difficulties attending the verification of the line of collimation of the mural circle, will always prevent its becoming a good transit instrument. It acts, however, better in this last office than the telescope of the mural quadrant, which slides along the limb of the quadrant, the plane of which cannot be made to be wholly in the plane of the meridian.

The mural circle is sufficient, as it is plain from its description, to determine, to the extent of 180 degrees, the differences of the declinations of stars that are to the south and the north of the zenith of the observer. There must be *two* quadrants to effect the same object. Besides this advantage (the advantage of a *single* instrument) the circle is better balanced, and its six microscopes, which are firmly fixed in a stone wall, together with the power of changing the position of its telescope (see p. 116,)

* This adjustment must be conducted by some formula which expresses the relation between the difference of the times, and the inclination of the line of collimation to the plane of the meridian.

must, when we take the mean results of a great number of observations, do away with, or, at the least, very considerably lessen the errors of division and of partial expansion.

But, it may be said, there being no plumb-line to mark the zenith point, the mural circle is defective inasmuch as it does not determine the zenith distances of stars: which distances are necessary to be known, if we would determine the refraction. The direct and special office of the mural circle is to determine the angular meridional distances of stars. If we extend the principle of its uses, and view the image of the pole star by reflection from a basin of quicksilver, we obtain the angular distance between the star and its image. Such angular distance is twice the elevation of the pole star above the horizon. Hence its zenith distance becomes known, and the zenith distances of other stars; the meridional angular distances of which, from the pole star, are determined by the instrument.

Since we can make observations, like the preceding, of the pole star both in its superior and inferior passage, we can thence determine (on an assumed law and quantity of refraction) the height of the pole itself above the horizon, which height (see p. 10.) equals the latitude of the place of observation.

We cannot with the mural quadrant view the reflected image of the pole star; nor can we at once, even if we use a plumb-line, determine by it the zenith distances of stars. These distances can only be truly known by knowing the error of collimation. The instrument *of itself* is unable to determine that error, and, in aid of its deficiences, we are obliged to have recourse (see pp. 67, &c.) to a *zenith sector*.

This latter instrument, by double observations of a star near the zenith, one set being made, with the face of the instrument towards the east, the other with the face towards the west, determines the star's *true* zenith distance (see pp. 63, 67, 68, &c.) γ Draconis is the star that has been most frequently observed at Greenwich. If we observe, on any particular day, either with the mural circle or mural quadrant, that star and other stars, we obtain their meridional angular distances, or the *differences* of their north polar distances. Hence, the zenith distance of γ Draconis being determined by the zenith sector, the zenith distances of the above observed stars become known.

Thus, suppose by observations in 1812, with the north mural quadrant, that the zenith distances of γ Draconis, β Ursæ Minoris, α Cassiopeæ appeared to be, respectively,

$$2'\ 20''.5, \quad 23^0\ 26'\ 49''.27, \quad 4^\circ\ 1'\ 41''.18\,;$$

but, with the zenith sector, the true zenith distance of γ Draconis appeared to be

$$2'\ 18''.5,$$

the true zenith distances of β Ursæ Minoris, and α Cassiopeæ, consequently, were

$$23^\circ\ 26'\ 47''.27, \text{ and } 4^\circ\ 1'\ 39''.18.$$

At the time the *instrumental* zenith distances are read off, the quadrant is adjusted to a certain position, by making the plumb-line (see the figures of pp. 59, 60.) pass over the two crosses that are on the face of the instrument. It is the office of this plumb-line to keep the quadrant in a given position; to be so kept, in order to use observations made of stars when we are unable to observe γ Draconis. The error of the line of collimation is presumed to be the same when the quadrant is adjusted by making the plumb-line pass over the two crosses.

But, it is plain, the zenith sector may be used as an auxiliary instrument to the mural circle as well as to the quadrant, and we may determine by their means the latitude of the place of the observation, and the zenith distances of stars. Thus, by the mean of a great number of observations made in 1812, at Greenwich, with the zenith sector, the zenith distance* of γ Draconis was found to be

$$0^0\ 2'\ 18''.5 \ldots\ldots = Z\gamma, \text{ see the Figure in the next page.}$$

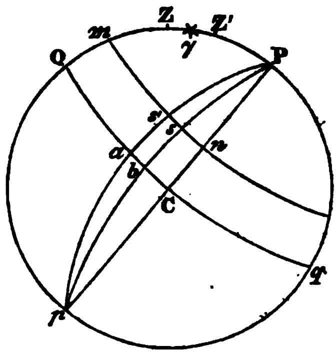
The north polar distance of the same star, found by the mural circle, and reduced to the same period, was equal to

$$38^0\ 29'\ 3'' \ldots\ldots = P\gamma\,;$$

$$\therefore\ ZP = Z\gamma + P\gamma = 38^0\ 31'\ 21''.5,$$

* The distance *reduced* to January 1812. The meaning of this phrase will be explained in the following Chapters.

the co-latitude (see pp. 9, 10, &c.) of Greenwich. Again, for



all stars north of the zenith, and between Z and P,

$$Z* = ZP - P*,$$

for stars south of the zenith,

$$Z* = P* - ZP,$$

from which formulæ, $Z*$ may be found, $P*$ being determined by the mural circle.

We may use then the zenith sector with the mural circle to determine the error of the line of collimation in the latter, and thence to determine the zenith distances of stars. But, if we observe stars by reflection, we may with the mural circle, and without the aid of another instrument, determine the latitude of the place of observation, and the zenith distances of stars. The peculiar office, however, of the mural circle is to determine the angular distances of those points at which the several stars pass the meridian.

These distances are used in correcting the existing catalogues of stars, and in determining, to greater degrees of exactness, their north polar distances. In this use the mural circle need not, like the quadrant, be brought by a plumb-line, or other means, to a given position. That operation is superseded by ascertaining the value of the *index error*. Thus, if at a certain period the *instrumental* polar distance of β Ursæ Minoris appeared to be

	15°	$4'$	$33''.04$		
and by the catalogue.	15	4	34.23		$-1''.19$,

$-1''.19$ would be the index error by that star; the *mean* index error is the sum of the several index errors divided by their number. If the position of the telescope be changed, or if the same number of microscopes (see p. 113,) be not used, the index error will be different: but whatever it is, it stands in lieu of, if we may so express ourselves, a *mechanical* adjustment of the instrument.

But we may use in the same way, and on the same principle, the two fixed mural quadrants. With the north mural quadrant we can observe (supposing Greenwich to be the place of observation) γ Draconis and other stars to the north of the zenith. With the south mural quadrant, were its limb an exact quadrant, we should be unable to observe γ Draconis: but (see pp. 59, 60, 64.) the limb being extended a little beyond the limits of an exact quadrant, we are enabled to observe γ Draconis: we can also observe with it (for this indeed, is its use) stars to the south of the zenith. By connecting, therefore, the two sets of observations, by means of the intermediate and common star γ Draconis, we can, without the plumb-line, determine the meridional angular distances of all stars visible at Greenwich. We may also, as with the mural circle, determine their north polar distances by the aid of catalogues, and the use of an index error *.

* It appears from the preceding matter, that neither mural quadrants nor mural circles are perfect instruments. The directions of their lines of collimation cannot be found without a zenith sector. Quadrants and circles with azimuth motions resemble that latter instrument, and are all capable of determining the directions of their lines of collimation, or of making observations independent of the errors of collimation. In principle then they are much more perfect instruments than fixed quadrants and circles. But large instruments are absolutely necessary in the present state of Astronomical Science, and for its future advancement, and it is difficult to construct large instruments capable of being turned half way round in azimuth on a vertical axis. Yet Ramsden constructed for the Dublin Observatory a circle of eight feet diameter turning round a vertical axis; and it seems natural to presume that such an instrument must have been defective, since, of late years, its construction has been abandoned,

We have in the preceding pages given a description of the *capital* Instruments of an Observatory, and which are used for the making of observations in the meridian. On such obser-

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abandoned, and fixed mural circles invented. But, theoretically viewed, there seem eminent advantages attached to the former instrument. Within the space of a few minutes it is capable of making a double observation on a star, one with its face towards the east, the other towards the west; the first before the star is on the meridian, the other after. Both observations must be *reduced* to the meridian by computation from the intervals of the times at which they were made, and the passage of the star over the meridian: which intervals may be most exactly known from the transit telescope and the Astronomical clock. The verticality of the axis, at each observation, is verified by a plumb-line. It may in practice be difficult to make these observations, but they have a singular and vast advantage in being free of all index error, and in determining, simply and directly, and within a short period of time, the zenith distance of a star. The index error of the mural circle, as it is proposed to be found, is a complex quantity, neither admitting of a brief definition, nor to be found by a single and simple process.

Small zenith sectors have an azimuthal motion round a vertical axis. The *reversion* of the face of the Greenwich zenith sector is obtained by moving the instrument from an eastern to a western wall. This is an operation not easily performed. Mr. Troughton now proposes to construct a zenith sector (or an instrument for like purposes) of twenty-five feet radius, and capable of being turned round a vertical axis. Its *range* will be small, not exceeding five minutes on each side of the zenith: it is specially designed for observations of γ Draconis which is distant, less than three minutes, from the zenith of Greenwich.

The observations to be made with this instrument will be nearly free of all inequality from refraction, and entirely free from index errors; they will also from the great length and power of the telescope be, it is probable, very exact, and will serve to determine, to a greater degree of exactness than has hitherto been done, the quantities of aberration and nutation. They may also settle the question, now agitated, of the existence and quantity of parallax.

The detection of this latter *inequality*, it may be here stated, has been made by an instrument, revolving, like the instrument just described, round

vations Astronomical Science is mainly founded. We must resort to the same source whether we seek for exact data to institute processes in Physical Astronomy, or to confirm their results.

Of the other Astronomical instruments there are some from which we derive neither the elements of Astronomical Science, nor the verification of the results of its processes: but which are employed in a practical application of those results. Of such character are, Hadley's Quadrant, the Sextant and Reflecting Circle, instruments of the same class and principle of construction. These are not instruments belonging to an observatory; but the equatoreal instrument does belong. Its use is to observe phenomena, such as comets, and new planets, when they are out of the meridian. Besides these instruments, there is a repeating circle, portable, and principally useful in determining the latitudes of different stations, and their *bearings* with respect to each other. An equal *altitude and azimuth instrument* of which, amongst others, one use is to ascertain the quantity and law of refraction. Some of these instruments will be briefly described in a future Chapter of this Work.

From the description of the construction and uses of instruments, we will proceed to consider the results of certain observations made with them: and the first observations that claim our attention are those made on the Sun at the time of his passing the meridian.

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round a fixed axis: or rather the proof of its existence depends on the accuracy of the Dublin circle (see p. 121.) The observations made with the mural circle of Greenwich do not verify such parallax. On this discordance of the two instruments, much controversy has arisen which is not yet settled; and, whatever be the excellence of the latter instrument, yet it must be allowed that the method of determining its *index error* involves many *debateable* points.

CHAP. VI.

Sun's Motion—Path—Ecliptic—Obliquity of Ecliptic.

By means of the Astronomical Quadrant, or of the circle, or by any instrument of a like class, we are able to observe the height either of the Sun's upper or his lower limb, and thence, of determining, by measurement and computation, the height of his centre. Let us first examine the observations of the north polar distances (N. P. D.) of the Sun's limbs,

		N. P. D.	Difference in two Days.
Jan. 2, 1816,	⊙ U. L.	$112^{\circ}\ 40'\ 22''.5$	
3, ...	⊙ L. L.	113 7 37.5	
4, ...	⊙ U. L.	112 29 21.7	$11'\ 0''.8$.

Again,

		N. P. D.	Difference in two Days.
March 31, 1816,	⊙ U. L.	$85^{\circ}\ 29'\ 26''.2$	
April 1, ...	⊙ L. L.	85 38 22.6	
April 2, ...	⊙ U. L.	84 43 27.5	$45'\ 58''.7$.

From these observations two inferences may be drawn: the first is, that the Sun, in the interval between January 2, and April 2, has approached the north pole by an angular ascent equal $27^{\circ}\ 56'\ 55''$ ($= 112^{\circ}\ 40'\ 22''.5 - 84^{\circ}\ 43'\ 27''.5$). The second is, that his daily portions of ascent are not equal: since, between January 2 and January 4, he ascended through $11'\ 0''.8$, and between March 31 and April 2, through $45'\ 58''.7$. So that the Sun's daily meridional ascent, or change of declination, or change of north polar distance (the fact is the same, but under different denominations) in the former period was about one-fourth of what it was in the latter.

We shall have like facts, and may make like inferences, in

the heights of the Sun's centre observed at Cambridge, in the months of January March and June of the year 1810.

	Altitudes.	Differences.
1810, Jan. 1.	14° 44′ 40″	5′ 4″
2.	14 49 44	5 31
3.	14 55 15	5 58
4.	15 1 13	

Therefore, the Sun, during these four days, was ascending in the meridian, but not by equal increases of altitudes, as it appears by the column of differences. Again, the altitudes of the Sun on four successive days in March and June, were

	Altitudes.	Diff.		Altitudes.	Diff.
Mar. 19	37° 5′ 46″	23′ 41″	June 20	61° 14′ 32″	29″
20	37 29 27	23 41	21	61 15 1	4
21	37 53 8	23 39	22	61 15 5	—21
22	38 16 47		23	61 14 44	

During the four days in March, then, the Sun was continually ascending, and by increments of ascent very nearly equal: in June he was still higher, and on the twenty-second at his greatest altitude; for, on the succeeding day, his altitude was diminished by twenty-one seconds. The increments of altitude, as appears by the column of differences, are, like those in January, unequal.

Thus far it appears then; the phenomenon of the Sun's continually varying altitude cannot be accounted for, by supposing the Sun to have an equable motion in the meridian; ascending for half the year from December 1809 to June 22, 1810. and then descending: let us next consider whether an explanation of other phenomena attending the Sun's transits over the meridian can be obtained, from attributing to the Sun an *unequal* motion in the direction of the meridian and merely in that direction *.

* In proving the Sun's motion in the meridian not uniform, we have supposed, what is not strictly true, the intervals between his successive transits over the meridian to be equal. But the result will be the same, that is, his motion will be found to be unequal, if we correct the supposition, and allow for the inequalities between successive transits.

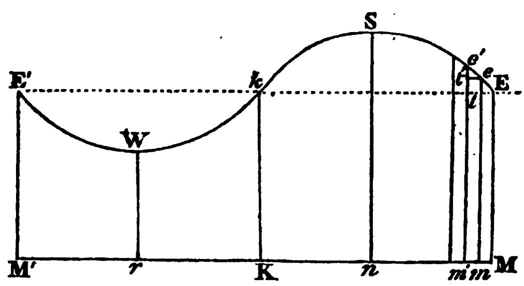
If the Sun had a motion merely in the meridian, then, since the Earth's rotation is supposed to be equable, the intervals of successive transits over the meridian would always be equal, one with another and, besides, would be equal to the intervals of the transits of a fixed star; or, of a star having neither a motion in the meridian nor one transversely to it. Now, neither of these conditions takes place; for on Aug. 21, 1810, *Regulus* was on the meridian 1 minute 20 seconds before the Sun: on the succeeding day 5 minutes 2 seconds: on the next, or twenty-third, 8 minutes 44 seconds: so that it is plain, during these intervals, the Sun must have shifted away from the meridian, and moved transversely towards the east of it. Hence, to account for the phenomena of the Sun on the meridian, namely, the changes there both in the places and in the times of his transits, two motions must be attributed to the Sun, one in the plane of the meridian, the other transverse to it; which two motions, (according to the doctrine of the composition of motion) are equivalent to, or may be compounded into, one single oblique motion.

From the preceding instance it appears, that the Sun moves to the east of the meridian, and of a fixed star (*Regulus*), through an angle which, in time, is equal to 3 minutes 42 seconds: but this angle is not constant: if, for instance, one of the stars of *Sagittarius* was, with the Sun, on the meridian, January, 2, 1810, the next day the Sun would come later, than the star, to the meridian, by 4 minutes 24 seconds; on January 4th, later by 8 minutes 48 seconds; on the 5th, by 13 minutes 12 seconds, &c. Hence, neither is the Sun's motion perpendicular to the meridian equable, nor, as it has appeared, his motion in the meridian. These two may be considered as the two resolved parts of the Sun's oblique motion.

The following Table exhibits the Sun's meridian heights on the 22d days of the several months of the year 1810.

January.	*February.*	*March.*	*April.*	
18° 2′ 7″	23° 55′ 19″	38° 16′ 47″	39° 17′ 28″	
May.	*June.*	*July.*	*August.*	
41° 32′ 27″	61° 15′ 5″	58° 10′ 54″	49° 44′ 19″	
September.	*October.*	*November.*	*December.*	*January,* 1811
38° 16′ 23″	26° 51′ 49″	17° 42′ 42″	14° 19′ 42″	17° 58′ 47″

From this Table we may determine, in a general way, the form of the curve in which the Sun may be supposed to move. For, if MM' be taken to represent the whole space from March 1810 to March 1811, and perpendiculars be erected respectively equal to the Sun's meridian altitudes on the several intervening days, the curve drawn through their extremities will



be $ESWE'$. If E be the Sun's place on March 20, e, e' on the two successive days, then ME, me, $m'e'$ must be taken respectively proportional to 37° 29′ 27″, 37° 53′ 8″, 38° 16′ 47″ (see p. 125.) The intervals Mm, mm', &c. are not exactly equal, since they are the spaces through which the Sun retires each

day, from his place on the meridian the preceding day [see Note, p. 125.]: and in the present case they are respectively equal, in degrees, &c. to $54' \, 33''$, $54' \, 31''.5$, $54' \, 31''$.

The spaces te, $t'e'$, &c., or the increments of the Sun's altitude in the meridian, are respectively equal to

$$23' \, 41'', \quad 23' \, 39'';$$

and the motions, in these directions, combined with the transverse motions in the directions Et, et, compound, as has been before remarked (p. 126,) the oblique motions Ee, ee', &c.

In the Figure ESE', there are two altitudes nS, rW, one the greatest, the other the least, which for the year 1810, (see p. 125,) would happen on June 22, and December 22d; and the mean of these two altitudes is

$$\tfrac{1}{2}\{(61^{\circ} \; 15' \; 5'') + (14^{\circ} \; 19' \; 42'')\} = 37^{\circ} \; 57' \; 23''.5,$$

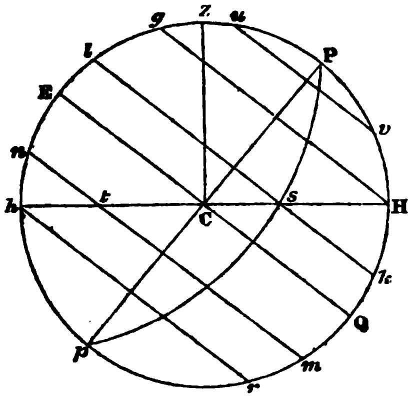
which is, very nearly, the Sun's altitude (ME) on March 21, or Kk his altitude, Sept. 22 *.

Now when the Sun is at these mean heights ME, Kk, he is in the equator. If, therefore, we knew when the Sun was in the equator, we could, by then observing the altitude of his centre determine that of the equator, which altitude (see p. 10.) is the co-latitude of the place of observation. Contrarywise, if, by observations other than those of the Sun, we determine the latitude of the place of observation, we are thence enabled to ascertain when the Sun is in the equator; which must happen when his zenith distance is equal to the latitude.

In pages 10 and 11, &c. we have given some instances of the method of determining the *differences* of the latitudes of places. But the latitude itself may be found, from the greatest and least altitudes of a circumpolar star above the horizon. Thus, if Hu, Hv denote those altitudes of a star, the parallel of which is vu,

* The greatest and least altitudes (nS, rW) are supposed to happen on the *noons* of June, and of Dec. 22; which is not exactly true, as it will be hereafter shewn.

the latitude $= PH = Hv + Pv$



$$= Hv + Pu$$
$$= Hv + Hu - HP$$
$$= Hv + Hu - \text{latitude}$$

consequently, the latitude $= \frac{1}{2}(Hv + Hu)$,

and the co-latitude equals half the sum of the greatest and least zenith distances. Thus, by observations made at Blackheath,

corr^{d}. least zen. dist. *Ursa min.* Bod. 4......	37^{0} $35'$ $55''$
corr^{d}. greatest zen. dist..................	39 27 57
	$\frac{1}{2}$)77 3 52
co-latitude of Observatory..........	38 31 56

Again,

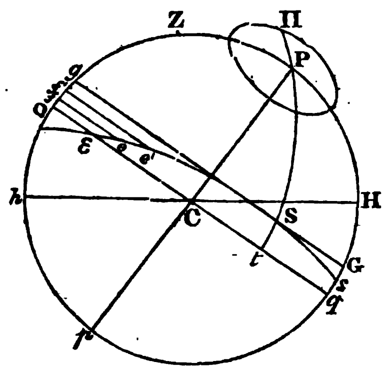
corr^{d}. least zen. dist. *o Cephei*.............	15^{0} $35'$ $21''$
corr^{d}. greatest zen. dist...................	61 28 31
	$\frac{1}{2}$)77 3 52
co-latitude........	38 31 56

By these means we should be able to recognise that twice in a year, in March and September, the Sun was in the equator. But, if the latitude were determined accurately, we should find no meridian altitude of the Sun to be exactly equal to the co-latitude: for instance, in the former cases, (pp. 125, &c.) the latitude

of the place of observation being supposed, by observations of the pole star, to be 52° 12′ 36″, its co-latitude is, consequently, 37° 47′ 24″. Now, amongst the altitudes stated in p. 125, there is no one exactly equal to 37° 47′ 24″: the altitude on March 20th is too small, that on the 21st too large: the reason of this is that, when the Sun was exactly in the equator, he was not on the meridian of the observer's station. There is some place to the east of Cambridge, at which the Sun was on the meridian when in the equator: and this place may easily be determined.

We may now pass from the Sun's tabulated places, obtained by daily observations of his meridian altitudes, to the explanation of the changes of places, as originating from, or explicable by, his oblique motion.

The line MM' (see Fig. p. 127,) is intended to represent the aggregate of the angular distances through which the Sun recedes each day from a fixed star, that was with the Sun on the meridian at e. This aggregate is 360°*; $MK = KM'$, moreover $ME = Kk = M'E'$ is the height of the equator, and a line EkE'



containing 360°, and extended on a plane, may be conceived to represent the equator. Reversely, the lines EkE', $ESkE'$ may be conceived to be wound round a sphere, the line EkE', coinciding

* This part being intended for general explanation only, the *precession of the equinoxes* is not taken account of.

with $Q \epsilon q$, &c. $E S k E'$ with $\epsilon S s$, &c., and the points ϵ, e, e' &c. in Fig. p. 127, with the points in Fig. p. 130, denoted by the same letters. Suppose now the Sun to be in the equator at ϵ; then, by the revolution of the sphere, the point ϵ and the Sun, would be transferred to the meridian at the point Q, and $hQ = ME$ will be the height of the equator: next, let the Sun recede through the space ϵe; the point e and the Sun will be on the meridian at f, and $fh = me$ (Fig. p. 127,) will be the meridian altitude: on the succeeding day let the Sun, having still farther receded through the space ee', be at e'; then his place on the meridian will be f', and his meridian altitude $f'h = m'e'$ (Fig. p. 127.): and similar circumstances will take place till the Sun has receded through the space ϵS (ϵS = a quadrant) when his place on the meridian will be at g, and his meridian altitude $gh = nS$ (Fig. p. 127,) then the greatest: after this the meridian altitudes will decrease.

By supposing therefore the Sun to move in the curve ϵS, &c. from ϵ towards S, whilst the sphere revolves in the opposite direction, from ϵ towards Q, all the phenomena indicated by observation admit of an adequate explanation. And, as the diurnal phenomena were shewn (p. 8,) equally explicable either by supposing the whole celestial sphere to revolve, the Earth being quiescent, or, the Earth to revolve in a contrary direction, the Heavens being at rest; so, these latter phenomena may be accounted for, either by supposing the Sun to move in an orbit such as $\epsilon S s$, &c., and the Earth to be at rest, or the Earth to move, but in a reverse direction, in an orbit similar to ϵS whilst the Sun remains at rest.

The above explanation does not depend, on the *real form* of the orbit $\epsilon S s$, which may be either circular or elliptical, or of any figure, provided that it lies in the same plane. For, the Sun is continually seen in the direction of a line drawn from him to the Earth; but, whatever be his place in that line, he will always, by the observer, be transferred to the imaginary concave spherical surface of the Heavens.

The imaginary path of the Sun in the Heavens is called the *Ecliptic:* the points E, E', (fig. p. 127,) of its intersection with the equator, are called the *Equinoctial* points: they are the *nodes* of

the equator; the points *S*, *W*, those of the greatest and least elevations above the horizon, or, the places of the Sun, at his greatest northern and southern declinations, are called the *Solstitial* points.

The points of the intersection of the equator and the ecliptic have been called the *Nodes* of the former; which they may be, by likening the equator to the orbit of a revolving body; for, generally, *nodes* are defined to be the intersections of the orbit of a planet, or other revolving body, with the plane of the ecliptic.

The planes in which the ecliptic and equator lie, are inclined to each other. The angle of their inclination is, for distinction, called the *Obliquity of the Ecliptic*: the angle of the inclination of the planes is the same as the angle made by two tangents, at the point ϵ, to the arcs ϵe, ϵq*. (see Fig. p. 130.)

If from *S* a solstitial point, a great circle *PS* be drawn perpendicular to the ecliptic, and ΠS be taken equal to a quadrant, then Π is the pole of the ecliptic †.

The circle *Gg*, a tangent to the ecliptic at the solstitial point *S*, and consequently parallel to the equator (and therefore a parallel of declination) is called a *Tropical* Circle. A similar one touches the ecliptic at the other solstitial point.

The small circle described round *P* in the circumference of which the pole of the ecliptic is always found, is called a *Polar* circle: sometimes the *Arctic Circle* (p. 38); and a similar one about the Earth's opposite pole is called the *Antarctic circle*.

A secondary (see p. 8,) to the equator, passing through *E*, the equinoctial point, is called the *Equinoctial Colure*: one passing through *S*, the *Solstitial Colure*.

Astronomers have divided the ecliptic into twelve equal parts called *Signs*: consequently, the ecliptic containing 360 degrees, each sign contains thirty degrees. Their names and characteristic symbols are,

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* *Trigonometry*, p. 128.

† Ibid. P. 89. l. 2. from bottom. This pole is situated in the sign of the *Dragon* between the stars δ and ζ, but nearer to the latter.

Northern.		Southern.	
Aries	♈	Libra	♎
Taurus	♉	Scorpio	♏
Gemini	♊	Sagittarius.	♐
Cancer	♋	Capricornus	♑
Leo	♌	Aquarius	♒
Virgo	♍	Pisces	♓

These signs are situated within an imaginary belt, called the *Zodiac*, extending eight degrees on each side of the ecliptic. To each of the signs, certain clusters, or groups of stars, called *Constellations* [*], are appropriated. But the signs, astronomically, serve merely to denote a certain number of degrees: thus, in the Nautical Almanack, the Sun's longitude for July 1, 1810, is stated to be 3 signs, 8 degrees, 54 minutes, 19 seconds; which is equivalent to 98 degrees, 54 minutes, 19 seconds.

The longitude is also sometimes expressed by means of the symbols of the constellations of the Zodiac. Thus, in Flamstead's catalogue of the fixed stars, the longitude of γ *Draconis* is expressed by:

$$♐\ 23^{0}\ 42'\ 48'',$$

which, since *Sagittarius*, represented by ♐ †, is the 9th sign, (the

---

[*] These groups of stars, or *constellations*, are by fancy imagined to form the outlines of the figures of animals and instruments, and are designated by their names. Thus, one group forms the figure of a Bear, another that of a Lion, a third of a Dragon, a fourth of a Lyre. So there are stars in the tail of the Bear, the head of the Dragon, the heart of the Lion: which are farther distinguished by Greek characters; the characters, according to their order, denoting the relative magnitudes of the stars. Thus, α Arietis designates the largest star in Aries, β Draconis, the second star of the Dragon. η Ursæ Majoris, the star the fifth in size of the greater Bear, &c.

† The particular stars of a constellation also are usually symbolically represented: thus α ♉ means the first or principal star in *Taurus* or the Bull; λ ♒, one of the inferior stars in *Aquarius;* β ♍, a star of the second magnitude in *Virgo;* γ ♎, a star of the third magnitude in *Libra.*

Aries

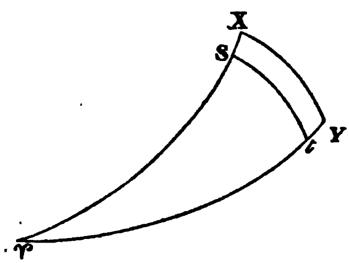
first point of which is accordingly distant from that of ~~ares~~ by 8 × ~~9~~0°, or 240°), denotes the longitude of γ *Draconis* to be
3

$$263° \ 42' \ 48''.$$

The term *Longitude,* which has been just introduced, means an angular distance measured or computed along the ecliptic, and from one of the intersections of the equator and ecliptic: which intersection is called the *First Point of Aries.*

After having passed through the 30° of *Aries,* the Sun enters *Taurus,* then *Gemini,* and, successively, the *signs* according to the order in which they were enumerated (p. 133). The motion of the Sun according to this order is said to be *direct,* or *in consequentia;* any motion in the reverse direction is said to be *retrograde,* or in *antecedentia.*

What longitude is with respect to the ecliptic, *right ascension* is with respect to the equator. It is angular distance, from the first point of *Aries,* (see l. 7,) measured along the equator. And, what declination is relatively to the equator, *latitude* is to the ecliptic: it is angular distance from the ecliptic, measured by that arc of a secondary to the ecliptic passing through the star, which lies between the star and the ecliptic. Thus, if ♈ be the first point of *Aries,* or denote the intersection of the equator and ecliptic, and St be perpendicular to the part ♈ t of a great circle,



St, ♈ t are, respectively, the latitude and longitude of S, if ♈ t be part of the ecliptic: or, they are, respectively, the declination and right ascension of S, if ♈ t be part of the equator. The Sun, being always in the ecliptic, has no latitude: at the first point of *Aries,* his declination, longitude, and right ascension, are nothing: at the solstitial points, his declination is the greatest, and his longitude and right ascension either 90°, or 270°.

The longitude of the Sun varying, in the year, from 0 to 360°, becomes successively, during that period, equal to the several longitudes of the stars. The longitude of *a Arietis* being in 1809, $1^s\ 4°\ 59'\ 31''$, that of the Sun was equal to it on April 25th. The longitude of *Regulus* being $4^s\ 27°\ 10'\ 27''$, that of the Sun was equal to it on August 20th. When this happens, the Sun is said to be in *conjunction* with the star. And, for conciseness of expression, Astronomers have invented another term called *Opposition,* which happens, when the longitude of the Sun differs from that of the star by 180°, or by 6 signs. The symbol for conjunction is ☌, for opposition ☍. Both the preceding terms are comprehended under a third, called *Syzygy*. Thus, the Sun having on Oct. 28th, a longitude of $7^s\ 4°\ 39'\ 54''$, was, during that day, in *opposition* to *a Arietis*. On April 25th, then, he was in *conjunction* with *a Arietis,* on Oct. 28th, in *opposition,* and on both days in *Syzygy* with that star.

The Sun was stated to be in conjunction with *a Arietis* on April 25th. But, the exact time of the day was not specified; that, however, may be found by a formula given in the Appendix: or, very nearly, after the following manner:

⊙ long^e. Apr. 25. .	$= 1^s\ 4°\ 49'\ 58''$.		$1^s\ 4°\ 49'\ 58''$
Apr. 26. .	$= 1\ \ 5\ \ 48\ \ 15$	long. of a ♈	$1\ \ 4\ \ 59\ \ 21$
Inc. of long. in 24^h	$58\ \ 17$	diff. of long.	$9\ \ 33$

$$\therefore\ 58'\ 17'' : 9'\ 33'' :: 24^h : 3^h\ 55^m\ 57^s;$$

consequently, the conjunction was April 25th, $3^h\ 55^m\ 57^s$, without estimating *the precession of the equinoxes*, by which the star's longitude was increased.

The Sun is said to be in *quadrature* with a star, or planet, when the difference of their longitudes is 90° or 3^s, or 270° or 9^s. For instance, the Sun was in quadratures with *a Arietis* when his longitude was either $4^s\ 4°\ 59'\ 31''$, or $10^s\ 4°\ 59'\ 31''$: which two events took place on July 28th, and January 24th. Again, the Sun was in quadratures with *Regulus,* when his longitude was either $7^s\ 27°\ 10'\ 27''$, or $1^s\ 27°\ 10'\ 27''$: that is, either on Nov. 19th, or May 18th. The symbol for quadratures is □; Thus ⊙ □ *a Aquilæ* denotes the Sun to be in quadratures with the first star in the *Eagle*.

The angle at t being a right one in the Figure of p. 134, we could determine ΥS, if Υt, and the angle Υ were known. If S be the Sun, ΥSX part of the ecliptic, ΥtY part of the equator, the angle at Υ is the *obliquity* of the ecliptic. If therefore this latter quantity were known, we could from it, and Υt the Sun's right ascension, find the Sun's longitude. We will now, then, briefly explain a method by which we may approximate to the value of the obliquity.

It appeared in p. 125, that the Sun's altitudes on four successive days were

$$61^\circ\ 14'\ 32'',\quad 61^\circ\ 15'\ 1'',\quad 61^\circ\ 15'\ 5'',\quad 61^\circ\ 14'\ 44'',$$

and the co-latitude being $37^\circ\ 47'\ 24''$, the corresponding declinations of the Sun, were

$$23^\circ\ 27'\ 8'',\quad 23^\circ\ 27'\ 37'',\quad 23^\circ\ 27'\ 41'',\quad 23^\circ\ 27'\ 20''.$$

If the greatest of these, that is, $23^\circ\ 27'\ 41''$, represented the Sun's greatest declination, it would measure the obliquity: for when ΥS, Υt are each equal to a quadrant, St is the measure of the spherical angle at Υ. But it plainly does not represent the greatest declination, since, if it did, the two adjacent declinations would be equal; which they are not: the greatest declination then must have happened sometime between the noons of June 21st, and June 22d, but nearer to the noon of the latter day. It is a quantity somewhat greater than $23^\circ\ 27'\ 41''$, and certainly not differing from it by four seconds. For, assume it to be the greatest declination, then, in fact, we assume the Sun's longitude to be (what it is at the Solstice) 3 signs or 90°. Now, this latter assumption cannot err $30'$ from the truth, since the change in the Sun's longitude for 12 hours is not quite equal to that quantity. Suppose it, however, to be $30'$, that is, in the Figure referred to, let X be the true place of the solstice, and $SX = 30'$, or $\Upsilon S = 89^\circ\ 30'$, then by Naper's rule *,

$$\text{rad.} \times \sin. St = \sin. \Upsilon \times \sin. S\Upsilon,$$

and $$\text{rad.} \times \sin. Xy = \sin. \Upsilon \times \sin. X\Upsilon;$$

consequently, eliminating sin. Υ, there results (since $\sin. X\Upsilon = 1$)

* Woodhouse's *Trigonometry*, p. 146.

$$\sin. Xy = \frac{\sin. St}{\sin. S\Upsilon} = \frac{\sin. St}{\cos. SX};$$

$$\therefore \log. \sin. Xy = 10 + \log. \sin. 23^\circ\ 27'\ 41'' - \log. \cos. 30',$$

$$\begin{array}{lr} \text{but, } 10 + \log. \sin. 23^\circ\ 27'\ 41'' \ldots = & 19.6000260 \\ \log. \cos. 30' \ldots\ldots\ldots = & 9.9999835 \\ \hline & 9.6000425 \end{array}$$

$$\therefore Xy = 23^\circ\ 27'\ 44''.5.$$

But since, in the case we have taken, the error in longitude must be less than 30′, the real obliquity must be some quantity between 23° 27′ 41″, and 23° 27′ 44″. And, if the error in longitude, instead of being 30′, were only 3′, the error in declination, instead of being 3″.5, would be only $3''.5 \cdot \frac{1}{(10)^2}$, or .035″ *. In the present instance the former error is about 20′, and therefore the latter is 1″.5 nearly, and consequently the obliquity † differs very little from 23° 27′ 42″.5.

We have thus, from the greatest observed altitude of the Sun and the latitude of the place of observation, deduced the greatest northern declination of the Sun: which declination is the measure of the obliquity. By a similar process we may observe the least meridional altitude of the Sun, and, if the Sun should not have exactly reached, or should just have ~~past~~ passed, the point of his greatest depression or declination, we may, as in the former instance, approximate to the time and value of such depression. This extreme southern declination of the Sun is, like the northern, a measure of the obliquity. And the *mean* obliquity

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* For the variations in declination near the solstice, are nearly, as the square of the variation in longitude: for, in the former Figure,

$$r \times \sin. p = \sin. \Upsilon . \sin. l \qquad (l = S\Upsilon,\ p = St)$$

$$\therefore r . dp . \cos. p = dl . \sin. \Upsilon . \cos. l \quad \text{(taking the differentials.)}$$

$$\therefore dp = \frac{dl}{r} \cdot \frac{\sin. \Upsilon}{\cos. p} \cos. l = \frac{dl}{r} . \tan. \Upsilon . \cos. l \text{ (since at sols., } p = \Upsilon \text{ nearly,)}$$

$$\therefore dp = \frac{dl}{r} \tan. \Upsilon . \sin. (90 - l) = \frac{dl}{r} \tan. \Upsilon . \sin. dl = \frac{(dl)^2}{r} \tan. \Upsilon,$$

since at the solstice $l = 90 - dl$ nearly.

† The obliquity thus determined is the *apparent* obliquity.

for any year, would be half the sum of the two extreme declinations computed for that year, or (in which case we do not need to know the latitude) would be half the difference of the greatest and least altitudes of the Sun, or half the difference of the least and greatest zenith distances*.

Thus, by observations made in 1807, at Blackheath,

Winter solstice, Sun's zenith distance	74° 55′ 56″.02
Summer....................	28 0 8.68
	2)46 55 47.34
Mean obliquity for 1807.........	23 27 53.67

According to received theories, the portions of the ecliptic that lie to the north and south of the equator are exactly similar to each other. The greatest southern declination of the Sun, then, ought to give, for the measure of the obliquity, the same quantity as the greatest northern declination gives. But there is some discordance of observations on this head. According to Dr. Maskelyne, Mr. Pond, Dr. Brinkley, M. Oriani, and M. Arago, the observations of the winter solstice give a *less* obliquity than observations of the summer solstice. M. Bessel, on the contrary, from his own observations finds the two measures of the obliquity concordant, and labours to shew that the latter observations of Bradley and those of Maskelyne made with the mural quadrant, and corrected for its errors, are of the same character.

The anomalous phenomenon (for such it is) of an inequality between the greatest northern and southern declinations of the Sun, *may* arise from some unknown modification of refraction. The question, certainly, is very intimately connected with the law and quantity of refraction. That source of inequality has not hitherto become the subject of consideration. This, therefore, would not be the place, did we possess the means, of solving the difficulty that has been stated. We will merely, in addition to what has been said, subjoin the various results of the *mean* obliquity that different Astronomers, with instruments of different size and construction, have arrived at.

* These distances, &c. must be *corrected* distances if the *mean* obliquity is to result from them.

Astronomers.	Instrument.	Year.	Summer Solstice.	Winter Solstice.
Bradley.	South Mural Quadrant of Greenwich......	1755	23° 28′ 15″.49	23° 28′ 15″.57
Maskelyne*	The above Quadrant...	1795	23 27 55.85	23 27 51.46
Piazzi.	Circle..............	1798	23 27 58.69	23 27 50.51
Oriani.	Repeating Circle of 3 feet diameter made by Reichenback....	1812	23 27 50.77	23 27 48.22
Pond.	Mural Circle of Greenwich..................	1813	23 27 50.0	23 27 47.34
Arago.	Repeating Circle of 3 feet diameter by Reichenback...........	1813	23 27 50.09	23 27 48.85
Bessel.	Circle by Cary of two feet..............	1814 1815	23 27 47.41 0 0 47.48	23 27 47.34 0 0 47.75
Brinkley.	Circle by Ramsden, 8 feet diameter.......	1813	23 27 50.99	23 27 48.14

We must remark on the preceding Table, that they are the largest, and, as they are generally esteemed to be, the best instruments that give discordant results in the two values of the obliquity. We refer to the mural circle of Greenwich, and the Dublin circle. The circle of M. Bessel, which makes the observations at the summer and winter solstice to accord so closely, is only two feet in diameter.

The obliquity of the ecliptic may be determined (see pp. 137, &c.) either from the greatest and least altitudes, or the least and greatest zenith distances. But that is not the sole method. For instance, half the difference between the greatest and least north polar distances of the Sun is the value of the obliquity. In fact, the method to be employed depends on the

* Bessel contends that these observations corrected for the errors of the quadrant (errors which the instrument became liable to after 1755) would render the quantities for the obliquity at the summer and winter solstice nearly equal, the seconds in the first being 57″.79, in the latter 57″.52.

instrument of observation. The Dublin circle, by a double observation, gives the true zenith distance. The *repeating* circle does the same: so, probably, does M. Bessel's two feet circle. The mural circle of Greenwich also gives the Sun's zenith distance, not, however, after the manner of the preceding instruments, but by the mediation of the zenith sector. By the mural circle, in which neither level nor plumb-line is employed, the Sun's north polar distance is determined.

One of the methods, that have been briefly described for determining the obliquity of the ecliptic, consists in deducting from the Sun's greatest altitude, found by computation from the greatest observed meridional altitude, the co-latitude of the place of observation: the latter quantity being determined (see pp. 129, &c.) from the greatest and least altitudes of circumpolar stars. The quantity remaining after the above deduction is the Sun's *greatest* declination. By a like method, we may, at any time, whether the Sun be on or past the meridian, find his declination. In the first case, the declination is merely the difference between the meridional altitude and co-latitude: In the second, the difference between the meridional altitude increased or diminished by the change of altitude proportional to the time from the passage over the meridian, and the co-latitude. For instance, in the first case,

June 21, 1810, Sun's U. L.	61°	29′	16″
L. L.	61	0	46
2)	122	30	2
Altitude of Sun's centre	61	15	1
Co-latitude of Cambridge	37	47	24
Sun's declination June 21, at 12^h app^t. time	23	27	37

To illustrate the second case, let it be required to find the Sun's declination on June 21, at three o'clock in the afternoon (civil time). Let the meridional altitude of the Sun's centre be found as above

On June 22, let it be	61°	15′	5″
Altitude on 21st	61	15	1
Increase of altitude in 24 hours .	0	0	4

Therefore, the increase in three hours (supposing the increase to be equable) is equal to $4'' \times \frac{3}{24} = \frac{4''}{8} = 0''.5$, consequently,

The alt. of Sun's centre, June 21, 3 o'clock	= 61° 15′ 1″.5
Co-latitude	37 47 24
Declination of Sun, June 21, 3 o'clock . . .	23 27 37.5

In the present Chapter, some instances have been given of the uses of the quadrant*, and transit instrument. The Sun has been observed on the meridian, and the attention of the Student directed to the changes both in the place and in the time of the Sun's passage. Twice a year, in March and September, the Sun is in the equator. From the first of these periods he continually, in his passages over the meridian, ascends towards the zenith till about the end of June when he becomes, with regard to his zenith distance, which is then the least, nearly stationary. From about the end of June to the latter end of September, the Sun's zenith distance, at his passage over the meridian, continually increases and with daily increments larger and larger. From his passage cross the equator, in September, the Sun's zenith distances increase till December, but at a diminished rate of increase; so that, towards the end of December, the Sun having reached his greatest zenith distance, becomes, with regard to such zenith distance, nearly stationary, or is at his *solstice*. The Sun's declination at this latter (our winter's) solstice is equal his declination at the other, the summer solstice, and either declination is the measure of the obliquity of the ecliptic.

The above are obvious inferences from the registered observations of the Astronomical quadrant. Like inferences may be made from the observations of the transit instrument and clock. If the Sun and a star are on the meridian together on a certain day, on the following day the star will pass before the Sun: but the interval of time by which it precedes the Sun will not be

* The *quadrant* is here, as in many other places, used as the generic term of all instruments that are used for determining meridional angular distances.

constant, whatever be the star, or, which amounts to the same thing, whatever be the day of observation. Thus, if on June 20, the Sun and a star are on the meridian together, on June 21, the star will pass the meridian about $4^m\ 9^s$ before the Sun; but a month after, the time of a like precedence, or acceleration of passage, of another star will not exceed $3^m\ 55^s$: a month after, it will be farther reduced to $3^m\ 41^s$; and, a month after, to $3^m\ 35^s$. Now this motion, to the east of a star, is a motion in right ascension. The Sun, therefore, has a motion in right ascension but not an equable one: he has also (see p. 141,) a motion in declination and not an equable one.

We will consider farther, in the next Chapter, the method of estimating the right ascension of an heavenly body.

We might also with the Quadrant and Circle make other observations of the Sun than those already mentioned. Thus, by moving the instrument itself round its axis, or (the instrument being steady) by means of a moveable horizontal wire placed in the focus of the eye-glass of the telescope of the instrument, we can measure the Sun's diameter. Now such measurements are found to vary according to the season of the year at which they are made. The inference from this is, that the Sun is, in different parts of the year, at different distances from the observer. So that, with respect to the Sun, the observations indicate a third inequality in addition to the two already mentioned.

But, it is to be remarked, the observations hitherto referred to of the Sun, whether of his north polar or zenith distance, or of the time of his passage over the meridian, are real observations (in the literal and natural signification of the term), such as faithful instruments ought to give us. They are, indeed, first in importance to the Astronomer, and the foundation of all his theories. But they soon become subservient to the deduction of another kind of right ascensions and declinations, more abstract in their nature, and independent of the circumstances of individual observations.

For instance, the zenith distance of Sirius (the great dog star) might be $68^\circ\ 43'\ 30''$ on one day, and $68^\circ\ 43'\ 22''$ on the succeeding day. Each distance might be truly given by the instrument, but either the one or the other, or each, must be viewed as a modification of the true distance (which in twenty-four hours

would not be changed) produced by some deranging cause. The Astronomer would contend, in this instance, the cause to be in the atmosphere, which, by bending the ray of light coming from the star, makes each zenith distance less than it would have been had not the light passed through a refracting medium: and he could go on to account for the difference (eight seconds) of the refraction of light from the same star from changes in the weight and temperature of the air. A change, for instance, of ten degrees of Fahrenheit's thermometer, and of 1 inch of the barometer *, would produce a change of eight seconds in the apparent altitude of Sirius: and other variations of the thermometer and barometer would account for the same fact: in this instance, then, the instrumental, or apparent zenith distance, is noted and *reduced,* by correcting it, to a *mean* zenith distance, or which would be such, did no other cause than that we have mentioned prevent the *apparent* and mean places of the star from coinciding.

Besides the one mentioned, there are, however, several other causes that produce the same effect. But, whatever they are, the observer, in the first instance, must be sure that his instrument is correct, and then must attend to its faithful report of phenomena. The observation is made just as it would be, were the observer placed in the centre of the Earth, at rest, and in an atmosphere that permitted light to pass through in right lines. What other phenomena, observations, so made, are indicative of, or proceed from, it is the business of Astronomical Science to explain. Towards such explanation our present course is now proceeding.

* In registering an observation the states of the thermometer and barometer are always put down, see pp. 98, 99.

CHAP. VII.

On the Methods of finding the Right Ascensions of Stars; from equal Altitudes near the Equinoxes, and from the Obliquity and Declination.—Latitudes and Longitudes of Stars.—Angles of Position.

The position of a star has been made to depend, as we have seen, on the arcs of two great circles perpendicular to each other. One of these circles is the equator, the other a great circle passing through its pole.

The declination of a star is its distance from the equator; and its measure is the arc, of a great circle passing through the pole of the equator and the star, intercepted between the star and equator. The *polar distance* is the complement of the declination: these terms are sufficiently significant, and the practical methods of instrumentally measuring, by observation, the quantities signified by the terms, admit of an easy explanation.

With regard to polar distances; there is no star in the pole from which we can, by our instruments, at once determine the angular distances of other stars: but (see pp. 120, &c.) we can always, by observations of circumpolar stars, determine where the pole is: that is, we are always able, at an assigned time, to state what number of the degrees, minutes and seconds of our instrument, the star Polaris, for instance, is distant from the north pole, and, consequently, since we can also by the same instrument observe the angular distances of Polaris and other stars, we can assign their north polar distances.

But with regard to the right ascensions of stars, the proceeding is not so natural and obvious. There is no point in the equator permanently marked by a star, or other phenomenon, from which we can take our departure in measuring right ascensions; nor, as in the former case, is there any point assignable by being the middle point between two phenomena. To find,

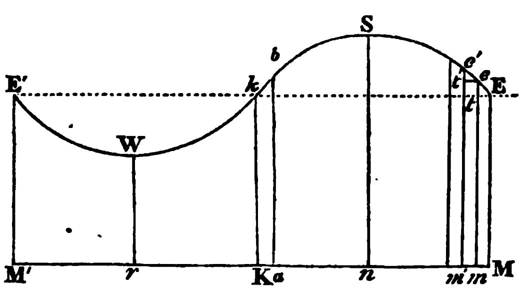
therefore, what we are in quest of, we must not confine our views to the stars and their apparent revolutions. If we look to the Sun, however, we shall find a convenient point, for dating right ascensions from, in the intersection of his path with the equator.

This point, in several of its *qualities* or in the circumstances attending it, is like that other point, the celestial pole, from which polar distances are measured. It is a point neither marked by a star nor capable of being permanently so marked. But though, like the pole, it be variable relatively to the stars, supposing them to be really fixed, yet it can, at any specified time, be assigned: that is, the Astronomer, if he knows his business, is able to tell in how many hours, minutes, seconds, and parts of seconds, after the passage of Sirius (for instance), this point, the intersection of the equator and ecliptic, shall also pass the meridian. If this can be done, the right ascensions of all stars become known from the intervals between their passages over the meridian and that of Sirius.

The place of the pole is determined from the zenith distances of a circumpolar star, at its superior and inferior passage over the meridian.

The star, at each passage, is at the same distance from the pole. The intersection of the equator and ecliptic, the *Vernal Equinox*, or as, still more technically, it is called, the *First Point of Aries*, may be determined from equal meridional altitudes of the Sun, and according to a method which we shall now proceed to describe.

In the subjoined Figure, E represents the vernal equinox, k



the autumnal, $ESkE'$ is the ecliptic, EkE' the equator, and

me, $m'e'$, &c. are the several meridional heights corresponding to the intervals of time Mm, Mm', &c.

Let e on a day near the time of the equinox be the Sun's place, me his meridional altitude: let b be his place, after an interval of nearly six months, then if ab were equal to me, Ka would equal Mm, since the portion of the curve SbK is supposed to be similar to SeE. But since the ordinates me, $m'e'$, ab, &c. represent meridional altitudes only, it will happen that there is no meridional altitude near to K exactly equal to me: ab may be very nearly equal to me, but it will be either a little greater or a little less: suppose it the next less, or that the preceding meridional altitude is greater than me: then the Sun's declination at b $(=ab-Kk)$ is less than the declination at e $(=me-ME)$, but equal to it, at some time between the noon at e and the previous noon: which time must be determined by computation.

Let X represent the Sun's right ascension at e, then if Ka were equal to Mm, 180^0-X, or 12^h-X would represent his right ascension at b. But Ka being less than Mm, 12^h-X represents the Sun's right ascension at some time previous to his being at b, and some greater quantity, 12^h-X+e, for instance, (e being a small quantity) will represent his right ascension at b. Let also Y represent the star's right ascension.

On the day at which the Sun is at e observe the transits of the Sun and star, and, by the clock, note the *difference* of their transits: represent the difference by d. Let also d' represent the difference of the transits of the star and Sun when the latter is at b: then, we have (supposing the star's apparent place not to have changed) these two equations,

$$Y - X = d,$$
$$12^h - X + e - Y = d',$$

whence

$$X = \frac{12^h - (d + d') + e}{2},$$

$$Y = \frac{12^h + d - d' + e}{2}.$$

Example from the Greenwich Observations of 1816. Transits of Pollux and the Sun.

$$\text{March 31,}\left\{\begin{array}{rrr} 7^h & 36^m & 0^s.5 \\ 0 & 41 & 9.31 \end{array}\right. \begin{array}{|c} \text{Pollux,} \\ \text{Sun's centre.} \end{array}$$

$$\text{App}^t\text{. diff. of transits} \ldots\ldots 6 \quad 54 \quad 51.19$$

$$\text{Sept. 12,}\left\{\begin{array}{rrr} 7^h & 33^m & 31^s.32 \\ 11 & 21 & 3.7 \end{array}\right. \begin{array}{|c} \text{Pollux,} \\ \text{Sun's centre.} \end{array}$$

$$3 \quad 47 \quad 32.37$$

The parts of the bottom line represent the apparent differences of the transits: but these differences must be corrected (see pp. 103, 104, &c.) on account of the clock's daily rate. Now on March 31, the clock's daily rate (estimated from three stars was $+.8$). On Sept. 12, from six stars, -1.45*: and the portions of these, proportional to the differences of the respective transits are $+.23$ and $-.226$. But, if the clock gains, the *difference* of transits shewn by it must be greater than the real difference, or than the difference of the right ascensions: and the contrary must take place, if the clock loses. Hence, diminishing the first difference by .23, and increasing the second by .226,

$$d = 6^h \ 54^m \ 50^s.96,$$
$$d' = 3 \ \ 47 \ \ 32.596;$$

* Sept. 11. Reduction of Wires.	Sept. 12. Reduction.	Rate.
14″.30	12″.90	1″.4
2.84	1.44	1.4
20.52	19.10	1.42
48.72	47.20	1.52
10.02	8.54	1.48
32.80	31.32	1.48
		8.7

$$\therefore \text{ mean rate} = \frac{-8.7}{6} = -1.45.$$

whence

$$d + d' = 10^h\ 42^m\ 23^s.556,$$
$$d - d' = 3\quad 7\quad 18.374.$$

We must now see what the altitudes of the Sun were on the noons of March 31, and Sept. 12th.

	Barometer.	Therm.		N.P.D.
March 31, 1816,	30.1	42	85° 29′ 26″.2	⊙ U. L.
Sept. 12.	29.95	57	85 34 28.1	⊙ U. L.

The above are the north polar distances of the Sun's upper limb observed at Greenwich with the mural circle. The first correction to be applied to them is the *index error*, (see pp. 112, &c.) Then, deducting the co-latitude (*ZP*, see fig. p. 7.) there remains the zenith distance of the Sun's upper limb: but this distance is too small, by reason of *refraction*: it requires, therefore, a correction on that account. This is sufficient to explain, in a general way, the following process:

	March 31, ⊙ U. L.	Sept. 12,
	= 85° 29′ 26″.2	= 85° 34′ 28″.1
Index error......	+ 4	0 0 2.5
	85 29 30.2	85 34 30.6
co-latitude........	38 31 21.5	38 31 21.5
app^t. Z. D. ⊙ U. L.	46 58 8.7	47 3 9.1
refraction.........	0 1 3.5	0 1 1.6
corrected Z. D.....	46 59 12.2	47 4 10.7
alt. ⊙ U. L......	43 0 47.8	42 55 49.3
Sun's semi-diameter	0 16 1.12	0 15 56.1
	42 44 46.6	42 39 53.2
parallax...........	0 0 6.4	0 0 6.5
true alt. Sun's centre.	42 44 53	42 39 59.7

The Sun's diameter which is subtracted from the altitude of his upper limb in order to obtain the altitude of the centre, may

be found by immediate observation, (see p. 98.) or may be taken from the Nautical Almanack. The correction for *parallax* will be explained in a subsequent Chapter.

We have now

$$\begin{array}{rrrr} me = & 42^\circ & 44' & 53'' \\ ab = & 42 & 39 & 59.7 \\ \hline & 0 & 4 & 53.3 \\ \hline \end{array}$$

the difference of meridional altitudes, or the difference of the Sun's declinations on the days of March 31, and Sept. 12. Now by observations on September 11th, the Sun's altitude was

$$43^0\ 2'\cdot 53''.7,$$

and his right ascension, on the same day, by the *clock* (allowing for its rate),

$$11^h\ 17^m\ 28^s.1.$$

Between the two apparent noons, then, of Sept. 11th, and Sept. 12th, the Sun's altitude from $42^0\ 52'\ 53''$ had decreased to $42^\circ\ 39'\ 59''.7$. The decrement then of altitude, which is also the decrement of declination, was $22'\ 54''$, whilst, in the same interval, the increment of right ascension was $3^m\ 35^s.6$.

Hence, $22'\ 54'' : 3^m\ 35^s.6 :: 4'\ 53''.7 : 46^s.023$ *.

The fourth term $46^s.023$, is the value of e.

Hence, (see p. 146,)

$$X(\text{or } \odot\text{'s R.A.}) = \frac{12^h - (10^h\ 42^m\ 23^s.556) + 46^s.023}{2} = 0^h\ 39^m\ 11^s.23,$$

$$Y(\text{or } *\text{'s R.A.}) = \frac{12^h + 3^h\ 7^m\ 18^s.374 + 46^s.023}{2} = 7^h\ 34^m\ 2^s.199$$

$$\begin{array}{lrl} * \text{ Log. } 3^m\ 35^s.6 & = & 2.3336488 \\ \text{log. } 4\ \ 53.3 & = & 2.4673121 \\ \hline & & 4.8009609 \\ \text{log. } 22\ \ 54 & & 3.1379867 \\ \hline & & 1.6629742 = \text{log. } 46.023. \end{array}$$

This determination of the star's right ascension is not quite exact; for, in the process by which it was obtained it was assumed, that the star's right ascension was the same on Sept. 12, as on March 31. This assumption, however, is not correct. The apparent right ascension of the star is different at the two periods: or, in other and plainer words, the index of the Astronomical Clock would not mark the same time when the Star, on Sept. 12th, was on the meridional wire of the telescope, as it did on March 31, supposing the clock, in the interval, adjusted to sidereal time to have preserved a perfectly equable motion.

The difference of the *apparent* right ascensions of the star at the two periods, is, indeed, but small, not exceeding eight-tenths of a second. If, as it will be in the present instance, the apparent right ascension be greater on Sept. 12th, than on March 31, the second of the equations of p. 146, instead of being

$$12^{h} - X + e - Y = d',$$

will become

$$12^{h} - X + e - (Y + y) = d',$$

$Y + y$, representing the star's right ascension on Sept 12th; or y representing the increase of right ascension; consequently, the resulting values of X and Y, will be

$$X = \frac{12^{h} - (d + d') + e}{2} - \frac{y}{2},$$

$$Y = \frac{12^{h} + d - d' + e}{2} - \frac{y}{2}.$$

Hence, if we make $y = 0''.71$ (which is nearly its value) we shall have

X, or Sun's R. A. $= 0^{h}\ 39^{m}\ 10^{s}.88$,
Y, or Star's R. A. $= 7\ \ 34\ \ 1.85$, nearly.

The student at present, must be content to take for granted that the value of y is rightly assigned. It is the result of four small *corrections* due to *inequalities* not yet explained. It, in truth, happens here as it will repeatedly happen again, that, in conducting an Astronomical process we are obliged to anticipate

the results of future demonstrations and to draw on funds not yet established.

The preceding value of y is very small, and, as it will be hereafter shewn, it can in no case be much larger. It is merely the *difference* of the *apparent* right ascensions of Pollux at the two periods of March 31, and Sept. 12, and it is only a portion (not a proportional portion) of the star's annual increase of right ascension. That there is such an increase may be easily shewn by finding from two observations, made at different periods, the corresponding right ascensions of the star: and, in order to obviate an objection that may be made against the preceding method, inasmuch as it is therein assumed, that the increase of the star's right ascension, during an interval of about six months, is either nothing or a small but undetermined quantity, we shall find the right ascension by a different method.

The method consists in finding the Sun's right ascension from two observed or known declinations: one the solstitial declination, or (see p. 136.) the obliquity of the ecliptic, the other an observed declination near the equinox. The star's right ascension will be the Sun's right ascension at the latter observation plus the difference of the times of the meridional transits of the Sun and star.

The two periods of observation are March 31, 1816, and March 24, 1768.

For the first of these periods, we have (see p. 149.)

me, or altitude of Sun's centre . . . =	$42^\circ\ 44'\ 53''$
co-latitude of Greenwich. . . . . . . =	38 31 21.5
Sun's declination March 31, 1816 . .	4 13 31.5

Let the obliquity of the ecliptic for that time be assumed equal to $23^\circ\ 27'\ 50''.8$.

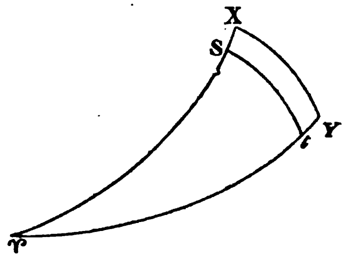
For the latter period we have from Maskelyne's Observations, reduced according to the methods of p. 148.

alt. Sun's centre.	$40^\circ\ 16'\ 2''$
co-latitude.	38 31 21.5
Sun's declination, March 21, 1768	1 44 40.5

Let the apparent obliquity at that period be assumed equal to $23^\circ\ 28'\ 14''.8$.

To find the right ascension, we have, in each case, this formula:

$$\text{rad.} \times \sin. ♈ t = \text{cotan.} \angle S ♈ t \times \tan. St,$$



or, rad. × sin. R.A. = cotan. obliquity × tan. dec.

March 31, 1816.

rad. . . . . =	− 10
tan. $4^\circ\ 13'\ 31''.5$ . . =	8.8685352
cot. 23 27 50.8. . =	10.3624424
(=log. 9 47 59)	9.2309776

March 24, 1768.

rad. . . . . =	− 10
tan. $1^\circ\ 44'\ 40''.5$ . . . . =	8.4837088
cot. 23 28 14.8. . . . . =	10.3623042
(=log. 4 1 21)	8.8460130

Hence, (reducing the angular measures into measures of time)

March 31, 1816, Sun's R.A. . . . . . . =	0^h	39^m	$11^s.8$
diff. of transits of Sun and star.	6	54	50.96
Star's R. A.	7	34	2.76
March 24, 1768, Sun's R. A.	0^h	16^m	$5^s.6$
diff. of transits (from Maskelyne's Obsv^{ns}.)	7	15	2.46
Star's R. A.	7	31	8.06

The difference between these two right ascensions is

$$2^m\ 54^s.7,$$

an increase that has taken place in forty-eight years, and, conse-

quently, if the same increase would always happen in every forty-eight years, the mean annual increase would be

$$3^s.63.$$

But it so happens that, of the inequalities causing the right ascension to vary, one inequality is variable both in degree and direction: it is not the same on March 24, 1816, and March 24, 1817: and, in the case before us, it *diminishes* the right ascension of Pollux by $1''.22$ in the March of 1816, and *increases* it by $1''.3$ in the March of 1768. The difference of these quantities is $2''.52$: so that, setting aside this variable inequality (which has its cause in the variable action of the Moon on the Earth) the increase of Pollux's right ascension in forty-eight years will be

$$2^m\ 54^s.7 + 2''.52;\ \text{or}\ 2^m\ 57''.22,$$

and the mean annual increase of right ascension,

$$3^s.69.$$

This augmentation of right ascension then exists: and it is part of this, (but not, as we said in p. 151, a proportional part) that causes the right ascension of Pollux on March 31, 1816, to be different from its right ascension on Sept. 12, 1816.

The first method which has been described for finding the right ascension is due to Flamstead. It is held by practical Astronomers to be a good method. The Sun is observed at equal zenith distances, and, therefore, any error assigned by the Tables in the quantity of refraction, or any error in the instrument, would equally affect each observation. We are tolerably sure of ascertaining (which is the essential part of the method) when the Sun is at equal distances from the zenith. It is less important to know the exact *quantities* of those zenith distances.

We must not hope to obtain, exactly, the right ascension of a star by one observation and process. On this, as on all like occasions, the process must be repeated, and the *mean* taken of several results; taken, as the true result, or as the result that is most nearly true. Thus, in the instance adduced, observations (should circumstances permit it) should be made on March 30, and Sept. 13: on March 29, and Sept. 14, &c.: and like sets of observations should be made on different years.

The right ascension of one star being settled, the right ascensions of other stars may thence be deduced. Thus, taking the

apparent right ascension of Pollux on March 31, 1816, to be $7^h\ 34^m\ 2^s.2$, let the index of the clock be set to that time when Pollux is on the meridional wire of the transit telescope. The clock, if it goes rightly, will denote the right ascensions of other stars when they are bisected by the meridional wire. Thus, on the above day,

Capella passing the meridional wire at	5^h	3^m	$5^s.5$
Aldébaran	5	3	21.2
Procyon	7	29	39.7
α Arietis	1	56	47.6

such times would be the *apparent* right ascensions of those stars.

α Arietis, the principal star in the constellation of the Ram, passes the meridian, as we see, at $1^h\ 56^m\ 47^s.6$. But the *first point of Aries*, is, as it has been already mentioned, a term altogether technical. It is, if we conceive the ecliptic and equator to be traced out in the Heavens, one of the intersections of those circles, namely, that in which the Sun would be at the time of the vernal equinox. When this point is on the meridian on March 31, 1816, the clock would note $0^h\ 0^m\ 0^s$, if, going regularly, it noted $7^h\ 34^m\ 2^s.2$ when Pollux was on the meridian; $7^h\ 34^m\ 2^s.2$ being supposed to be the truly computed apparent right ascension of that star.

In illustrating the preceding method of finding the right ascensions of stars, we have employed the star Pollux: but, it is plain, there are many other stars that would, equally well, have served that purpose. Dr. Maskelyne found, according to the preceding methods (or at least on their principle) the right ascension of α Aquilæ: that was, what he called, his fundamental star, the right ascension of which regulated the right ascensions of other stars.

But it may be here noted that, whatever the star and the time of observation, the result of the process merely gives the *apparent* right ascension of the star at that time: and, consequently, the right ascensions of other stars, deduced from that of the fundamental one by means of the differences of their transits, will be merely their *apparent* right ascensions for the same time. The day after the observation, the right ascensions will, in strictness of theory, be different, although imperceptibly so. But, a month after the time of the first observation, the right ascen-

sions of the stars will be found to have altered, or, the clock, going rightly, will no longer indicate the original times of their transits.

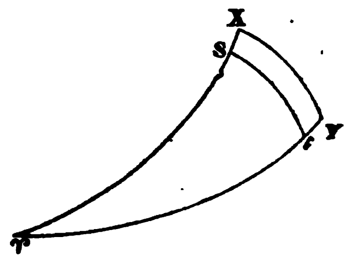
This has been (see pp. 106, &c.) already adverted to; the full explanation of the phenomenon cannot be given till all the *inequalities* that prevent the times of the recurrence of a star to the same horary wire, or to the meridional wire, of a transit telescope, from being entirely regulated by the time of the Earth's rotation. The present purpose of mentioning the circumstance is to shew that a catalogue of the right ascensions of stars made from the observations of two, three, or four years, and even by the best instruments, would, without the aid of theory, or of results obtained during long periods of time, be an imperfect catalogue. The results from the observations of a few years are quite insufficient. For, although we might by such establish as a fact that both the right ascensions and declinations did not remain the same, but, upon the whole, did either continually increase or diminish, still the *mean* values of such increases and diminutions would be entirely vitiated, by the operation of certain variable and recurring inequalities. We have already seen an instance of this in the case of Pollux. The interval of forty-eight years is not sufficiently large to give accurately the mean value of the annual increase of the right ascension of that star. If, however, we possessed good observations distant from each other by two hundred years, then, since the utmost effect of the variable inequality, of which we have spoken, must be less than 3^s, the mean annual increase of the star's right ascension found as it was in p. 153, cannot be erroneous beyond $0^s.015$. But this mean increase is only one point gained in the formation of the catalogue of stars. We may know the right ascension of Pollux on March 31, 1816, and its annual change, and still not be able to determine its right ascension on March 31, 1817, or on Sept. 12, 1816.

But although the exact determination of these and other points is not within our present reach, still, enough has been done for the elucidation of the general principles of the methods by which the places of the pole and of the *first point of Aries*, are determined. Both these points are perpetually changing their positions; but, such is the advanced state of Astronomical Science, they can always be exactly found.

We will now proceed to subjects related to the one which has been just discussed.

The right ascensions and declinations of the Sun and stars are deduced from observation. Their longitudes and latitudes, not being subjects of immediate observation, are deduced, by computation, and by processes purely mathematical, from right ascensions and declinations. In the case of the Sun, the computation is very easy, resting on the solution of a right-angled triangle. One or two examples will be sufficient for the illustration of this part of the subject.

Thus, let ♈ S be part of the ecliptic, and ♈ t part of the equator, and let St be part of a circle of declination: and let the



Sun's longitude Nov. 28, 1810, be required, his declination being 21° 16′ 4″, and right ascension $16^h\ 14^m\ 58^s.4$, or in space, 243° 44′ 36″.

By Naper's rule, $r \times$ cos. ♈ $S =$ cos. ♈ $t \times$ cos. St;

∴ log. cos. ♈ t or log. cos. 243° 44′ 36″ ...... = 9.6458083

log. cos. St, or log. cos. 21 16 4 .... = 9.9693672

10 + log. cos. ♈ S 19.6151755

∴ ♈ S = 245° 39′ 10″ the longitude required;

or = 8^s 5° 39′ 10″.

2dly, Required the Sun's longitude Nov. 29, from his declination = 21° 26′ 35″, and obliquity = 23° 27′ 41″.3.

By Naper, $r \times \sin. st = \sin. \Upsilon S \times \sin. S \Upsilon t$;

$$\therefore \log. r + \log. \sin. 21^\circ\ 26'\ 35'' \ldots\ldots = 19.5629781$$
$$\log. \sin. 23\ \ 27\ \ 41.3 \ldots\ldots = \underline{\ 9.6000276}$$
$$\log. \sin. \Upsilon S \ldots\ldots\ldots\ldots = \underline{\ 9.9629505}$$

$\therefore$ longitude $= 246^\circ\ 40'\ 6''$, or $8^s\ 6^\circ\ 40'\ 6''$.

3dly, Required the Sun's longitude Nov. 30, from his Æ$=$ $16^h\ 23^m\ 34^s$, and the obliquity of the ecliptic$=23^\circ\ 27'\ 42''.3$.

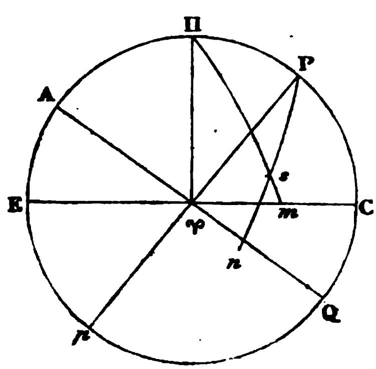
By Naper, $r \times \cos. S \Upsilon t = \text{cotan}. S \Upsilon \times \tan. \Upsilon t$;

$$\therefore \log. r + \log. \cos. 23^\circ\ 27'\ 42''.3 = 19.9625237$$
$$\log\ \tan. 16^h\ 23^m\ 34^s.1 = \underline{10.3492191}$$
$$\log. \text{cotan}. \Upsilon S \ldots\ldots\ldots = \ \ 9.6133046$$
$$\therefore \text{longitude} \ldots\ldots\ldots\ldots = 247^\circ\ 40'\ 56'',$$
$$\text{or} = 8^s\ 7^\circ\ 40'\ 56''.$$

The longitude in these examples is computed from the right ascension and declination, conditions given by observation. But, in the construction of the Nautical Almanack, the reverse operation takes place. The *Solar Tables* give the Sun's longitude : thence, and from the obliquity of the ecliptic, the right ascension and declination are computed, by trigonometrical operations, similar to the preceding.

The longitudes and latitudes of stars (their respective angular distances from the first point of Aries and the ecliptic) are found from their right ascensions and declinations; by processes, however, less simple than the preceding.

Let P, π be the poles of the equator AQ and of the ecliptic



EC: then since $PQ=90^\circ$, and $\pi C=90^\circ$, $P\pi=CQ$, but CQ is the measure of the *obliquity*; therefore $P\pi$ is.

Let s be a star, Psn part of a circle of declination passing through it, πsm part of a circle of latitude. Ps is the star's north polar, πs is the complement of the star's latitude; the angle $sP\pi$ *depends* on the star's right ascension, and the angle $s\pi P$ on the star's longitude.

In the present figure, ♈ represents the first point of Aries, and the order of the signs is from ♈ to C: therefore, the star's longitude measured according to that order will be ♈ m, the measure of the angle ♈ πm, which latter angle equals $90^\circ - \angle s\pi P$; consequently, in this case (the star being in the first quadrant),

$$\text{the longitude } (L) = 90^\circ - \angle s\pi P,$$
$$= 90^\circ - C \quad (C = \angle s\pi P).$$

The star's right ascension is ♈n, which measures $\angle$ ♈Pn, which angle equals $\angle \pi Ps - 90^\circ$; consequently, in this case,

$$\text{the right ascension } (\text{Æ}) \ldots = \angle \pi Ps - 90^\circ,$$
$$(A = \angle \pi Ps) \ldots\ldots\ldots\ldots = A - 90^\circ,$$
$$\text{and, consequently, } A = 90^\circ + \text{Æ}.$$

This is the case in the *first quadrant*, as it is called, or, the above equation is true when the star (s) is situated within the quadrant P♈Q, or, when the star's right ascension is less than six hours of sidereal time. The relation of A to the right ascension is different in the other quadrants;

in the 2d quadrant...... $A = 270^\circ - \text{Æ}$, and $\cos. A = -\sin. \text{Æ}$
in the 3d quadrant...... $A = 270^\circ - \text{Æ}$, and $\cos. A = -\sin. \text{Æ}$
in the 4th quadrant..... $A = \text{Æ} - 270^\circ$, and $\cos. A = -\sin. \text{Æ}$
in the 1st (as we have seen) $A = 90^\circ + \text{Æ}$, and $\cos. A = -\sin. \text{Æ}$

In all these cases, $\cos. A = -\sin. \text{Æ}$: which equation will enable us to lay down a simple and general formula for the value of the star's latitude.

In the oblique spherical triangle $s\pi P$,

$$\cos. \pi s = \cos. P\pi . \cos. Ps + \sin. P\pi . \sin. Ps . \cos. \pi Ps.$$

Let $\pi s = \Delta$, $P\pi = I$, $Ps = \delta$, then

$$\cos. \Delta = \cos. I . \cos. \delta - \sin. I \sin. \delta \sin. \text{Æ}.$$

Hence, (see *Trigonometry*, pp. 39, 171.)

$$1 - 2\sin.^2\frac{\Delta}{2} = \cos.I.\cos.\delta - \sin.I.\sin.\delta$$

$$+ 2\sin.I.\sin.\delta.\sin.^2\left(\frac{90 - Æ}{2}\right)$$

$$= \cos.(I+\delta) + 2\sin.I\sin.\delta.\sin.^2\left(\frac{90 - Æ}{2}\right),$$

and

$$2\sin.^2\frac{\Delta}{2} = 2\sin.^2\frac{(I+\delta)}{2} - 2\sin.I.\sin.\delta.\sin.^2\left(\frac{90^\circ - Æ}{2}\right).$$

Let $\sin.I.\sin.\delta.\sin.^2\left(\frac{90 - Æ}{2}\right) = \sin.^2 M$

then

$$\sin.^2\frac{\Delta}{2} = \sin.^2\left(\frac{I+\delta}{2}\right) - \sin.^2 M,$$

$$(\textit{Trig.}\ \text{p. } 31.) = \sin.\left(\frac{I+\delta}{2} + M\right).\sin.\left(\frac{I+\delta}{2} - M\right),$$

and, logarithmically expressed,

$$\sin.\frac{\Delta}{2} = \frac{1}{2}\left\{\log.\sin.\left(\frac{I+\delta}{2} + M\right) + \log.\sin.\frac{I+\delta}{2} - M\right\}.$$

Hence, we have this rule for finding the latitude of a star from its right ascension and north polar distance, and the obliquity of the ecliptic.

1st. *Add twice the logarithmic sine of half the difference between the right ascension and* 90°, *to the sum of the logarithmic sines of the obliquity, and the star's polar distance: half this whole sum diminished by* 20 *will be the logarithmic sine of an auxiliary angle* M.

2d. *Form two arcs by adding* M *to the half sum of the obliquity and north polar distance, and by taking* M *away from that half sum; half the sum of the logarithmic sines of these two latter arcs is the logarithmic sine of half the complement of the star's latitude.*

EXAMPLE 1.

Required the latitude of *a Arietis*, its *mean* right ascension (for 1815) being. . . . . $1^h\ 56^m\ 45''.9$
its *mean* north polar distance. . . . . $67^\circ\ 25'\ 1''.7$
and the *mean* obliquity of the ecliptic*. 23 27 46.3

Reduce Æ to degrees at the rate of twenty-four hours to 360°, or of 1^h to 15° : then

1st,

$$
\begin{array}{rlr}
Æ = 29^\circ\ 11'\ 28''.5 & & \\
90 - Æ = 60\ \ 48\ \ 31.5 & & \\
\frac{1}{2}(90 - Æ) = 30\ \ 24\ \ 15.75 \ldots\ldots & \text{log. sin.} & 9.7042361 \\
 & & 2 \\
\hline
 & & 19.4084722 \\
\text{N.P.D.} = 67\ \ 25\ \ 1.7 \ldots\ldots & \text{log. sin.} & 9.9653546 \\
I = 23\ \ 27\ \ 46.3 \ldots\ldots & \text{log. sin.} & 9.6000517 \\
 & 2\ \text{log. rad.} & 20 \\
\hline
\frac{\text{N.P.D.} + I}{2} = 45\ \ 26\ \ 24 & & 2)18.9738785 \\
\hline
M = 17\ \ 52\ \ 12.3 & \text{log. sin.}\ M & 9.4869392
\end{array}
$$

2dly,

$$\therefore M = 17^\circ\ 52'\ 12''.3$$

$$
\begin{array}{lr}
\frac{\text{N.P.D.} + I}{2} + M = 63\ \ 18\ \ 36.3 \ldots\ldots\ \text{log. sin.} & 9.9510705 \\
\frac{\text{N.P.D.} + I}{2} - M = 27\ \ 34\ \ 11.7 \ldots\ldots\ \text{log. sin.} & 9.6654221 \\
\hline
 & 2)19.6164926 \\
\hline
(\text{log. sin. } 40^\circ\ 1'\ 11''.3) \ldots & 9.8082463
\end{array}
$$

Hence, the complement of latitude = $80^\circ\ 2'\ 22''.6$
and the latitude = 9 57 37.4.

---

* This is very nearly the mean obliquity for 1815, supposing, according to Bradley, the mean obliquity for 1750 to be $23^\circ\ 28'\ 18''$, and also that the secular diminution of the obliquity is $50''$. The true value of the mean obliquity is, probably, two seconds greater.

Example 2.

Required the latitude of Pollux in 1815,

$$\text{Æ} = 7^h\ 33^m\ 58^s.7 = 113^\circ\ 29'\ 40''.5,$$
$$\text{N.P.D} \cdots\cdots\cdots\cdots = 61\ \ 32\ \ 12.4.$$

1st,

$$\text{Æ} = 113^\circ\ 29'\ 40''.5$$
$$\tfrac{1}{2}(\text{Æ} - 90) = 11\ \ 44\ \ 50.25 \cdots\cdots \text{log. sin.} = 9.3087681$$
$$2$$
$$18.6175362$$
$$\text{N.P.D.} = 61^\circ\ 32'\ 12''.4 \cdots\cdots\cdots \text{log. sin. } 9.9440497$$
$$I\ (\text{obliquity}) = 23\ \ 27\ \ 46.3. \cdots\cdots\cdots \text{log. sin. } 9.6000517$$
$$-\,2\ \text{log. rad.} -20$$
$$\text{N.P.D} + I = 84\ \ 59\ \ 58.7 \qquad 2)\,18.1616376$$
$$(\text{log. sin. } M)\ \ 9.0808188$$
$$\frac{\text{N.P.D.} + I}{2} = 42\ \ 29\ \ 59.3 \qquad M = 6^\circ\ 55'\ 5''.75$$

2dly,

$$M = 6^\circ\ 55'\ 5''.75$$
$$\frac{\text{N.P.D.} + I}{2} + M = 49\ \ 25\ \ 5.05 \cdots \text{log. sin.} = 9.8805143$$
$$\frac{\text{N.P.D.} + I}{2} - M = 35\ \ 34\ \ 53.55 \cdots \text{log. sin. } 9.7648191$$
$$2)\,19.6453334$$
$$(\text{log. sin. } 41^\circ\ 39'\ 50''.8)\ \ 9.8226667$$

Hence the complement of star's latitude is .. $83^\circ\ 19'\ 41''.6$

and the star's latitude ………………… $6\ \ 40\ \ 18.4.$

Example 3.

Required the latitude of Spica Virginis (in 1815.)

its $Æ = 13^h\ 15^m\ 27^s.53 = 198^0\ 51'\ 52''.95$

N. P. D. . . . . $= 100\ \ 11\ \ 28.9$

1st,

$Æ = 198^0\ 51'\ 52''.95$

$\frac{Æ - 90}{2} = 54\ \ 25\ \ 56.47$ . . . . log. sin. $= 9.9103198$

$\underline{\qquad\qquad 2}$

19.8206396

N. P. D. $= 100^0\ 11'\ 28''.9$ . . . . . . log. sin. 9.9930933

$I = 23\ \ 27\ \ 46.3$ . . . . log. sin. 9.6000517

-2 log. rad. -20

N. P. D. $+ I = 123\ \ 39\ \ 15.2$ $\qquad 2)\overline{19.4137846}$

$\frac{\text{N.P.D.} + I}{2} = 61\ \ 49\ \ 37.6$ (log. sin. M) 9.7068923

$\therefore\ M = 30^0\ 36'\ 39''.1$

2dly,

$M = \underline{30^0\ 36'\ 39''.1}$

$\frac{\text{N.P.D.} + I}{2} + M = 92\ \ 26\ \ 16.7$ . . . . log. sin. $= 9.9996068$

$\frac{\text{N.P.D.} + I}{2} - M = 31\ \ 12\ \ 58.5$ . . . . log. sin. $= 9.7145557$

$2)\overline{19.7141625}$

(log. sin. $46^0\ 1'\ 12''.4$) 9.8570812

Hence the star's distance from the pole of the ecliptic is $92^0\ 2'\ 24''.8$, and the star's *south* latitude is $2^0\ 2'\ 24''.8$.

Example 4.

Required the latitude of α Aquilæ for the year 1815.

$$\text{Æ} = 295° \ 26' \ 17''.7$$
$$\text{N. P. D.} = 81 \ 36 \ 40.6$$

1st,

$$\text{Æ} = 295° \ 26' \ 17''.7$$

$$\frac{\text{Æ} - 90°}{2} = 102 \ 43 \ 8.8 \ldots\ldots\ldots\ldots \text{log. sin.} = 9.9892102$$

$$2$$

$$19.9784204$$

$$\text{N. P. D.} = 81° \ 36' \ 40''.6 \ldots\ldots\ldots\ldots \text{log. sin.} \ 9.9953285$$
$$I = 23 \ 27.46.3 \ldots\ldots\ldots\ldots \text{log. sin.} \ 9.6000517$$
$$-2 \text{ log. rad.} = -20$$

$$\text{N. P. D.} + I = 105 \ 4 \ 26.9 \qquad 2)\overline{19.5738006}$$

$$\frac{\text{N.P.D.} + I}{2} = 52 \ 32 \ 13.45$$

$$(\text{log. sin. } 37° \ 44' \ 38''.09) \ 9.7869003$$

2dly,

$$M = 37° \ 44' \ 38''.09$$

$$\frac{\text{N. P. D.} + I}{2} + M = 90 \ 17 \ 11.5 \ldots\ldots\ldots\ldots \text{log. sin.} \ 9.9999946$$

$$\frac{\text{N. P. D.} + I}{2} - M = 14. \ 47 \ 15.4 \ldots\ldots\ldots\ldots \text{log. sin.} \ 9.4069438$$

$$2)\overline{19.4069384}$$

$$(= \text{log. sin. } 30° \ 20' \ 42''.25) \ 9.7034692$$

Hence, the star's distance from the pole of the ecliptic is 60° 41′ 24″.5, and, consequently, the star's latitude is 29° 18′ 35″.5

EXAMPLE 5.

Required the latitude of α Pegasi (in 1815).

its Æ $= 343^\circ\ 53'\ 15''.75$,

and N.P.D. $= 75\ \ 47\ \ 12.7$.

1st,

$$Æ = 343^\circ\ 53'\ 15''.75$$

$$\frac{Æ - 90}{2} = 126\ \ 56\ \ 37.87 \text{ .... log. sin.} = 9.9026692$$

$$2$$

$$19.8053384$$

$$\text{N.P.D.} = 75^\circ\ 47'\ 12''.7 \text{ .... log. sin. } 9.9864981$$

$$I = 23\ \ 27\ \ 46.3 \text{ .... log. sin. } 9.6000517$$

$$-\ 2 \text{ log. rad.} - 20$$

$$\text{N.P.D.} + I = 99\ \ 14\ \ 59 \qquad 2)19.3918882$$

$$(\text{log. sin. } M) \text{ ...... } 9.6959441$$

$$\frac{\text{N.P.D.} + I}{2} = 49^\circ\ 37'\ 29''.5 \qquad \therefore M = 29^\circ\ 46'\ 14''.25$$

2dly,

$$M = 29^\circ\ 46'\ 14''.25$$

$$\frac{\text{N.P.D.} + I}{2} + M = 79\ \ 23\ \ 43.75 \text{ .... log. sin. } 9.9925185$$

$$\frac{\text{N.P.D.} + I}{2} - M = 19\ \ 51\ \ 15.25 \text{ .... log. sin. } 9.5310040$$

$$2)19.5235225$$

$$(\text{log. sin. } 35^\circ\ 17'\ 40'')\ 9.7617612$$

Hence, the distance of the star from the pole of the ecliptic is $70^\circ\ 35'\ 20''$: and, consequently, the star's latitude is $19^\circ\ 24'\ 40''$.

Example 6.

Required the latitude of γ Draconis in 1750.

its Æ .... $= 267^\circ\ 42'\ 7''$
its N. P. D .... $= 38\ \ 28\ \ 16$
and obliquity .... $= 23\ \ 28\ \ 18$

$$Æ = 267^\circ\ 42'\ 7''$$
$$\underline{\quad 90 \quad}$$
$$2)\,177\ \ 42\ \ 7$$

$\frac{Æ - 90^0}{2} = 88\ \ 51\ \ 3.5$ ...... log. sin. $= 9.9999126$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 19.9998253$

N. P. D. $= 38^\circ\ 28'\ 16''$ .... log. sin. 9.7938741
$I = 23\ \ 28\ \ 18$ .... log. 9.6002054
N. P. D $+ I = 61\ \ 56\ \ 34$ $\qquad 2)\,19.3939048$

$\frac{\text{N.P.D.} + I}{2} = 30\ \ 58\ \ 17$ $\qquad$ (log. sin. M) 9.6969524

$M = 29\ \ 50\ \ 48.4$ .... $M = 29^\circ\ 50'\ 48''.4$

$\frac{\text{N.P.D.} + I}{2} + M = 60\ \ 49\ \ 5.4$ ...... log. sin. $= 9.9410524$

$\frac{\text{N.P.D.} + I}{2} - M = 1\ \ 7\ \ 28.6$ ...... log. sin. $= 8.2928518$

$\qquad\qquad\qquad\qquad\qquad\qquad\qquad 2)\,18.2339042$

(log. sin. $7^0\ 31'\ 18''.55$) 9.1169521

consequently, the complement of star's latitude $= 15^\circ\ 2'\ 37''.1$
and star's latitude .... $74\ \ 57\ \ 22.9$

Example 7.

Required the latitude of γ Draconis in 1815, its right ascension being $\cdots\cdots\cdots$ $268^\circ\ 4'\ 40''.2$

its north polar distance $\cdots\cdots$ $38\ 29\ 4.95$

and the obliquity of the ecliptic $\cdots\cdots$ $23\ 27\ 52.5$

$$Æ = 268^0\ 4'\ 40''.2$$

$$\frac{Æ - 90^0}{2} = 89\ \ 2\ \ 20.1 \ldots\ldots \text{log. sin. } 9.9999389$$

$$2$$

$$19.9998778$$

$$\text{N. P. D.} = 38^\circ\ 29'\ 4''.95 \ldots\ldots \text{log. sin. } 9.7940038$$

$$I = 23\ \ 27\ \ 52.5 \ldots\ldots \text{log. sin. } 9.6000817$$

$$- 2 \text{ log. rad.} - 20$$

$$\text{N. P. D.} + I = 61\ \ 56\ \ 57.45 \qquad 2)19.3939633$$

$$\frac{\text{N. P. D.} + I}{2} = 30\ \ 58\ \ 28.72 \qquad (\text{log. sin. } M)\ 9.6969816$$

$$M = 29\ \ 50\ \ 56.43$$

$$\frac{\text{N.P.D.} + I}{2} + M = 60\ \ 49\ \ 25.15 \ldots\ldots \text{log. sin.} = 9.9410757$$

$$\frac{\text{N.P.D.} + I}{2} - M = 1\ \ 7\ \ 32.29 \ldots\ldots \text{log. sin.} = 8.2932485$$

$$2)18.2343242$$

$$(\text{log. sin. } 7^\circ\ 31'\ 31''.7)\ 9.1171621$$

complement of the star's latitude is $15^0\ 3'\ 3''.4$

and star's latitude $\ldots\ldots$ $74\ 56\ 56.6.$

If instead of the above value for the obliquity, we had assumed it equal to $23^0\ 27'\ 46''.3$, the resulting value of the star's latitude would have been $74^\circ\ 56'\ 51''$.

EXAMPLE 8.

Required the latitude of Polaris in 1800: its right ascension being . . . . 13° 5′ 15″

its north polar distance (δ) . . . . 1° 45′ 34″.5

and the obliquity of the ecliptic. 23 27 54.8

$$Æ = 13° \ 5' \ 15''$$

$$\frac{90^0 - Æ}{2} = 38 \ 27 \ 22.5 \ldots\ldots \quad 9.7937323$$

$$2$$

$$19.5874646$$

$$\delta = 1° \ 45' \ 34''.5 \ldots\ldots \text{log. sin. } 8.4872189$$

$$I = 23 \ 27 \ 54.8 \ldots\ldots \text{log. sin. } 9.6000929$$

$$-2 \text{ log. } r = -20$$

$$\delta + I = 25 \ 13 \ 29.3 \qquad 2)17.6747764$$

$$\text{log. sin. } (3° \ 56' \ 35''.7) \ 8.8373882$$

$$\frac{\delta + I}{2} = 12° \ 36' \ 44''.6$$

$$M = 3 \ 56 \ 35.7$$

$$\frac{\delta + I}{2} + M = 16 \ 33 \ 20.2 \ldots\ldots \text{log. sin.} = 9.4547617$$

$$\frac{\delta + I}{2} - M = 8 \ 40 \ 8.9 \ldots\ldots \text{log. sin. } 9.1781950$$

$$2)18.6329567$$

$$(\text{log. sin. } 11° \ 57' \ 38''.9) \ 9.3164783$$

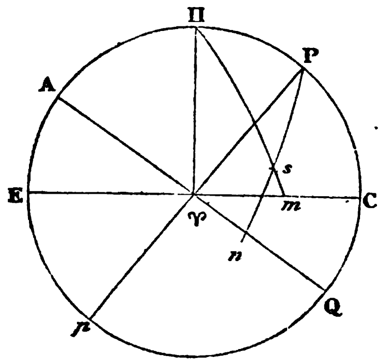
$\frac{1}{2}$ complement of star's latitude . . . . . . = 11° 57′ 38″.9

∴ complement . . . . = 23 55 17.8

∴ star's latitude . . . . = 66 4 42.2

We will now proceed the investigate a formula for the star's longitude.

The angle $s\pi P\,(C)$, as it was mentioned in p. 158, depends on the longitude. In the subjoined Figure $L=90^0-C$;



consequently,

in the 1st quadrant $C=90^\circ-L$; $\quad\therefore$ cos. $C=$sin. L
in the 2d ...... $C=L-90^\circ$, $\quad$ cos. $C=$sin. L
in the 3d ...... $C=L-90^\circ$, $\quad$ cos. $C=$sin. L
in the 4th...... $C=360^\circ+90^\circ-L$, $\quad$ cos. $C=$sin. L.

Now,

$$\cos.\,s\pi P=\frac{\cos.\,sP-\cos.\,s\pi\,.\cos.\,P\pi}{\sin.\,s\pi\,.\,\sin.\,P\pi};$$

$$\therefore\ \sin.\,L\ (=\cos.\,C)=\frac{\cos.\,\delta-\cos.\,\Delta\,.\cos.\,I}{\sin.\,\Delta\ \sin.\,I}.$$

But, (see *Trigonometry*, p. 39.)

$$2\sin.^2\left(\frac{90+L}{2}\right)-1=\sin.\,L,$$

consequently,

$$2\sin.^2\left(\frac{90+L}{2}\right)=\frac{\cos.\,\delta-(\cos.\,\Delta\ \cos.\,I-\sin.\,\Delta\ \sin.\,I)}{\sin.\,\Delta\ \sin.\,L},$$

and (see *Trigonometry*, pp. 30, 33.)

$$\sin.^2\left(\frac{90+L}{2}\right)=\frac{\sin.\left(\frac{\Delta+I+\delta}{2}\right)\sin.\left\{\left(\frac{\Delta+I+\delta}{2}\right)-\delta\right\}}{\sin.\ \Delta\ .\sin.\ I},$$

and, logarithmically expressed,

$$\log.\ \sin.\left(\frac{90+L}{2}\right)=$$

$$\frac{1}{2}\left\{\begin{array}{c}20+\log.\ \sin.\left(\frac{\Delta+I+\delta}{2}\right)+\log.\ \sin.\left(\frac{\Delta+I+\delta}{2}-\delta\right)\\ -\log.\ \sin.\ \Delta\ -\ \log.\ \sin.\ I\end{array}\right\}.$$

EXAMPLE 1.

To find the longitude of α Arietis for the beginning of the year 1815.

(see p. 160.) $\Delta = 80^\circ\ 2'\ 24''.56$.... log. sin. = 9.9934050
$I = 23\ \ 27\ \ 46.3$...... log. sin. = 9.6000517
$\delta = 67\ \ 25\ \ 1.7$ (d) 19.5934567

2)170 55 12.56

$\frac{1}{2}$ sum .... 85 27 36.28...... log. sin. = 9.9986352
$\frac{1}{2}$ sum $-\ \delta$ ...... 18 2 34.58...... log. sin. = 9.4909872
2 log. rad. = 20

39.4896224
(d) 19.5934567
2)19.8961657
(log. sin. $62^\circ\ 32'\ 20''.5$) 9.9480828

$90 + L = 125^\circ\ 4'\ 41''$
$L = 35\ \ 4\ \ 41$
$= 1^s\ \ 5\ \ 4\ \ 41.$

EXAMPLE 2.

Required the longitude of Pollux for 1815.

(see p. 161.)	$\Delta = 83^\circ\ 19'\ 41''.6$ ……..	log. sin. $= 9.9970490$
	$I = 23\ \ 27\ \ 46.3$ ……..	log. sin. $= 9.6000517$
	$\delta = 61\ \ 32\ \ 12.4$	(d) 19.5971007
	$2)168\ \ 19\ \ 40.3$	
	$\frac{1}{2}$ sum $= 84\ \ 9\ \ 50.15$ ……..	log. sin. $= 9.9977431$
	$\frac{1}{2}$ sum $- \delta = 22\ \ 37\ \ 37.7$ ……..	log. sin. $= 9.5851587$
		2 log. rad. $= 20$
		39.5829018
		(d) 19.5971007
		$2)19.9858011$
		9.9929005

Now 9.9929005 is the logarithmic sine of $79^0\ 40'\ 6''$: it is also the logarithmic sine of $100^\circ\ 19'\ 6''$ the *supplement* of the former: and this latter angle is the proper angle, since the star (see p. 158, 161,) is situated in the second quadrant.

Hence,

$$90^\circ + L = 200^0\ 39'\ 48'',$$

$$L = 110\ \ 39\ \ 48$$

$$\text{or} = 3^s\ 20\ \ 39\ \ 48.$$

EXAMPLE 3.

Required the longitude of Spica Virginis in 1815.

$$
\begin{array}{rrrrll}
\text{(see p. 162.)}\ \Delta = & 92^{\circ} & 2' & 24''.8 & \ldots\ldots \text{log. sin.} = & 9.9997247 \\
I = & 23 & 27 & 46.3 & \ldots\ldots \text{log. sin.} = & \underline{9.6000517} \\
\delta = & \underline{100} & \underline{11} & \underline{28.9} & (d) & 19.5997764 \\
2)& 215 & 41 & 40 & & \\
\tfrac{1}{2}\ \text{sum} = & 107 & 50 & 50 & \ldots\ldots\ldots \text{log. sin.} = & 9.9785809 \\
\tfrac{1}{2}\ \text{sum} - \delta = & 7 & 39 & 22 & \ldots\ldots\ldots \text{log. sin.} = & 9.1245924 \\
 & & & & 2\ \text{log. rad.} = & \underline{20} \\
 & & & & & 39.1031733 \\
 & & & & (d) & \underline{19.5997764} \\
 & & & & & 2)\underline{19.5033969} \\
 & & & & & 9.7516984
\end{array}
$$

Now 9.7516984 is the logarithmic sine of 34° 22′ 14″.5, and of the supplement 145° 37′ 45″.5, and, since the star's Æ (see p. 163,) is greater than 12^h, it must be the latter of these angles that is the true one; consequently,

$$
\begin{array}{rrrr}
90^{0} + L = & 291^{\circ} & 15' & 31'' \\
\text{and } L = & 201 & 15 & 31 \\
\text{or } = & 6^{s}\ 21 & 15 & 31.
\end{array}
$$

EXAMPLE 4.

Required the longitude of α Aquilæ in 1815.

$$\begin{array}{rrrrll}
\Delta = & 60^\circ & 41' & 24''.5 & \ldots\ldots\text{log. sin.} = & 9.9405090 \\
I = & 28 & 27 & 46.3 & \ldots\ldots\text{log. sin.} = & \underline{9.6000517} \\
\delta = & \underline{81} & \underline{36} & \underline{40.6} & & 19.5405607 \\
 & 165 & 45 & 51.4 & & \\
\tfrac{1}{2}\text{ sum} = & 82 & 52 & 55.7 & \ldots\ldots\text{log. sin.} = & 9.9966401 \\
\tfrac{1}{2}\text{ sum} - \delta = & 1 & 16 & 15.1 & \ldots\ldots\text{log. sin.} = & 8.3459399 \\
 & & & & 2\text{ log. rad.} = & \underline{20} \\
 & & & & & 38.3425800 \\
 & & & & & \underline{19.5405607} \\
 & & & & & 2)\underline{18.8020193} \\
 & & & & & 9.4010096
\end{array}$$

Now 9.4010096 is, the logarithmic sine of the arcs

$$\begin{array}{rrl}
14^\circ & 34' & 56''.9 \\
165 & 25 & 3.1 \\
374 & 34 & 56.9 \\
\&c. & &
\end{array}$$

Now, if either of the two first arcs were taken, the star's longitude would be less than 9 signs; whereas, since its right ascension is $19^h\ 41^m\ 45^s$, it must be greater. Taking, therefore, the third arc, we have

$$\begin{aligned}
90^\circ + L &= 749^\circ\ 9'\ 53''.8 \\
\text{and } L &= 659\ \ 9\ \ 53.8 \\
&= 360^\circ + 299\ \ 9\ \ 53.8,
\end{aligned}$$

and thence, rejecting 360°, we have the longitude

$$= 9^s\ 29^\circ\ 9'\ 53''.8.$$

EXAMPLE 5.

Required the longitude of α Pegasi in 1815.

$$
\begin{array}{rrrrlr}
\text{(see p. 164.) } \Delta = & 70^\circ & 35' & 20'' & \ldots\ldots\text{log. sin.} = & 9.9745846 \\
I = & 23 & 27 & 46.3 & \ldots\ldots\text{log. sin.} = & 9.6000517 \\
\hline
\delta = & 75 & 47 & 12.7 & & (d)\ 19.5746363 \\
\hline
2)\,& 169 & 50 & 19 & & \\
\hline
\tfrac{1}{2}\text{ sum} = & 84 & 55 & 9.5 & \ldots\ldots\text{log. sin.} = & 9.9982902 \\
\tfrac{1}{2}\text{ sum} - \delta = & 9 & 7 & 56.8 & \ldots\ldots\text{log. sin.} = & 9.2006233 \\
& & & & 2\text{ log. rad.} = & 20 \\
\hline
& & & & & 39.1989135 \\
& & & & & (d)\ 19.5746363 \\
\hline
& & & & & 2)\,19.6242772 \\
\hline
& & & & & 9.8121386
\end{array}
$$

Now 9.8121386 is the logarithmic sine of the arcs $40^\circ\ 27'\ 15''.5$, $180^\circ-(40^\circ\ 27'\ 15''.5)$, $360^\circ+40^\circ\ 27'\ 15''.5$, &c. Assuming the third, for reasons such as are stated in the last Example,

$$
\begin{aligned}
90 + L &= 2 \times (400^\circ\ 27'\ 15''.5) \\
&= 800^\circ\ 54'\ 31'' \\
\text{and } L &= 710\ \ 54\ \ 31 \\
&= 360^\circ + 350^\circ\ 54'\ 31'';
\end{aligned}
$$

and rejecting 360°,

$$\text{the longitude} = 11^s\ 20^\circ\ 54'\ 31''.$$

EXAMPLE 6.

Required the longitude of γ Draconis in 1750.

$$
\begin{array}{rrrrl}
\text{(see p. 165,)}\ \Delta = & 15^{0} & 2' & 37''.1 \ldots\ldots & \text{log. sin. } 9.4142288 \\
I = & 23 & 28 & 18 \ldots\ldots & \text{log. sin. } \underline{9.6002054} \\
\delta = & \underline{38} & \underline{28} & \underline{16} & (d)\ 19.0144342 \\
2) & \underline{76} & \underline{59} & \underline{11.1} & \\
\tfrac{1}{2}\text{ sum} = & 38 & 29 & 35.5 \ldots\ldots & \text{log. sin. } 9.7940847.7 \\
\tfrac{1}{2}\text{ sum} - \delta = & 0 & 1 & 19.5 \ldots\ldots & \text{log sin. } 6.5859420 \\
 & & & & 2 \text{ log. rad. } \underline{20} \\
 & & & & 36.3800267.7 \\
 & & & & \underline{19.0144342} \\
 & & & & 2)\underline{17.3655925} \\
 & & & & 8.6827962
\end{array}
$$

Now 8.6827962 is the logarithm of $2^{\circ}\ 45'\ 40''$ and of $(180^{\circ} - 2^{\circ}\ 45'\ 40'')$, and if, for reasons such as have been alledged, we take the latter arc, we have

$$
\begin{array}{rl}
90^{\circ} + L = & 354^{\circ}\ 28'\ 40'' \\
\text{and } L = & 264\ \ 28\ \ 40 \\
= & 8^{s}\ 24\ \ 28\ \ 40.
\end{array}
$$

EXAMPLE 7.

Required the longitude of Polaris in 1800.

$$
\begin{array}{rrrrll}
\Delta = & 23^{\circ} & 55' & 17''.8 & \ldots\ldots \text{ log. sin.} = & 9.6079754 \\
I = & 23 & 27 & 54.8 & \ldots\ldots \text{ log. sin.} = & 9.6000930 \\
\delta = & 1 & 45 & 34.5 & & \overline{19.2080684} \\
2)\!\! & 49 & 8 & 47.1 & & \\
\tfrac{1}{2}\text{ sum} = & 24 & 34 & 23.5 & \ldots\ldots \text{ log. sin.} = & 9.6189423 \\
\tfrac{1}{2}\text{ sum} - \delta = & 22 & 48 & 49 & \ldots\ldots\ldots \text{ log. sin.} = & 9.5885345 \\
 & & & & 2 \text{ log. rad.} = & 20 \\
 & & & & & \overline{39.2074768} \\
 & & & & & 19.2080684 \\
 & & & & & 2)\overline{19.9994084} \\
 & & & & (\text{log. sin. } 87^{\circ}\ 53'\ 8'')\ldots\ldots\ldots\ldots & 9.9997042
\end{array}
$$

$\therefore\ L + 90^{\circ} = 175^{\circ}\ 46'\ 16''$,

and $L = 85^{\circ}\ 46'\ 16''$, or $2^{s}\ 25^{\circ}\ 46'\ 16''$.

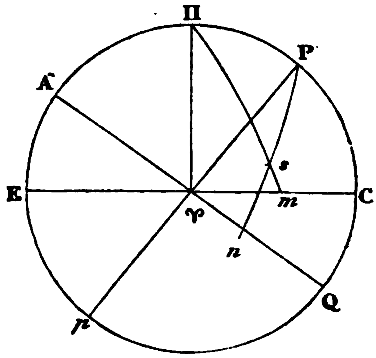
The longitudes and latitudes of stars are of some, but not of frequent use, in Astronomy. They are useful in the Theory of the *Aberration of Light*, and in certain methods founded on the occultations of stars by the Moon. They are also useful in the comparison of catalogues of stars made at different epochs, and afford us, as we shall hereafter see, the most direct mode of finding the quantity of the *Precession of the Equinoxes* *.

There are certain angles, technically called *Angles of Position*, dependent, like the latitudes and longitudes of stars, on their right ascensions and declinations, and the obliquity of the ecliptic, and thence deducible.

---

* There are Tables of the latitudes and longitudes of stars in Lalande's *Astronomy*, edition 3. In the *Connoissance des Tems* of 1788, for the Epoch of 1756: and in that of 1804, for the Epoch of 1800.

Now, the *angle of position* of a star, or of any point in the Heavens, is the angle formed at the star or point, by the arcs of a circle of declination and of a circle of latitude passing through that star or point. In the subjoined Figure it is the angle $\pi s P$.



In the Chapter on the Aberration of Light, we shall see the uses of these *angles of position*. Our present business is concerning the method of computing them.

Let P denote the angle $\pi s P$, then, (see *Trig*. p. 141.)

$$\sin. P \times \sin. \delta = \sin. s\pi P \times \sin. I;$$

$$\text{but (see p. 168,) } \sin. s\pi P = \cos. L;$$

therefore, to compute P, we have,

$$\sin. P \times \sin. \delta = \cos. L \times \sin. I,$$

and, in logarithms,

$$\log. \sin. P = \log. \cos. L + \log. \sin. I - \log. \sin. \delta;$$

or we may compute the angle of position thus,

$$\sin. P : \sin. sP\pi :: \sin. I : \sin. \Delta;$$

but (see p. 156,) $\sin. sP\pi = \pm \cos. Æ$;

$$\therefore \sin. P \times \sin. \Delta = \pm \cos. Æ \times \sin. I,$$

and

$$\log. \sin. P = \log. \cos. Æ + \log. \sin. I - \log. \sin. \Delta.$$

EXAMPLE 1.

Required the angle of position of α Arietis for 1815, (see p. 169,)

$$\begin{array}{lr} \text{log. cos. } 1^s\ 5^{\circ}\ 4'\ 41'' \ldots\ldots\ldots = & 9.9129496 \\ \text{log. sin. } 0\ 23\ 27\ 46.3 \ldots\ldots\ldots = & 9.6000517 \\ \hline & 19.5130013 \\ \text{log. sin. } 0^s\ 67^{\circ}\ 25'\ 1''.7 \ldots\ldots = & 9.9653546 \\ \hline (\text{log. sin. } 0\ 20\ 39\ 52.3) \ldots\ldots = & 9.5476467 \end{array}$$

Therefore the angle of position is

$$20^{\circ}\ 39'\ 52''.3.$$

EXAMPLE 2.

Required the angle of position of γ Draconis in 1815, (see p. 166.)

In this Example we will use the second formula of computation

$$\begin{array}{lr} \text{log. cos. } 268^{\circ}\ 2'\ 40''.2 \ldots\ldots\ldots\ldots = & 8.5255869 \\ \text{log. sin. } 23\ 27\ 52.5 \ldots\ldots\ldots\ldots = & 9.6000816 \\ \hline & 18.1256685 \\ \text{log. sin. } 74^{\circ}\ 56'\ 56'' \ldots\ldots\ldots\ldots\ldots & 9.4144395 \\ \hline (\text{log. sin. } 2\ 56\ 53.2) \ldots\ldots\ldots\ldots & 8.7112290 \end{array}$$

The angle of position, therefore, is

$$2^{\circ}\ 56'\ 53''.3.$$

EXAMPLE 3.

Required the angle of position of Polaris in 1800, see pp. 167, 175.

log. cos. 13^0 $5'$ $15''$............ =	8.8677093
log. sin. 23 27 54.8 .... =	9.6000936
	18.4678029
log. sin. 1 45 34.5 .... =	8.4872199
(log. 72 59 39.3)...........	9.9805830

therefore the angle of position is

$$72^0\ 59'\ 39''.3.$$

If a star be situated on the solstitial colure, its right ascension is either 90^0 or 270^0: in each case, cos. Æ = 0, consequently, since

$$\sin. P.\sin. \Delta = \cos. Æ.\sin. I,$$

$P=0$. γ Draconis, as we have seen in p. 166, is very near to the solstitial colure (its longitude = $8^s\ 24^0\ 28'\ 40''$), and its angle of position is less than three degrees. The mean right ascension of a star not continuing the same from year to year, and even the star's latitude and the obliquity of the ecliptic being subject to certain minute changes (*secular* variations) the angle of position must vary. Lalande's *Astronomy*, vol. I. p. 488, and the *Connoisance des Tems* for 1804, contain the angles of positions of several stars, together with their annual variations. The values of those angles thus then become known for several years adjacent to the year for which they are computed. The most simple method of computing the *variations* is to take the fluxion or differential of some expression involving P, Æ, δ, &c. Thus, we have (see p. 176,)

$$\sin. P.\sin. \Delta = \cos. Æ.\sin. I;$$

$$\therefore dP.\cos. P \sin. \Delta + d\Delta.\sin. P.\cos. \Delta =$$

$$- d Æ.\sin. Æ \sin. I + dI.\cos. Æ.\cos. I.$$

If we neglect, by reason of their smallness, the second and fourth terms, there remains for computing dP, this equation,

$$dP = -d\,\text{Æ} \times \frac{\sin.\ \text{Æ}\ .\ \sin.\ I}{\cos.\ P\ .\ \sin.\ \Delta}.$$

Example I.

It is required to find the annual variation of the angle of position of γ Draconis in 1815 (see pp. 166, 177.)

$$\begin{aligned}
\sin.\ \text{Æ} &= \sin.\ 268^\circ\ 4'\ 40'' = -\ .9994 \\
\sin.\ I &= \sin.\ \ 23\ \ 27\ \ 46 = \ \ .398 \\
\cos.\ P &= \cos.\ \ \ 2\cdot 56\ \ 53 = \ \ .9986 \\
\sin.\ \Delta &= \sin.\ \ 15\cdot 3\ \ \ 3 = \ \ .2597
\end{aligned}$$

and $d\,\text{Æ}$ (to be subsequently computed) $= 20''.7$;

$$\therefore\ dP = 20''.7 \times \frac{.9994 \times .398}{9986 \times .2596} = 31''.73, \text{ nearly.}$$

Example II.

Required the annual variation of the angle of position of a Arietis, the epoch being the year 1815, (see pp. 160, 177,)

$$\begin{aligned}
\sin.\ \text{Æ} &= \sin.\ 29^\circ\ 11'\ 28'' = .487 \\
\sin.\ I &= \sin.\ 23\ \ 27\ \ 46 = .398 \\
\cos.\ P &= \cos.\ 20\ \ 39\ \ 52 = .935 \\
\sin.\ \Delta &= \cos.\ \ 9\ \ 57\ \ 35 = .985 \\
\text{and } d\,\text{Æ} &= 50''.25\,^* ;
\end{aligned}$$

$$\therefore\ dP = -50''.25 \times \frac{.487 \times .398}{.935 \times .985} = -10''.5 \text{ nearly.}$$

Hence, the angle of position for the year 1800, would be

$$20^\circ\ 39'\ 52''.3 + 10''.5 \times 15;$$

$$\text{that is, } 20^\circ\ 42'\ 29''.8.$$

---

* The variation of the right ascension will be computed in a subsequent Chapter.

This result does not exactly agree with that which is given at p. 431, of the *Connoisance des Tems* for 1804. The angle of position for 1800 is there set down at 20° 42′ 44″. *Part* of the difference between the two results arises from the obliquity of the ecliptic being assumed of different values in the two processes. M. Chabrol (the computer in the French Almanac) has assumed the value of the mean obliquity equal to

$$23^\circ\ 27'\ 58''.$$

If we take the secular diminution of the obliquity to be 45″.7, then, in fifteen years (the interval between 1800 and 1815) the diminution would amount to 6″.85: consequently, the obliquity in 1815, on the above grounds, would be

$$23^\circ\ 27'\ 58'' - 6''.85 = 23^\circ\ 27'\ 51''.15\,^*,$$

whereas (see p. 160,) we have assumed it equal to

$$23^\circ\ 27'\ 46''.3.$$

Now, if the obliquity be lessened, the other quantities, such as the right ascension and north polar distance, remaining the same, the angle of position will also be lessened: and its diminution may be computed from this formula, (see p. 178,)

$$dP = dI \times \frac{\cos. Æ . \cos. I}{\cos. P . \sin. \Delta}.$$

If we take $dI = 5''$, cos. $Æ = .873$, cos. $I = .917$, we have

$$dP = 5'' \times \frac{.873 \times .917}{.935 \times .985} = 4''.3.$$

This quantity *added* to 29° 42′ 29″.8 (see p. 179,) will make the angle of position equal

$$29^\circ\ 42'\ 34''.1.$$

---

* The mean obliquity for 1813 is stated by Mr. Pond, (*Phil. Trans.* 1813,) to be 23° 27′ 50″. Therefore if the secular diminution be 45″.7, the mean obliquity for 1815 is 23° 27′ 49″.196.

By formulæ, in principle like the preceding, may the latitude of a star computed for one value of the obliquity, be changed into the latitude due to another value of the obliquity, supposing the obliquity alone to vary and to vary by a small quantity.

The latitudes and longitudes of stars, the *angles of position*, their annual variations, &c. are, as we have already said, mere matters of computation. They are useful, like other Astronomical formulæ, in the elucidation of theories, in the succinct expressions of results, and in the construction of Tables. The quantities on which they depend, or from which they are derived, are the obliquity of the ecliptic, the right ascensions and declinations of stars. These latter are determined by observations, not, indeed, from single observations, nor from those of one or two years, but from observations made at different and distant epochs and continued through a series of years. The longitude of a star may be computed in a few minutes; but it requires the observations of fifty years to settle its right ascension.

We wish to make one remark more before we quit this subject. The preceding latitudes, longitudes, &c. are intended to be the *mean* latitudes and longitudes, and are computed from the *mean* values of the obliquity, and of the right ascensions and declinations. This, for the present, must be taken as a mere statement. We have not hitherto advanced far enough to give a distinct explanation of those *mean* quantities which are, indeed, (it may be here premised) the fictions of Astronomers: abstract quantities never seen nor observed, but which would be, if our theories be right, were certain obstructive or deranging circumstances removed.

In order to aid the computation of formulæ by which the *variations* of the latitude, longitude, and angle of position of a star may be expressed, we subjoin a few formulæ which the attentive Student, by the aid of the annexed references, may easily investigate:

P the angle of position,
λ the latitude of a star,
L its longitude,
δ its north polar distance,
Æ its right ascension,
I the obliquity of the ecliptic.

Then,

1. $\tan. \text{Æ} = \tan. L . \cos. I - \tan. \lambda \sec. \lambda \sin. I$ ·· *Trig.* p. 117,
2. $\cos. \delta = \sin. L . \cos. \lambda \sin. I + \sin. \lambda \cos. I$ 140
3. $\tan. L = \sin. I \cot. \delta \sec. \text{Æ} + \tan. \text{Æ} \cos. I$ 157
4. $\sin. \lambda = \cos. \delta \cos. I - \sin. \delta \sin. I . \sin. \text{Æ}$ 140
5. $\cot. P = \cos. \delta . \tan. \text{Æ} + \sin. \delta . \sec. \text{Æ} . \cot. I$ 157
6. $\cot. P = \cos. \lambda \sec. L \cot. I - \sin. \lambda \tan. L$ 157
7. $\cos. \text{Æ} \sin. \delta = \cos. L . \cos. \lambda$ .... *Trig.* p. 141
8. $\sin. P \sin. \delta = \sin. I . \cos. L$ 141
9. $\sin. P \cos. \lambda = \sin. I . \cos. \text{Æ}$ 141
10. $\sin. \text{Æ} = \sin. L . \cos. P + \cos. L \sin. P . \sin. \lambda$.
11. $\cos. I \cos. \text{Æ} = \cos. L . \cos. P - \sin. L \sin. P \sin. L$.

CHAP. VIII.

Comparison of the Catalogues of Stars made for different Epochs. The annual Increments of the Longitudes of all Stars nearly the same.—The Precession of the Equinoxes.—Comparison of the Latitudes of Stars computed for different Epochs.—The Latitudes of Stars subject only to slight Variations.—Comparison of the North Polar Distances and Right Ascensions of Stars.—Suggested Formulæ of their Variations.—Consequences respecting the Length of the Year, &c. that follow from the fact of the Precession.

In the preceding Chapter, the terms *Mean* and *Apparent* Right Ascension, *Mean* and *Apparent* North Polar Distance, &c. have frequently occurred, without a formal definition of their meanings. Indeed, a definition is not easily given: for, in order to its being intelligible, it ought to enumerate the several circumstances that make a star's mean place to differ from its apparent: which enumeration depends on what is to follow.

The *mean* place of a star differs from its *apparent* place at a given epoch, not for one cause only, but for several. The mean place of a star at one epoch, differs from its mean place at another epoch, almost solely, from one cause: with the explanation of this latter point our course of explanation will begin. We will first shew that the place of a star is different in the year 1815 from what it was in 1760.

The place of a star depends on its distance from the *first point of Aries* and from the pole of the equator: or (for so also may its position be determined) from the first point of Aries and the pole of ecliptic. We may determine then whether a star's place is changed or not, by comparing together its registered right ascensions and polar distances for two different epochs: or, by comparing together its longitudes and latitudes computed

(see pp. 160, &c.) from those right ascensions and polar distances. We will begin with the latter comparison, although it may seem to be more simple to compare together right ascensions and polar distances, which, indeed, may be considered as objects of immediate observation.

In order to deduce the variations of the longitudes and latitudes of some of the principal stars, we will compare our results (see pp. 160, &c.) with Delambre's Catalogue of Longitudes and Latitudes inserted in the *Connoisance des Tems* for the year 1756. The following Tables contain this comparison (see pp. 160, 161, &c.)

Longitudes.				
Stars.	1815.	1756.	Diff. of Long. in 59 Years.	Mean Annual Increase.
α Arietis.	$1^s\ 5^0\ 4'\ 41''$	$1^s\ 4^0\ 15'\ 3''$	$49'\ 38''$	$50''.47$
Pollux.	3 20 39 48	3 19 50 55	48 53	49.7
Spica Virginis.	6 21 15 31	6 20 26 20	49 11	50.1
α Aquilæ	9 29 9 53.8	9 28 20 6*	49 57.8	50.8
α Pegasi.	11 20 54 31	11 20 5 19	49 12	50.1

Here it appears, notwithstanding the different positions of these stars with regard either to the pole of the equator or ecliptic, that their longitudes are increased by *nearly* the same

* The star's longitude would be $9^s\ 28^0\ 20'\ 1''.1$, the elements of the computation being

Æ .... =	$294^\circ\ 43'\ 7''.36$
N. P. D............... =	81 45 27.92
and obliquity............... =	23 28 14.86.

quantity. If we divide the numbers in the fourth column by 59 †, the results will be (as they are expressed in the fifth column) the mean annual increases of the longitudes.

By the mere comparison then of the longitudes of stars at different epochs, we arrive at the important fact of the *nearly equal increases of those longitudes* at the rate of about 50″ annually. We may *account* for it (or assign a probable reason for the fact) either by supposing the whole sphere, on which the stars seem placed, to be slowly turned (in addition to its diurnal rotation round the prolonged axis of the Earth) round an axis passing through the poles of the ecliptic, or by supposing the intersection of the equator and ecliptic, the *first point of Aries* as it technically is called, to have *retrograded.*

This *retrogradation* (the fact in its relation to Astronomical calculations is the same in either supposition) is technically called the *Precession of the Equinoxes.* Its mean value estimated from the five preceding stars is $\frac{251.17}{5} = 50''.23$.

But the *Precession* is an Astronomical element of too much importance to be estimated from a few observations, or (we should say, if we did not know the past state of science) from observations not distant from each other by more than sixty years. If the observations of Hipparchus, who lived one hundred and twenty years before the Christian æra were as accurate as the observations now made, or as the observations made in Flamstead's time, we should be able thence to determine the mean quantity of the precession with the greatest precision. But the antient observations are very little to be relied on.

† Since the number of years is 59 (= 60 − 1), we may easily compute the annual increase from the whole increase: thus, if the latter be

$$1^\circ\ 23'\ 34'',$$

$$\text{the former is} \left\{\begin{array}{rrrr} 1' & 23'' & 34''' & \\ & 1 & 23 & 34 \\ & & 1 & 23, \text{\&c.} \\ \hline 1 & 24 & 58 & 59 \end{array}\right.$$

That they are inaccurate, we have evidence from the very statements that have come down to us from Hipparchus and Ptolemy*.

The longitude of Spica Virginis (see p. 171,) in 1815 was $6^s\ 21^\circ\ 15'\ 31''$; therefore, since the longitude of the *autumnal* equinox is 6^s, it may be said, that the latter *precedes* the star by $21^\circ\ 15'\ 31''$. Hipparchus (according to Ptolemy) says that, in his time, the star *preceded* the autumnal equinox by 6° instead of 8°, which it did, according to the observations of Timocharis, made in the year 295 before our æra. Now, M. Delambre very justly observes, first, that these round numbers of 6 and 8 degrees throw great doubts on the precision of the observations; secondly, that the quantity 2° of the precession, at the rate of $50''$ annually, would give an interval of time equal to 144 years instead of 160 or 170 years that intervened between the two observations: so that, it is probable, the observations made or computed were inaccurate to the amount of a quarter of a degree. Now such an error diffused over even as great an interval as 1800 years would still be of moment: it would amount to $0''.833$, and altogether vitiate the investigation.

On this account it is better to compute the precession by comparing together observations that are now making, with observations made about the year 1750 by those distinguished Astronomers Bradley and Lacaille. And this M. Delambre has done; by comparing a great number of his own observations with those of Mayer and of the two last-mentioned Astronomers, he finds the mean quantity of the precession to be $50''.1$.

M. Lalande, in his Astronomy, has computed the *precession*, by comparing the longitude of Spica Virginis as assigned by Hipparchus with the longitude of the same star computed in 1750. Thus,

5

128. A.C. Longitude of Spica Virginis..	$=5^s$	$\cancel{2}4^\circ$	$0'$
1750. A.D........................	$=6$	20	21
difference of longitudes..............	$=0$	26	21

* Ptolemy lived in the year 137 of our æra.

Therefore the mean annual precession =

$$\frac{26^\circ\ 21'}{1878} = 50''\ 30''' = 50''.5.$$

By a number of like comparisons, the same author finds the *secular* precession, that is, the amount of the accumulated precessions for 100 years, to be $1^\circ\ 23'\ 34''$. The *mean* annual precession corresponding to this is $50''.34$; and the sum of such annual precessions amounts to 1° in $71\frac{1}{2}$ years.

If we suppose the precession to be $50''.1$, then, in $25869\ \left(=\frac{360^\circ}{50.1}\right)$ years, the *first point* of Aries will have retrograded through an entire circle.

The quantity $50''.1$, which is the *mean* value of the precession, is obtained from the differences of the longitudes of a great many stars (three or four hundred for instance) computed at different epochs. This mean quantity may not agree with the mean quantity derived from the observations of a single star, however many, or accurately made, those observations may be. It will not be the case with Pollux, the second star in the preceding Table. The difference, however, between the mean quantities of the precession, as they result from 300 stars or a single star, is, in all cases, very small. Still the difference, which is proved to exist, points out to some peculiarity in the single star. It cannot be, like most of the other stars, entirely fixed, but must have, what is called, (or what we are obliged to call from default of a knowledge of its cause) a *proper* motion. For this reason, namely, that the mean longitude of a star is not altered *solely* from the *regression* of the intersection of the equator and ecliptic, or by the precession, Astronomers employ the term of *Annual Variation*, comprehending under it the effect both of precession and of annual proper motion. This subject will be more fully treated of in a subsequent Chapter.

The comparison of the longitudes of stars computed for the two epochs of 1750 and 1815 establishes, as we have seen, the important fact of the precession of the equinoxes. Let us now compare the latitudes.

Latitudes.			
Stars.	1815.	1756.	Diff. in 59 Years.
α Arietis.....	9° 57′ 37″.4 N	9° 57′ 32″	+ 5″.4
Pollux......	6 40 18.4 N	6 40 3	+15.4
Spica Virginis.	2 2 24.8 S	2 2 6	+18.8
α Aquilæ....	29 18 35.5 N	29 18 44*	− 8.5
α Pegasi.....	19 24 40. N	19 24 44	− 4

It appears from this Table that the changes of latitudes are very small; in no case amounting, annually, to 0″.4. The Astronomical fact then is, a minute annual change of latitude with a considerable annual change of longitude. With regard to the *cause* of the former change, we may *conjecture* that it arises either partly from the precession of the equinoxes and partly from other causes, or, that is altogether independent of the precession. We shall consider this matter again; at present we have not sufficient means to remove our uncertainty.

In the mean time we will examine (what are indeed the *foundations* of the preceding Tables of longitudes and latitudes) the right ascensions and north polar distances of stars observed at the two different epochs of 1756 and 1815. To the former stars we shall add some others for the farther elucidation of the subject.

---

* The latitude computed, from the star's Æ =294° 43′ 7″.36
its N. P. D.= 81 45 27.92
and obliquity= 23 28 14.86
would be 29° 18′ 39″.2.

North Polar Distances.

Stars.	1756.	1815.	Variation in 59 Years.	
	° ′ ″	° ′ ″	′ ″	″
γ Pegasi...	76 10 25.85	75 50 40.6	−19 45.25	−20.08
α Arietis............	67 42 12.86	67 25 1.69	−17 11.17	−17.4
Aldebaran (α Tauri)	74 0 14.89	73 52 19.46	− 7 55.43	− 8
η Geminorum......	67 26 52	67 27 1.18	+ 0 9.18	+ 0.15
Pollux (β Gemin.)	61 24 26.8	61 32 12.34	+ 7 45.54	+ 7.9
δ Ursæ majoris ...	31 36 32.75	31 56 17.95	+19 45.2	+20.09
Spica Virginis (α)	99 52 46.76	100 11 28.9	+ 18 42.14	+19
γ Draconis.........	38 28 20.294	38 29 5.05	+ 0 44.756	+ 0.75
δ Sagittarii.........	119 54 15.535	119 53 35.28	− 0 40.255	− 0.68
α Aquilæ...........	81 45 27.92	81 36 40.54	− 8 47.38	− 8.9
α Pegasi............	76 6 11.51	75 47 12.84	− 18 58.67	−19.3

Right Ascensions.

Stars.	1756.	1815.	Variation in 59 Years.	Annual Variation.
	° ′ ″	° ′ ″	′ ″	″
γ Pegasi........	0 10 27.511	0 55 46.95	45 19.44	46.09
α Arietis..... .	28 22 7.201	29 11 27.3	49 20.1	50.1
Aldebaran	65 29 12.999	66 19 42.9	50 29.9	51.34
η Geminorum ...	90 2 14.924	90 55 39.44	53 24.51	54.3
Pollux.........	112 35 16.989	113 29 39.6	54 22.61	55.3
δ Ursæ majoris ..	180 48 23.344	181 33 8.22	44 44.87	45.5
Spica Virginis...	198 5 34.856	198 51 52.95	46 18.1	47.09
γ Draconis......	267 44 11.386	268 4 40.2	20 28.81	20.82
δ Sagittarii	271 20 38.827	272 16 42.9	56 4.06	57
α Aquilæ.......	294 43 7.36	295 26 17.7	43 10.34	43.9
α Pegasi	343 9 20.86	343 53 15.15	43 54.29	44.65

We will first examine the Table of North Polar Distances. The first star, γ Pegasi, is subjected to the greatest diminution in north polar distance, α Arietis suffers less, and Aldebaran still less. The north polar distance of the fourth star (η Geminorum) is *augmented*, but by a very small quantity. The north polar distance of Pollux is augmented by a greater quantity, and δ Ursæ majoris by the greatest ($20''.09$). The north polar distance of γ Draconis is very slightly augmented. Those of the remaining stars are diminished, and the last star (α Pegasi) suffers a diminution of north polar distance nearly equal to that of γ Pegasi.

Now the slightest inspection of the Table will shew us that these variations of north polar distances, whether we regard their quantities or their directions, are entirely independent of the north polar distances themselves. We must look, therefore, elsewhere for a clue to lead us to the detection of the law (if any such should exist) that regulates these variations of polar distance. If we look to the second Table we shall easily perceive a connexion between the above-mentioned variations and the right ascensions.

For instance, γ Pegasi which has the smallest right ascension, suffers the greatest diminution in north polar distance. The change in the north polar distance of η Geminorum is very small and positive, and its right ascension is a little *more* than 90^0. It is, therefore, immediately suggested to us that, if its right ascension had been exactly 90^0, its polar distance would have been unaltered. Again, of the following stars, the north polar distance of δ Ursæ majoris is augmented by the largest quantity (by a quantity equal the diminution of the north polar distance of γ Pegasi) and its right ascension a little exceeds 180^o. We have next γ Draconis; its right ascension is a little *less* than three quadrants, and the variation of its north polar distance *small* and *additive;* then, δ Sagittarii, its right ascension is a little *greater* than three quadrants whilst the variation of its north polar distance is nearly equal to that of the former star but *subtractive*. The right ascension of α Pegasi is about seventeen degrees less than 360^0, and the variation of its north polar distance is of the same sign with that of γ Pegasi, but somewhat less.

To those who are acquainted, in the slightest degree, with the properties of Trigonometrical lines, it will be obvious that the above variations are *analogous* to the variations of the cosine of an arc that passes through all its degrees of magnitude from $0°$ to $360°$. Suppose, then, we should conjecture the variation in north polar distance to be comprised under this formula

$$- C . \cos. Æ,$$

in order to examine the truth of the conjecture, we have (taking the mean of the two cosines),

$$\begin{array}{llll} \cos. & 0° & 33' & 6'' = .9999 \\ \cos. & 28 & 46 & 47 = .8764 \\ \cos. & 65 & 54 & 27 = .4028 \\ \cos. & 90 & 28 & 57 = .0083. \end{array}$$

Equate $- C . \cos. 0^0\ 33'\ 6''$, and $- 20''.08$, and we have

$$C = \frac{20''.08}{.9999} = 20.082, \text{ nearly,}$$

and, accordingly,

$$\begin{array}{lll} -20''.082 \times & .8764 = & - \ 17''.5, \text{ nearly,} \\ -20''.082 \times & .4082 = & - \ 8.1 \\ -20''.082 \times & -.0083 = & \ \ 0''.16, \end{array}$$

which results agree, very nearly, with those of the last column of of the Table, p. 189. And if we were to make a like experiment of the truth of the formula $- C . \cos. Æ$, with the remaining stars, we should find a like near* agreement between its results and the numbers of the above-mentioned last column.

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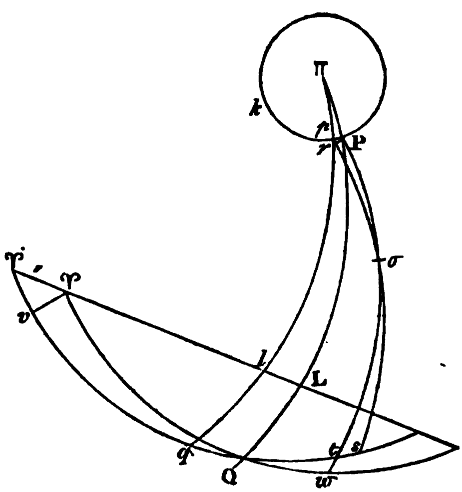
* We cannot expect an *exact* agreement. In order to *try* whether the *conjectural* formula ($C . \cos. Æ$) were true, we took the cosine of the mean of the two arcs and multiplied it into $\frac{1}{59}$th part of the whole variation of north polar distance between the years 1756 and 1815. But this mode of proceeding was adopted merely, as we said, for the purpose of procuring a test of the truth of the formula. If $C . \cos. Æ$ be a true formula $-20''.08 \times \cos. 29°\ 11'\ 27''.3$ is the *annual variation* of *a* Arietis for the year 1815 : which will differ (a little indeed) from

$$\frac{17'\ 11''.17}{59} \times \cos. \tfrac{1}{2}(67°\ 42'\ 12''.86 + 67°\ 25'\ 1''.69).$$

We have now then, from the mere examination of the registered right ascensions and declinations of stars and their computed latitudes and longitudes, established, or rendered probable the existence of, three important facts. The first is, the nearly equal increases of the longitudes of all stars at the rate of about fifty seconds of space annually; the second is, the very small annual changes of their latitudes, or the nearly permanent position of the pole of the ecliptic: the third is, the annual variations of the declinations of stars regulated, both in their directions and quantities, by the right ascensions.

The variations of the right ascensions of stars (see Table, p. 189,) we have not examined for the purpose of finding out their law, which is not so obvious, nor so easy to be detected, as that of the variations in declination. We have, however, found out sufficient to enable us to make further conjecture concerning the cause (if there should be only one) which produces those variations that have resulted from the preceding investigations.

For instance, let π be the pole of the ecliptic ♈ L, P the



pole of the equator ♈ $v\,Q$. Let the retrogradation of the intersection (♈) of the equator and ecliptic be expressed by ♈ ♈′: then Q the solstitial point, 90° distant from ♈, must also *regress* to some point q, and the position of the solstitial colure, from

that of πPLQ, will become that of πplq. Now, it has appeared (see p. 179,) that the obliquity of the ecliptic is subject to very slight variation. It may be supposed nearly constant during small portions of time (during portions of a year, for instance,) in which case, P may be supposed to have regressed to p through a circular arc Pp, the radius of which is πP. Let σ be a star: $P\sigma$ is its polar distance when the pole is at P, $p\sigma$ its polar distance when the pole is transferred to p: and $p\sigma - P\sigma$ will be its change of polar distance arising from *precession*.

The right ascension of the star, when the pole is at P, will be $wQ\Upsilon$, and, when the pole is transferred to p, $stq\Upsilon'$. The increase, therefore, of right ascension arising from precession, will be

$$st + tq\Upsilon' - wQ\Upsilon$$

$$\text{or } st + tqv + \Upsilon'v - wQ\Upsilon,$$

(Υv being perpendicular to $\Upsilon'v$),

$$\text{or (since } tqv = wQ\Upsilon)$$

$$st + \Upsilon'v.$$

We have then, on the supposition that the motion of the pole of the equator is rightly represented by the preceding scheme, a mode of computing the variation of right ascension. If its variation and that of the polar distance, computed according to the same scheme, should accord with the results of observations, there would arise a *presumption* that the scheme was true, or that it adequately represented the nature and law of the change of the intersection of the equator and ecliptic.

We may add farther that, according to the above method of representing the change of the position of the equator, the latitudes of stars will remain unaltered: a consequence (see p. 188.) which accords, very nearly, with the results of observation.

In a future Chapter we shall enter more fully into this subject. It is sufficient for our present purpose to have shewn, that the mere examination and comparison of registered observations is sufficient to *suggest* the laws of the variations of the polar distances and right ascensions of stars and of schemes and

formulæ for representing and computing them. Whether the laws and formulæ so suggested be true or not must be decided by the test of observations. We can do nothing else than try whether or not the results of the formulæ of computations accord with those of observations. Science furnishes us no surer clue than this to guide our researches. And Astronomy, exacted as it may now seem, is merely a system built up by like trials and processes.

Before we proceed to this verification, we wish to deduce a few results that necessarily follow from the *Astronomical fact* of the precession of the equinoxes.

The intersection of the equator and ecliptic *happens* when the altitude of the Sun's centre is equal to the co-latitude of the place. The instant of time, therefore, of the intersection, or of the Sun's being in the equator is that, at which the above equality takes place. Now, with fixed instruments, it is the Sun's meridional altitude only which is observed. It may happen, indeed, but it is very unlikely to happen, that the Sun's meridional altitude shall be equal to the co-latitude of the place. The instant of time then of the Sun's being in the equator, must, in almost every case, be determined by computation: by observing one altitude less than the co-latitude, and, on the succeeding noon, an altitude greater than the co-latitude, and then by computing the time between the two successive noons, at which the Sun was at that intermediate altitude which is equal the co-latitude of the place.

For instance, by Observations at Greenwich in March 1770,

	Z. D. Sun's Centre.	Mean Time.
March 20	51° 29′ 59″.5	12ʰ 7ᵐ 36ˢ.5
March 21	51 6 18.5	12 7 18.5
	0 23 41	0 0 18

But latitude of Greenwich	=51° 28′ 38″.5
Zenith distance of the Sun on 20th	=51 29 59.5
	0 1 21

Hence, (since the interval of time, corresponding to the change of $23' \, 41''$ in the Sun's zenith distance, is

$24^h - 18^s = 86382^s$), we have

$23' \, 41''(=85260''') : 86382^s :: 1' \, 21'' : 1^h \, 22^m \, 3^s.2$;

consequently,

at $12^h \, 7^m \, 36^s.5 + 1^h \, 22^m \, 3^s.2$, or, at $13^h \, 29^m \, 39^s.7$ mean solar time, the Sun's zenith distance was equal $51^\circ \, 28' \, 38''.5$; or, in other words, the Sun was then in the equator.

Find, by a like process, the time of the Sun's being in the equator in 1771, and the interval of the times is the length of an *equinoctial* year. Or, by finding the time of the Sun's being in the equator at some other epoch, in 1820 for instance, we may find the interval of time due to fifty *equinoctial* years, and thence the mean value of one equinoctial year. For instance,

	Sun's Zenith Distance.	Mean Time.
March 20, 1820	$51^\circ \, 32' \, 52''.5$	$12^h \, 7^m \, 37^s.8$
March 21	51 9 11.5	12 7 19.5
	0 23 41	0 0 18.3

Sun's zenith distance	$51^\circ \, 32' \, 52''.5$
Equator's zenith distance	51 28 38.5
	0 4 14

Hence, $85260''' : 86381^s.7 :: 4' \, 14'' : 4^h \, 17^m \, 18^s$.

The Sun therefore entered the equinox on March 20, 1820, at $16^h \, 24^m \, 55^s.8$, and, consequently, the interval of time, between the equinoxes of 1770 and 1820, equals

$$50 \text{ years} + 16^h \, 24^m \, 55^s.8 - 13^h \, 29^m \, 37^s.1:$$

now, out of the fifty years, twelve are *Bissextiles*, or contain 366 days, consequently, the above interval equals

$$50 \times 365^d + 12^d \, 2^h \, 55^m \, 18^s.7,$$

and one-fiftieth of this sum, or the mean value of one year equals

$$365^d \, 5^h \, 49^m \, 6^s.374.$$

This is an easy consequence when the interval between two equinoxes is already known. But if, independently of previous results, we sought, by direct processes, the length of the year, we could more simply effect it, thus:

	Sun's Zenith Distance.	Mean Solar Time.
March 21, 1820	$51^{0}\ 9'\ 11''.5$	$12^{h}\ 7^{m}\ 19^{s}.5$
March 21, 1770	51 6 18.5	12 7 18.2
	0 2 53	0 0 1.3

We must then enquire at what time on March 21, 1770, (civil time) the Sun's zenith distance was $51^{0}\ 9'\ 11''.5$: because, the interval between two equal zenith distances observed in 1770 and 1820, and each equal to the latitude of the place, must equal the interval between any other two equal zenith distances observed, respectively, in 1770 and 1820, and towards the same equinox. The enquiry, then, is reduced to the finding of the time due to a decrease of $2'\ 53''$ in zenith distance. Now, (see p. 194,) the Sun's zenith distance on March 20, 1770, at $12^{h}\ 7^{m}\ 36^{s}.5$ was $51^{0}\ 29'\ 59''.5$; therefore the decrease in zenith distance in $24^{h} - 18^{s}.3$ was $23'\ 41''$. Hence, as before,

$$23'\ 41'' : 86381^{s}.7 :: 2'\ 53'' : 2^{h}\ 55^{m}\ 14^{s}.94;$$

consequently, the interval of time between two equal zenith distances of the Sun, near the same equinoctial point, in 1770 and 1820, is

$$365^{d} \times 50 + 12^{d} + 2^{h}\ 55^{m}\ 16^{s}.24,$$

which is, nearly, the same result as was obtained before in p. 195.

Instead of equal zenith distances of the Sun's centre, we may use (which will be a more simple operation) equal zenith distances of his upper limb, or of his lower limb: and, since the equal zenith distances may be any where assumed (provided they are referred to the same equinoctial point) we may find, by processes like to the preceding, the length of the year from observed equal distances of the Sun's centre, or of one of his limbs, at or near the solstices.

The following is an instance of the determination of the length of the year from observations of the altitudes of the Sun's upper limb at Paris.

Altitude of Sun's Upper Limb.

March 20, 1672. .	41° 43′ 0″		
March 20, 1716. .	41 27 10	. . . .	41° 27′ 10″
		March 21, 1716. .	41 51 0
	0 15 50		0 23 50

But $\dfrac{15' \; 50''}{23' \; 50''} \times 24^h = 15^h \; 56^m \; 39^s$, nearly.

Hence, since in this interval of forty-four years there were ten bissextile years, the whole interval between the equal altitudes of the Sun's upper limb is

$$44 \times 365^d + 10^d \; 15^h \; 56^m \; 39^s,$$

and one forty-fourth part, or the mean length of one of the years is

$$365^d \; 5^h \; 49^m \; 0^s.88.$$

This value is different from the preceding one of p. 195: and, if we were to select and operate on the observations of other epochs, the resulting value of the length of a mean year would, most probably, differ from both of the preceding values. The difference is too considerable to be attributed to the errors of observation. There exists, as it will be shewn in the solar theory, a real difference, which arises from the motion of the Earth in an ellipse, the *ellipse itself being moveable.*

Since then the Sun, after quitting the equinoctial, or the solstitial point, does not continue to return to the same points in intervals of time exactly equal, the lengths of all real equinoctial years or *tropical* years, as they sometimes are called, cannot be equal. The preceding value then of the length of a year, (see pp. 195, 197,) whether it be the fiftieth or forty-fourth part of the time absolved between similar positions of the Sun, ought not, if we would preserve the analogies of language, to be called the length of a *mean* solar year. It may be called, as, indeed, it usually is, the mean length of an *apparent* solar year. The length of a *mean* solar year, if we would derive it solely from observation, must be obtained from the comparison of observa-

tions, distant from each other by an interval of time equal to that in which the *apogee* of the Earth's orbit *progresses* through 360° *.

But, as it will be shewn hereafter, there are other means of computing the length of a *mean* solar year, than those which are founded on the comparison of observations separated from each other by so long an interval. M. Delambre estimates the length of the year at $365^d\ 5^h\ 48^m\ 51^s.6$ (=365.226396593684).

A *sidereal* year is the time elapsed from the Sun's quitting a particular star to his next return to the same star; or the interval between two successive periods, at each of which the difference of longitude between the Sun and a star was the same quantity. This year must differ from the equinoctial year, by reason of the *precession:* it must be greater than the latter year by the time taken up by the Sun in describing $50''.1$. The length of a sidereal year, then, can be determined by no method more obvious or more correct, than that of adding, to the computed length of the equinoctial year, the time of describing $50''.1$: or, which rests on the same principle, than that of finding the time of describing 360° from the previously ascertained time of describing $360^0 - 50''.1$. Thus, T denoting the length of the equinoctial year,

$$360^\circ - 50''.1 : 360^\circ :: T : T \times \frac{360^\circ}{360^\circ - 50''.1},$$

the length of the sidereal year: which therefore equals

$$365^d.2236396593684 \times \frac{1296000}{1295949.9};$$

$$\text{or } 365^d\ 6^h\ 9^m\ 11^s.5,$$

the difference, therefore, of the length of a sidereal and equinoctial year is

$$20^m\ 19^s.9.$$

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* The *progression* of the apogee is not the sole cause of the inequality of solar years (see vol. II., pp. 453, &c.)

But as it appears by p. 198, the length of a sidereal year may be *immediately* determined from observations. Thus,

April 1, 1669, at $0^h\ 3^m\ 47^s$,

longitude of Procyon — Sun's longitude $= 3^s\ 8^0\ 59'\ 36''$.

In April 2, 1745, at $11^h\ 10^m\ 45^s$, there was the same difference of longitudes.

The interval of the two epochs is 76 years (18 of which were bissextiles) $+ 1^d\ 11^h\ 6^m\ 58^s$, or $27759^d\ 11^h\ 6^m\ 58^s$. But since an exact number of sidereal ~~years had~~ elapsed, which number can be no other than 76, we have the mean length of one year equal to

$$\frac{27759^d\ 11^h\ 6^m\ 58^s}{76}; \text{ that is, } 365^d\ 6^h\ 8^m\ 46^s.27.$$

In the Chapters on the Solar Theory and the Calendar, two other kinds of years the *Anomalistic* and the *Julian* will be considered, and their lengths found. Indeed, in now treating of the equinoctial and sidereal years, we have anticipated what will be the subject of future enquiries. But the quantity of the *precession* of the equinoxes being found, the digression towards the subject of the equinoctial years was natural and of little difficulty.

But there are subjects of primary importance that now claim our attention. If the Astronomical doctrine 'that there are *fixed* stars,' be true, it must be shewn why the apparent places of such stars do not remain the same: why the apparent place of a star is sometimes not the same on two successive days; in different months of the same year; on the same days of different years. We must, in fact, go into that investigation, the entrance to which was merely pointed out in pp. 44, &c. The objects of investigation are the causes that render *unequal* the places of stars at different temperatures, in different seasons, in different years, and in different positions of the observer. The knowledge of these causes, and the determination of their quantities and laws, constitute, what is called, the *Theory of Corrections*,

by which an observed, or *apparent* place of a star is reduced to a *mean* place; and by which, if we illustrate it by an instance, an observation of α Aquilæ made at Palermo on April 1, 1800, may be compared with an observation of the same star made at Greenwich on June 15, 1819.

CHAP. IX.

THEORY OF CORRECTIONS.

On the Corrections to be made to the observed or Apparent Right Ascensions and Declinations of Stars in order to reduce them to the Mean.—Refraction.—Aberration.—Precession.—Inequality of Precession.—Nutation.—Parallax.

THE *inequalities* that cause the apparent places of stars to differ, sometimes from themselves and always from their mean places, are not discernible without the aid of instruments. They are exceedingly minute, and their existence does not affect that explanation of the general phenomena of the Heavens, (see pp. 7, 8, &c.) which is founded on the rotation of the Earth and the permanence of the places of certain stars on the apparently concave surface of the Heavens.

To the fact of the mere rotation of the Earth round its axis, which is sufficient for the general explanation of phenomena, we must add another, quite essential to their minute and particular explanation, which is the *uniformity* of the Earth's rotation. This latter is a fundamental principle admitting neither great nor slight modifications, and proved to be true by its being the basis of a large class of Astronomical calculations, and by the agreement of their results with observations.

We cannot add to this, as fundamental principles of the same kind and invariability, the *parallelism* of the Earth's axis of rotation and the permanence of the places of the *fixed* stars. Both these latter principles are very nearly, but not exactly, invariable. They require certain modifications which it will be part of the business of the present Chapter to explain.

The *fixed* stars are so called from preserving (what nearly takes place) the same distances from each other. If their mutual distances were strictly invariable, still it would not follow that their *places* were invariable: for, as we have seen, (see pp. 46, 152, &c.) the *place* of a fixed star is referred to two points, one, the pole of the equator, the other, the intersection of the equator and the ecliptic; which two points are not fixed.

The inequalities that arise from the motions of the pole and the equinoctial point affect a star's place, and continue to do so by the same kind of effect. That is, if the north polar distance of a star be diminished from March to June, it will be still farther diminished in the next October, and continue to be diminished during succeeding portions of time. There is no periodical variation nor *recurrence* of effect, except after exceedingly long periods. But there are inequalities, if we may so express ourselves, of a simpler kind, that affect, and irregularly, the star's polar distance: on one day diminishing that distance by a certain quantity: on the next, perhaps, by a larger quantity, and on the third day, perhaps, by a quantity less than that of the first diminution. The inequality we allude to is *refraction;* depending, at a given altitude of the observed star, on the weight, temperature, and probably, moisture of the amosphere, and, consequently, to be computed by means of the barometer, thermometer, and hygrometer.

We will begin with this inequality: first explaining, in a general way, its cause, and then establishing its existence as a phenomenon, by means of one or two simple observations.

The atmosphere which surrounds the Earth is to be considered as a medium of variable density, decreasing as the distance from the Earth's surface increases. A ray of light then from a star, in its progress through the atmosphere to the eye of the observer, continually passes from a rarer to a denser medium, and, consequently, according to optical laws, is continually deflected. The *deflections* take place towards a perpendicular to the medium at the point of the light's impact, and, consequently, the variation of the density of the atmosphere being gradual, the

path of the star's light through the atmosphere will be curvilinear and, if we may so express it, convex towards the star. The star, then, will seem to be in the direction of a tangent to that last portion of its curvilinear path which enters the eye of the observer: and, consequently, the star will appear to be elevated above its true place.

This deviation must take place in a plane perpendicular to the atmosphere, and passing through the star and the spectator. At each point of the light's progress, the medium to the right and left, in the direction of a perpendicular to the above plane, being supposed, for small distances, to be the same, no *lateral* deviation can take place. Hence the refraction takes place, and entirely, in the plane of a vertical circle. The vertical plane becomes the plane of the meridian when the star is to the north or south of the spectator. Hence, in observations made with a mural quadrant or circle, the whole effect of refraction takes place in declination, whilst the right ascension continues unaltered. The observations, therefore, made with a transit instrument are independent of refraction, so that, if its middle wire be *meridional*, the time of a star's transit will be the same at whatever point of the wire it passes (see p. 83.).

This is a brief explanation of the inequality of refraction from a consideration of its cause. We will now shew, by a simple instance or two, its existence as a fact or phenomenon.

If the angular distance of *Polaris* and γ *Andromedæ* (both being below the pole) be observed at Cambridge, it will be found to be about* 46° 41′: but if the same stars be observed when γ *Andromedæ* passes the meridian to the south of the zenith, its distance from *Polaris* will be greater than before, by about eight minutes, and be very nearly 46° 50′.

If similar observations be made at Paris of the same stars, then, in the first case, their distance will be about 46° 22′, and, in the latter, about 46° 45′.

* The variations in the state of the air prevent the distance from always being the same.

We have then the angular distance of two stars (which ought, were their light not impeded, to remain unaltered) represented by four different instrumental angles. But the inequalities in these angles are perfectly explicable on the principles that have been laid down. For instance, by the observation at Paris, the distance of the two stars, below the pole, was 46° 22′, whereas it was 46° 41′ at Cambridge. But the latitude of Paris being 48° 50′ 13″.3, and the north polar distance of γ Andromedæ 48° 33′ 12″, the latter star, when on the meridian below the pole, would not be much more than 17′ above the horizon; whereas, in a similar position at Cambridge, the latitude of which is 52° 13′ 24″ the star's elevation would be more than 3° 40′. Now, at the former elevation, the light from γ Andromedæ would suffer much greater refraction than at the latter, and, consequently, the apparent distance between it and Polaris would be less at Cambridge than at Paris, although Polaris itself would be a little more elevated by refraction at the latter than at the former place.

The other cases admit of a like explanation; the distance between the stars when they are above the pole must be greater than when below, because γ Andromedæ, in the first case, is not more than 10° from the zenith when its light will not suffer much refraction; whereas, in the latter, it is very near the horizon when its light will be refracted as much as it can be.

It is easy to find other instances of like nature: for example, in the Greenwich Observations of Dec. 8, 1815, we have

Barometer.	Thermometer. In.	Out.		Zenith Distance.
29.88	27	24	Polaris.	36° 50′ 25″
29.94	29	21	Polaris, S. P.	40 10 44.4
29.88	27	23	β Ursæ minoris, S. P.	53 35 37.8
29.96	28	25	β Ursæ minoris.	23 25 17.9

Now (see p. 129,) half the sum of the greatest and least zenith distances of a circumpolar star, is equal to the co-latitude of the place of observation. Hence the co-latitude of Greenwich from the above two observations of Polaris is

$$\tfrac{1}{2}(77^\circ\ 1'\ 9''.4),\ \text{or}\ 38^\circ\ 30'\ 34''.7,$$

from the observations of β Ursæ minoris,

$$\tfrac{1}{2}(77^0\ 0'\ 55''.7),\ \text{or}\ 38^0\ 30'\ 27''.85:$$

the co-latitude, then, which should be an unalterable quantity, is represented, by reason of some inequality, by two different quantities. But of this circumstance, as of the former, the explanation (at least the general explanation) easily follows from the cause that has been propounded. Each star, in both its positions, will be apparently elevated by the deflection of its rays; its two zenith distances, therefore, will be less than they ought to be, and, consequently, half their sum, which is to represent the co-latitude, will be less than its true value. Again, the defect of this half sum from the true value of the co-latitude is greater in β Ursæ minoris than in Polaris; because the former star in its greatest distance from the zenith is distant from it nearly 54°, and therefore the course of its light is much more bent than that of the light from Polaris.

But it is not only that different circumpolar stars give, according to the preceding method, different values for the latitude, but the same star will, on two different days, give different values. Thus, by the Greenwich Observations of 1812.

	Barometer.	Thermometer. In.	Thermometer. Out.		Zenith Distance.
Oct. 14.	28.82	50	47	*a* Cephei. . . .	10° 19′ 15″.6
	29.10	47	45	*a* Cephei, S. P.	66 41 7.9
Oct. 16.	29.50	50	47	*a* Cephei. . . .	10 19 17.2
		42	39	*a* Cephei, S. P.	66 41 3.6

Half the sum of the zenith distances on Oct. 14, is 38° 30′ 11″.7

——————————————on Oct. 16, is 38 30 10.4

Two results not only differing from each other, but from those that were given in p. 205. But here, as before, the differences in the results may be probably accounted for, if we admit the preceding principles of explanation. In the first place, *a* Cephei at its lowest altitude will be much nearer to the horizon than *β* Ursæ minoris, and, therefore, will be more elevated (more than proportionally elevated). In the second place, the observations of *a* Cephei, below the pole, on the sixteenth and fourteenth, were made under *different circumstances.* These different circumstances are the *weight* and *temperature* of the air. On the latter day the barometer was four-tenths of an inch *higher*, and the thermometer six degrees *lower* than on the former day. For both reasons then (for each instrument shewed the air to be denser) the star would be more elevated, and its zenith distance more diminished, on the 16th than on the 14th; which is the fact shewn by the observations. If the above were a solitary instance, no great reliance ought to be placed on the preceding reasonings; for, the errors of one or two seconds of space may be the errors of the instrument, or of the observer. But the Greenwich Observations contain numerous similar instances, all tending to the same conclusion and to exclude the supposition of instrumental or accidental error.

This inequality of *refraction* then causes the north polar distances, and the zenith distances of stars to be apparently unequal on contiguous days. The *law* of the inequality is only that which can be inferred from experiments and observations on the state of the atmosphere.

The knowledge of the theory of refraction, and of the expression of its laws by formulæ, enables us to divest observations in altitude of one kind of inequality. Two north polar distances or two zenith distances, then, of the same star on two following days, or on days distant from each by short intervals, if unequal by the instrument, ought when *corrected* for refraction, to appear equal. In so small a portion of time, as that of two or three days, the effect of other inequalities would not be sensible by the instrument. Again, two observations of the zenith distances of the same star, made at the interval of two, three, or four months, and

corrected for refraction, if then unequal, would be so from some other cause or causes. If we exclude, by our supposition, error from the instrument and observation, and suppose the *correction* for refraction properly made, two such observations as those that have been just mentioned would be unequal from *precession*. For, if this latter inequality causes, as it does in certain stars, an *annual* variation of 20″ in their north polar distances, it would, supposing its operation uniform, cause a variation of 10″ in half a year, and of 5″ in three months.

These two inequalities, then, of *refraction* and *precession* being known and their effects rescinded, if the zenith distances of the same star, observed at the interval of three, or of six months, should still be unequal, it would be necessary to investigate the source of the inequality. By such steps as have been described, that is, by *correcting* observations for all known and ascertained causes, and then by comparing the observations so corrected, Bradley found the north polar distances and right ascensions of stars to be different at different parts of the same year, but to be the same again in like periods of different years. If the north polar distance of a star were increased in March, it would be diminished in September. In June the inequality would affect the star's right ascension, and if it then increased the right ascension, it would in January, diminish it. This inequality is now succinctly designated by the term *Aberration*. Bradley discovered its cause, (and the discovery is altogether a wonderful one) in the combination of the motion of light with the motion of the Earth in her orbit. His explanation is founded mainly on the fact which Roemer had established from observations of the eclipses of Jupiter's satellites, namely, that light is not instantaneously transmitted but successively communicated and propagated, or that some portion of time is necessary, (no matter how small that portion,) for the light to pass along so short a space as that, for instance, of the tube of a telescope.

The method of detecting a fourth or fifth inequality, would resemble the method already used. If observations of γ Draconis made during March 1815, and *corrected* for the inequalities of *refraction*, *precession*, and *aberration*, differed from the corrected

observations of the same star made during July 1820, that is, if its north polar distance and right ascension were represented in the two periods by different quantities, there would be some new cause or causes of inequality to be detected. Such would be the fact. Observations, like those described, would not agree by reason of an inequality called *Nutation*. We will briefly explain its cause.

We have hitherto (see pp. 185, &c.) viewed the precession of the equinoxes as an Astronomical fact, established by the comparison of the longitudes of stars at different epochs. This effect may be conceived to take place, from the intersection of the ecliptic and equator being endowed with a regressive motion, describing, thereby, gradually and equably the whole arc of the precession.

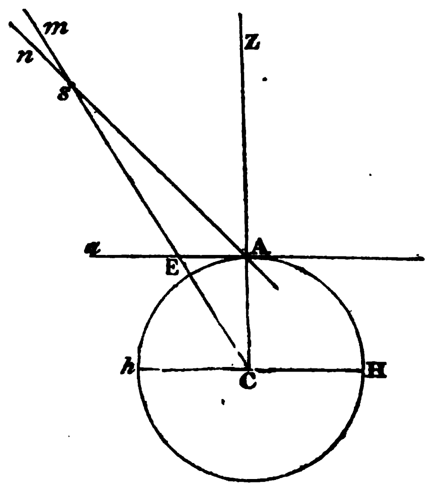
Now the cause of this regression is the action of the Sun and Moon on the bulging equatoreal parts of the terrestrial spheroid. The *actions* of these two luminaries vary, that of the first from its declination, that of the latter from the inclination of its orbit to the ecliptic. From the variableness, then, of these two actions there will arise two inequalities, one called the *Solar Inequality of Precession*, the other the *Nutation* *.

When the preceding *inequalities* are known, a star's *apparent* place may be divested of their effects and *reduced to* its mean place. But another *correction* is still wanting in the case of the Sun and planets. An Observer at the Cape of Good Hope, and an Observer at Greenwich, would, on the same day, refer a planet to different parts of the Heavens. But observations, made at the above-mentioned places, ought to be true to all the world. Each observer then applies, in addition to the five enumerated corrections, a sixth correction, and reduces his observations to the centre of the Earth. This last correction is called

* In the Tables published by Dr. Maskelyne in the *Greenwich Observations*, the *Nutation* is separated into two *equations:* one called the *Equation of the Equinoxes*, the other *deviation* in right ascension, and deviation in north polar distance.

Parallax: it cannot be said to arise from any *inequality:* but is used for the sake of simplicity and the convenience of Astronomical computations.

In simple cases the effect of parallax may be easily shewn. A spectator at *A* would see a star *s*, in the direction *Asn*. Another spectator, situated in the point at which *Cs* cuts the



Earth's surface, or situated at *C* the Earth's centre, would see the star *s* in the direction *Csm*. The difference of the star's places, seen from *A* and *C*, is measured by the angle *AsC* called the *Parallax*.

This is the parallax arising from the situation of the observer on the surface of the Earth. But there is a parallax called an *Annual* Parallax; which is the difference of the places of a star seen respectively from two opposite points of a diameter of the Earth's orbit.

We have now enumerated, and briefly described, the causes of those *corrections* by which the apparent places of stars at one epoch may be *reduced* to their *mean* places, at the same, or at any other, epoch. The first, *refraction*, is independent of the time of the day and year, and varies with the state of the atmo-

sphere and the altitude of the observed body. A knowledge of its laws enables us to translate the angular distance, shewn by the Astronomical instrument, into another distance, such as the instrument would shew, did light pass through a perfectly pervious medium.

The next inequality is that of *precession*, which changes the place of a star by changing, relatively to the Heavens, the place of the observer. It depends on time, inasmuch as, if it augments the north polar distance of a star in May 1816, it will still farther augment it in December, and still farther after an increased lapse of time*. The former inequality affects only the declinations of stars, but this alters both their declinations and right ascensions.

The third inequality, *Aberration*, depends not on the year, but on the time of the year. If it diminishes the right ascension of a star in May 1, 1816, by a certain quantity, it will equally diminish it in May 1817, and at similar times of succeeding years. In the month of October it will augment the right ascension: and, if in these months of May and October, its diminishing and augmenting effects on the star's right ascension are the greatest, in the intermediate months of August and January, its greatest effects will be on the star's declination.

The term *Nutation* was originally meant to be significant, agreeably to its import, of a like motion in the pole of the equator, produced by the variable action of the Moon on that protuberant shell of matter by which the Earth is made greater than a sphere. The variableness of the action depends on the Moon's distance from the equator: which depends, for its mean quantity, on the inclination of the lunar orbit to the ecliptic: which again, as it will hereafter appear, depends on the longitude of the node of the Moon's orbit. The inequality, then, of nutation will be the same, when the longitude of the node is: it will vary, whilst the place of the node continues (which is the fact) to *regress*, and will have experienced all its vicissitudes of augmentation, maximum, diminution and mean state, during a period of *regression;* which period is about eighteen years and an half.

* We exclude from this statement all extraordinary cases of exception.

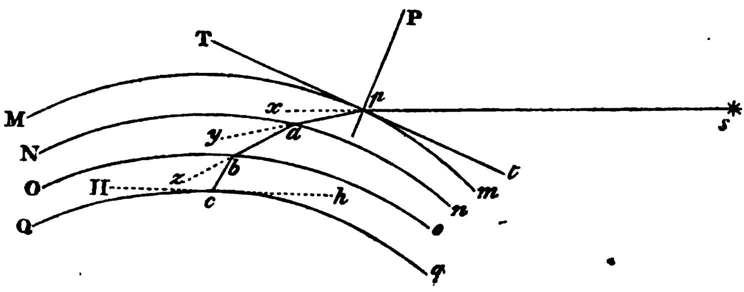
It is proposed to consider each of these inequalities in a separate Chapter, and to obtain (except which we do nothing) their *mean* values, and the formulæ of the laws of their variations. These being obtained, Tables may be constructed which will conveniently exhibit, at any given epoch, the respective quantities of the several inequalities affecting any particular star. The numbers in the Tables are, technically, called *Corrections*, and, when they are applied, with their proper signs, to the *apparent* places of stars, the latter are said to be *reduced* to their mean places. The *apparent* places are what the observer sees and what his instruments shew to him. The *mean* places can never be the objects of observation; but are, as it has been already said, abstract quantities, the results of computations, the last conclusions which Astronomical Science, in its progress towards perfection, has arrived at.

CHAP. X.

REFRACTION.

Refraction.—General Explanation of its Effects.—Computation of its Effects on the Supposition of the Earth's Surface being plane, and the Laminæ of the Atmosphere parallel to it.—Error produced by that Supposition at 80° Zenith Distance.—Tycho Brahe's, and Bradley's Methods of determining the Refraction.—Method of determining the Refraction by Observations of Circumpolar Stars.—Different Formulæ of Refraction.—Dependence of the Value of the Latitude of the Observatory on the Mean Refraction at 45° of Zenith Distance.—Corrections of Refraction due to the Thermometer and Barometer.—Instances and Uses of the Formulæ and Tables of Refraction.—Explanation of certain Phenomena arising from Refraction.

In the preceding Chapter, we have explained, in a general way, how stars become apparently elevated by the deflections of their rays of light in their passages through the atmosphere. The general effect is the same, whether the atmosphere be of an uniform or of a variable density. Suppose s to be a star, and that a ray of light sp falls on Mpm, the boundary between a

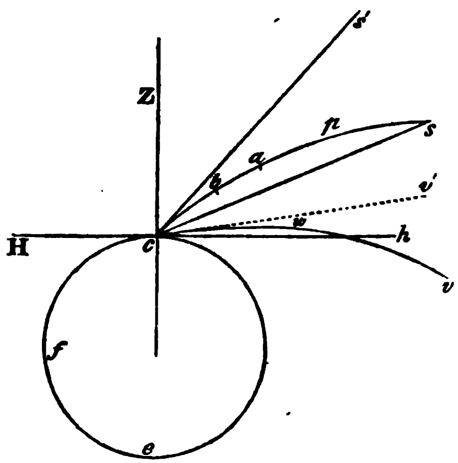


denser and a rarer medium, the former being beneath Mpm. Let Pp be a perpendicular to this bounding surface; then, the ray instead of pursuing the course spx, is deflected at the point p,

into the direction pa. Let Nn be a second boundary similar to the former: then the ray, after the first refraction at p, instead of pursuing the course pay, is deflected, at a, into that of ab. Again, the ray is, a third time, deflected at b from the course abz into bc. The eye of the spectator, supposed to be at c, sees the star in the direction of cb. The inclination of the lines cb and ps is the whole refraction, or deflection, which the ray has undergone.

In what has preceded, the medium contained between Mpm and Qcq has been parcelled out into different strata. But circumstances, similar to those that have been described, would take place, if the medium had been distributed into a greater number of strata. The deflections would have been more, but all the same way: and, if we suppose the parcelling out of the whole medium between Mpm and Qcq into minute portions to be *indefinitely* continued, the course of the ray $pabc$ will become curvilinear, of which pa, ab, bc, &c. are the elements.

We may, therefore, thus represent the course of the ray. Let s be the star, c the spectator, and suppose a plane perpen-



dicular to the Earth's surface to pass through s and c; then (see p. 203.) the media at the several points, on both sides of the plane, being supposed to be the same, the refraction will take place entirely in the plane; that is, entirely in the plane of a

vertical circle. The refraction also taking place, at every point of the light's course, on the supposition that it is made through a medium of a continually varying density, the path of the ray of light will be such as *spabc* is; and, cs', being a tangent to such curve at its extreme element at *c*, will be apparently the star's direction. Or, s' will be the apparent place of the star *s*, and the angle scs' will be the whole refraction.

Let $A =$ the angle *sch*, equal to the star's elevation above the horizon, then, if *r* be the refraction due to that elevation, the star's apparent elevation $= A + r$; and, if $Z = 90 - A$, Z being the zenith distance, the star's apparent zenith distance $= Z - r$.

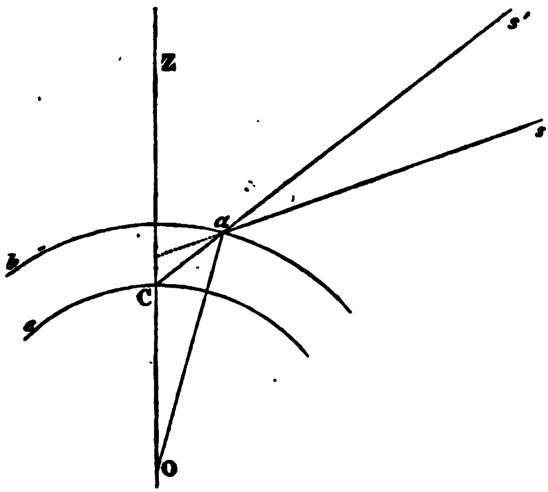
If the star be in the zenith, $r = 0$; since the light, in its descent, cuts the tangent plane of each succeeding stratum of the atmosphere perpendicularly; consequently, there is no reason why it should be deflected towards one part rather than towards any other.

In the zenith then there is no refraction, in the horizon, the greatest. In intermediate points, the refraction is of some mean values, but not proportional to the angular distance of those points from the horizon. The question we have now to consider is, since the refraction varies with the star's elevation, that is, since it is greater (other things being equal) the less the zenith distance, what *function* of the zenith distance, or, what terms involving the sine or tangent of such zenith distance, will represent the *law* of refraction. This is one part of the enquiry; the other part respects the actual *quantity* of refraction at some certain zenith distance, at 45°, for instance, and at some state of the air, held to be its *mean* state.

We have already in a previous Chapter, (see pp. 203, &c.) established the general fact of refraction, and the facts of the increase of refraction with the increase of zenith distance, and of the *small quantities* of refraction at zenith distances less than 90°. On this latter fact as a condition, and the constant ratio existing between the sines of incidence and refraction, we will found a simple formula * of atmospheric refraction.

* This is principally taken from Dr. Brinkley's Investigations, (see *Irish Transactions*, for 1814.)

Let O be the Earth's centre, OC its radius, ab a boundary



between two media (such as were spoken of in p. 213,) sa a ray of light refracted at a into the direction Ca.

Let

$OC = a$

$Oa = a + \delta$, δ very small relatively to a,

$\angle ZCa = Z$,

$I =$ angle of incidence

$R =$ angle of refraction,

$r =$ refraction measured by sas',

and $\sin. I = m . \sin. R$;

$$\text{then } \sin. I = m . \sin. OaC = m \sin. OCa . \frac{OC}{Oa}$$

$$= m \sin. Z\left(1 - \frac{\delta}{a}\right), \text{ nearly.}$$

$$\text{But } \sin. I = \sin. (R + r) = \sin. R \cos. r + \cos. R . \sin. r$$
$$= \sin. R + r \sin. 1'' . \cos. R,$$

since, r being very small, $\sin. r = r \sin. 1''$, and $\cos. r = 1$, nearly.

Substitute now for $\sin. I$, $\sin. R$, $\cos. R$,

and we have

$$m.\sin. Z.\left(1-\frac{\delta}{a}\right) = \sin. Z.\left(1-\frac{\delta}{a}\right)$$

$$+ r.\sin. 1''.\sqrt{\left[1-\sin.^2 Z\left(1-\frac{\delta}{a}\right)^2\right]};$$

but

$$1-\sin.^2 Z.\left(1-\frac{\delta}{a}\right)^2 = 1-\sin.^2 Z.\left(1-\frac{2\delta}{a}\right), \text{ nearly,}$$

$$= \cos.^2 Z + \frac{2\delta}{a}\sin.^2 Z.$$

Hence, the square root $=\cos. Z \times \left(1+\frac{\delta}{a}\tan.^2 Z\right)$, nearly,

and

$$r = \frac{(m-1)\left(1-\frac{\delta}{a}\right).\sin. Z}{\sin. 1''.\cos. Z.\left(1+\frac{\delta}{a}\tan.^2 Z\right)}$$

$$= \frac{(m-1)\left(1-\frac{\delta}{a}\right)}{\sin. 1''}.\tan. Z \times \left(1-\frac{\delta}{a}\tan.^2 Z\right), \text{ nearly,}$$

$$= \frac{m-1}{\sin. 1''}.\tan. Z - \frac{(m-1)\delta}{\sin. 1''.a}(\tan. Z + \tan.^3 Z),$$

or, since

$$\tan. Z + \tan.^3 Z = \tan. Z\sec.^2 Z,$$

$$r = \frac{m-1}{\sin. 1''}.\tan. Z - \frac{m-1}{\sin. 1''}.\frac{\delta}{a}.\tan. Z.\sec.^2 Z.$$

If the terms (see l. 2,) had been farther expanded, terms involving $\tan.^5 Z$, &c. would have been introduced into the preceding formula, which would then have been of this from

$$r = A\tan. Z + B\tan.^3 Z + C.\tan.^5 Z + \&c.$$

In the former expression, if a be made infinite, the second term vanishes, and other terms, had they been introduced, would

also have vanished, since they would have involved the powers of $\frac{\delta}{a}$. In this case then

$$r = \frac{m-1}{\sin. 1''} . \tan. Z;$$

and this is the expression for the refraction, supposing the Earth's surface to be a plane, and light to be transmitted through a stratum of uniformly dense air parallel to the Earth's surface.

But, if the Earth were a plane, and light were transmitted through a number of parallel strata of increasing densities, the refraction would be the same, as if the light, with its first angle of incidence, impinged immediately on the last stratum of air, or on that which is nearest to the Earth's surface. The refraction in that case would be represented by

$$\frac{m-1}{\sin. 1''} . \tan. Z.$$

The other terms of the series, therefore, arise, from the spherical form of the Earth, and from the supposition of concentric laminæ of the atmosphere. Let us estimate the value of the second term, namely, of

$$\frac{m-1}{\sin. 1''} . \frac{\delta}{a} . \tan. Z . \sec.^2 Z,$$

when $Z = 80^\circ$.

Let δ, the height of an uniform atmosphere of the same density as at the Earth's surface, $= 5.095$ miles, a, the Earth's radius, $= 3979$.

m (the ratio of the sines of incidence and refraction, the barometer being $= 29.6$, and the thermometer $= 50^\circ$) $= 1.0002803$;

then, since $\frac{\delta}{a} = .00128$, nearly,

we have

$$*\frac{m-1}{\sin. 1''}\cdot\frac{\delta}{a}\tan. Z.\sec.^2 Z = 13''.92, \text{ nearly.}$$

In computing then the refraction, on the supposition of the Earth being a plane, we fall, at 80° zenith distance, into an error of about 14″, the first term of the refraction being 5′ 27″.9.

At 45°, when $\tan. Z = 1$, the first term, namely,

$$\frac{m-1}{\sin. 1''} = 57''.817,$$

and the second, $\frac{m-1}{\sin. 1''}\cdot\frac{\delta}{a}\tan. Z \sec.^2 Z = 0''.148.$

Hence the mean refraction (barometer = 29.6 inches, and thermometer = 50°) is equal to

$$57''.817 - 0''.148 = 57''.67, \text{ nearly.}$$

At distances from the zenith less than 45°, the second term will bear a still less proportion to the first term: so that, we may safely conclude, for all zenith distances less than 45°, the refraction will vary nearly as the tangent of the zenith distance, and its mean quantity will be expounded by the term

$$\frac{m-1}{\sin. 1''}.\tan. Z, \text{ equal to } 57''.82.\tan. Z.$$

In determining the value of the coefficient $\frac{m-1}{\sin. 1''}$, no reference has been made to *astronomical* refractions. The value of m was assumed equal to 1.0002803, which value was taken from certain direct experiments on the refractive power of air. We shall, however, see that the observations of circumpolar stars

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* Log. .0002803 .... =	6.4476251
log. tan. 80° .... =	10.7536812
2 log. sec. 80 .... =	21.5206596
log. .00128 .... =	7.1072100
arithmetical complement of log. sin. 1″ =	5.3144251
	41.1436010

take away 40 and 1.143601 = log. 13.919.

will enable us to compute the coefficient $\frac{m-1}{\sin. 1''}$, and also the coefficients of other terms, supposing the refraction to be represented by

$$A . \tan. Z + B . \tan.^3 Z + C \tan.^5 Z + \&c.$$

We will now briefly describe the methods by which Tycho Brahé and Bradley determined, *astronomically*, the quantities of refraction.

Let H denote the latitude of the Observatory,
I the obliquity of the ecliptic,
δ the polar distance of a circumpolar star,
Z, Z' its two apparent meridional zenith distances,
S the Sun's apparent summer solstitial zenith distance,
S', his winter;

if ρ, ρ', r, r', be the quantities of refraction due, respectively, to the last apparent distances, then (see pp. 129, 140, 145, &c.)

$$H - I = S + r,$$
$$H + I = S' + r',$$
$$180^0 - 2H = Z + \rho + Z' + \rho',$$

adding these three equations together, we have

$$180^\circ = S + S' + Z + Z' + r + r' + \rho + \rho'.$$

If the refraction varied as the tangent of the zenith distance, or (see p. 217,) could adequately be expressed by $A . \tan. Z$, the first term of the series, we should have, by substituting in the preceding equation,

$$180^\circ =$$
$$S + S' + Z + Z' + A (\tan. S + \tan. S' + \tan. Z + \tan. Z'),$$

from which equation, A would immediately become known, since S, S', &c. are known from observations.

But the first term, A tan. Z, of the formula of refraction will not represent the refraction with sufficient exactness when the observed star is far from the zenith.

The Sun, for instance, at the winter solstice, if Greenwich be the place of observation, will be distant from the zenith by $51^0\ 28'\ 39''.5 + 23^\circ\ 27'\ 50''$, or nearly by 75°. At such a distance, in order to represent the refraction with sufficient exactness, we must take account, at least, of the second term of the formula. If $B.\tan.^3 Z$ represent that second term, the preceding equation of p. 209, l. 24, will be augmented by

$$B(\tan.^3 S + \tan.^3 S' + \tan.^3 Z + \tan.^3 Z'),$$

in which case, if we had observations of only one circumpolar star (the pole star, for instance,) we should have one equation involving two indeterminate quantities A and B.

We cannot, therefore, by the preceding method, and with the aid of only one circumpolar star, determine the formula of refraction, if we suppose it to consist of two terms. If we compute B from the formula of p. 216, supposing m to be known, by direct experiments on the refractive power of air, it will be equal to $0''.073$, nearly.

Since

$$A = \frac{180^\circ - (S + S' + Z + Z') - B(\tan.^3 S + \tan.^3 S' + \tan.^3 Z + \tan.^3 Z')}{\tan. S + \tan. S' + \tan. Z + \tan Z'}$$

if we attribute to B certain small values, such as $0''.05$, $0''.1$, &c. we may deduce, from the above expression, corresponding values of A. But, with these changes in the values of A and B, the latitude will also vary: for (see p. 219.)

$$2H = 180^0 - [Z + Z' + A(\tan. Z + \tan. Z') + B(\tan.^3 Z + \tan.^3 Z')];$$

consequently,

$$2dH = -dA(\tan. Z + \tan. Z') - dB(\tan.^3 Z + \tan.^3 Z').$$

From such expressions, and from Bradley's Observations (observations determining the values of Z, Z', S, S') M. Delambre has formed a small Table exhibiting the alterations which will take place in the resulting values of the latitude, from the differences of values assigned to the coefficient of the principal term of the formula of refraction. This Table is subjoined.

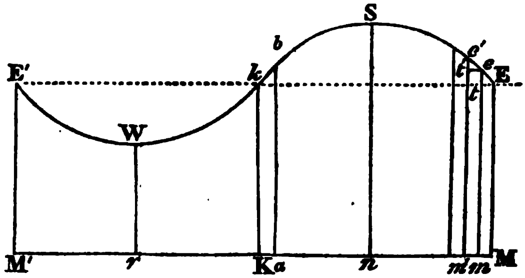
B.	*A.*	*H.*	*dA.*	*dH.*	Refraction at 45°.
−0″.05	56″.92	51° 28′ 39″.65	+	−0″.33	56″.87
.10	57.36	0 0 39.32	0″.44	0.32	57.26
.15	57.81	0 0 39.00	0.45	0.33	57.66
.20	58.25	0 0 38.67	0.44	0.33	58.05
.25	58.70	0 0 38.34	0.45	0.33	58.45
.30	59.14	0 0 38.01	0.44	0.33	58.74

We may perceive in this Table a circumstance worthy of notice; which is that, if we diminish by a certain quantity (0″.4 in the present instance) the refraction at 45°, we increase, by nearly as much (by 0″.33), the latitude of the place, and *vice versâ*; when we speak, therefore, of the latitude of a place, we must be supposed to speak of it as computed by a certain Table of refractions, or according to a certain theory of refractions. The latitude of the Greenwich Observatory computed by the French Tables of refractions, in which the mean refraction, at 45° zenith distance, is 57″.5, is 51° 28′ 38″; by Bradley's Refractions, in which the mean refraction, at 45°, is 56″.9, it is 51° 28′ 39″. Nor is there, as we shall hereafter see, any infallible method of determining the latitude of a place, the problem of refractions being, to a certain extent, an indeterminate one.

The preceding method of determining the refraction from observations of the Sun at the solstices is due to Tycho Brahé. It was imitated, and made better, by Bradley, whose method we will now explain.

Instead of observing the Sun near the solstices, Bradley observed the Sun near the equinoxes. For instance, he observed the Sun (see fig. p. 222.) at *e* and *b*, when it had equal zenith distances; or rather (as the principle has been explained in pages 146, 147, &c.) he observed a zenith distance about the end of March; and then, by observing, on two successive days in September, two other

zenith distances, one less, the other greater than the original one,



he was able to compute at what time between the two latter days the Sun, could it have been observed on the meridian, would have had the same zenith distance as at the first observation in March. By observing also the difference of transits between the Sun and a star, Bradley was enabled to compute the arc comprehended between *e* and *b* (supposing those to be the positions of the Sun when he was at equal distances from the zenith): when *am* then became 180°, the Sun was in the equator: and by computing (see pp. 149, &c.) the changes of zenith distance proportional to the changes of right ascension, the illustrious Astronomer, of whom we are speaking, was enabled to compute the Sun's zenith distance when he was in the equator; or, those two equal zenith distances which were distant from each other by twelve hours of right ascension. Now these observed zenith distances were less than the true, by reason of refraction. Let S represent the zenith distance, and r its refraction, then the true zenith distance is

$$S + r,$$

and S, according to Bradley, was $51^{0}\ 27'\ 28''$*.

The half sum of the observed zenith distances of Polaris, above and below the pole, was found † equal to

$$38^{0}\ 30'\ 35''.$$

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* This result was the mean of several observations in 1746 and 1747.

† The mean apparent zenith distance of the pole was obtained by a multitude of observations, of the pole star above and below the pole, made between 1750 and 1752, and reduced by being corrected for Precession, Aberration, and Nutation, to January 1751, Old Stile, (see Bradley's Observations, p. 9.)

This, if there were no refraction (r') would be the value of ZP (see fig. p. 7.) the co-latitude; the other value

$$51^\circ\ 27'\ 28'',$$

ought, were there no refraction, to be the latitude, or the value of the arc ZE: their sum

$$89^\circ\ 58'\ 3'',$$

is less than 90°, by $1'\ 57''$; which expresses the sum of the equatoreal and polar refractions. Now, as before, (see p. 218,) if the refraction varied as the tangent of the zenith distance, we should have

$$r = A \cdot \tan.\ 51^\circ\ 27'\ 28'',$$
$$r' = A \cdot \tan.\ 38\ \ 30\ \ 35,$$

and, adding the equations together,

$$1'\ 57''\ (= r + r') = A\ (\tan.\ 51^\circ\ 27'\ 28'' + \tan.\ 38^\circ\ 30'\ 35'').$$

Hence, since $\tan.\ 51^\circ\ 27'\ 28'' \ldots\ldots\ldots = 1.2553$, nearly,
and $\tan.\ 38\ \ 30\ \ 35 \ldots\ldots\ldots = \ .7956$

$$\overline{2.0509}$$

we have

$$A = \frac{117}{2.0509} = 57''.045,$$

and accordingly,

the equatoreal refraction $= 57''.045 \times 1.2553 = 71''.61$,
the polar refraction... $= 57''.045 \times\ .7956 = 45''.39$.

Hence, the apparent zenith distance of the equator being (see l. 3,)

	51°	27′	28″	
and the refraction	0	1	11.6	
	51	28	39.6	is the latitude.

and $57''.045$ is the mean refraction at the zenith distance of 45°.

But these results depend, as it is clear, on the refraction being represented, with sufficient exactness, by the term

$57''.045$ tan. Z, at the apparent zenith distances of $51° 27' 28''$, and $38° 30' 35''$; or, which amounts to the same thing, on the smallness of the coefficient (B) of the second term of the following formula,

$$\text{refraction} = A \tan. Z + B . \tan.^3 Z.$$

Suppose B to equal $-0''.1$, then the sum of the terms involving $\tan.^3 51° 27' 28''$, and $\tan.^3 38° 30' 35''$, (which terms are neglected in Bradley's Computation) will be $0''.2482$, and (see p. 223,) A will equal to $57''.167$, and the latitude will equal to

$$57''.167 \times \tan. 51° 27' 28'' - 0''.1 . \tan.^3 51° 27' 28'',$$

that is to

$$51° 28' 39''.56.$$

The quantity $1' 57''$ is the sum of the equatoreal and polar refractions; the exactness of which depends, on the accuracy of the observations of the zenith distances of the Sun and of the pole star. If we suppose an error of $1''$ in each observation, the sum of the refractions may remain the same, or be $117'' \pm 2''$. But (see p. 223,),

$$A = \frac{117'' \pm 2''}{2.0509} = \frac{117''}{2.0509} \pm \frac{1''}{1.0254}$$
$$= 57''.045 \pm 0''.975$$
$$= 58''.02, \text{ or } 56''.07,$$

in the first of which cases the resulting value of the latitude will be greater than $51° 28' 39''.5$, and, in the second, less.

When we suppose, in the preceding method, the refraction to be represented by the single term,

$$57''.045 \tan. Z,$$

we determine the refractions for all zenith distances that are *less* than the latitude of Greenwich. But how shall the refraction be determined when the zenith distances are greater than that latitude? The above term, it is clear, will not apply to all zenith distances: for it fails when $Z = 90°$. Bradley determined

the quantities of refraction, at zenith distances greater than the latitude of his Observatory, by means of circumpolar stars. An instance will best illustrate his method:

Zenith Distance of *a* Cassiopeæ above the pole =	4°	23′	20″
refraction = $57''.04 \times \tan. 4° 23' 20''$ ..... =	0	0	4.5
true zenith distance	4	23	24.5
but co-latitude	38	31	39.5
north polar distance	34	8	15
zenith distance below the pole	72	39	54.5
observed zenith distance	72	36	55.5
refraction at last zenith distance	0	2	59

By these means, the *quantity* of refraction, at the zenith distance of 72° 36′ 55″, was determined: but the term

$$57''.045 \times \tan. 72° 36' 55'',$$

gives a larger quantity. That single term, therefore, does not represent the *law* of refraction at zenith distances equal to or greater than 72° 36′ 55″. If we assume a formula of two terms to represent the refraction, we have from the above observations,

$$2' 59'' = 57''.045 \,.\, \tan. 72° 36' 55'' - B \,.\, \tan.^3 72° 36' 55'',$$

and if from such equation we determine B, we determine it (see p. 220,) on an assumed value of the latitude, the errors in the determination of which may exceed B.

But, if we assume the refraction to be represented by a formula of two terms with indeterminate coefficients, and suppose the latitude also to be undetermined, we must have, at least, the observations of three circumpolar stars to furnish us with three equations to determine the three above-mentioned unknown quantities: or, if we suppose that a formula of three terms will represent, more correctly, the refraction, there will be need of four circumpolar stars. In the second Volume of the *Systême du Base Metrique*, &c. we have an instance of the determination of the latitude and refraction, by assuming the latter to be represented by the following empirical formula,

$$\text{refraction} = A \tan. Z + B \tan.^3 Z + C \,.\, \tan.^5 Z.$$

The four circumpolar stars, were Polaris, β Ursæ minoris, α Draconis, and ζ Ursæ majoris, and the place of observation was Mountjoy. Now, see p. 219, if H be the latitude, Z, Z' the least and greatest zenith distances of a circumpolar star, ρ, ρ' the refractions corresponding to Z, Z',

$$180^0 - 2H = Z + Z' + \rho + \rho'.$$

Now, with the pole star, $Z + Z' = 97^0\ 14'\ 21''.47$
with β Ursæ minoris...... $= 97\ 13\ 58.17$
with α Draconis......... $= 97\ 12\ 58.6$
with ζ Ursæ majoris...... $= 97\ 9\ 31.65$

computing, therefore, $\rho + \rho'$, from the formula,

$$A.(\tan. Z + \tan. Z') + B(\tan.^3 Z + \tan.^3 Z') + C.(\tan.^5 Z + \tan.^5 Z'),$$

we obtain four values of $180° - 2H$, or four equations involving four indeterminate quantities, namely, H, A, B, C.

The four equations are

$$180 - 2H$$
$$= 97^0\ 14'\ 21''.47 + 2.27634\,A - 2.98440\,B + 3.9740\,C$$
$$= 97\ 13\ 58.17 + 2.67976\,A - 8.47044\,B + 33.31284\,C$$
$$= 97\ 12\ 58.6 + 3.75994\,A - 36.50251\,B + 400.06998\,C$$
$$= 97\ 9\ 31.65 + 7.86330\,A - 439.26963\,B + 25581.807\,C.$$

The resulting values of the coefficients are

$$A = 61''.1766$$
$$B = 0''.2648$$
$$C = 0''.002485.$$

Substitute these values in the four preceding equations, and those equations become

$$180° - 2H = 97°\ 16'\ 39''.95$$
$$180 - 2H = 97\ 16\ 39.95$$
$$180 - 2H = 97\ 16\ 39.95$$
$$180 - 2H = 97\ 16\ 39.94.$$

Taking, then, the mean,

$$180° - 2H = 97°\ 16'\ 39''.9475,$$
$$\text{and } H = 41\ 21\ 40.02625.$$

The formula of refraction is

$$r = 61''.1766 \text{ tan. } Z - 0''.2648 \text{ tan.}^3 Z + 0''.002485 \text{ tan.}^5 Z,$$

and making $Z = 45°$,

$$r = 60''.914285, \text{ instead of } 57''.045,$$

which it would be, nearly, according to Bradley.

In the above instance, we have, as M. Delambre justly observes, a formula of refraction derived (with regard to the numerical values of its coefficients) entirely from observations, and satisfying eight observed zenith distances. The formula, however, gives a mean refraction at 45° much greater than Bradley's formula gives.

If we were to correct the apparent zenith distance of the pole (rather the half sum of the apparent zenith distances of the pole star above and below the pole) found by Bradley's method (see p. 223,) by the preceding formula, we should have

$$\begin{aligned} \text{true co-latitude} &= 38° \, 30' \, 35'' + 61''.176 \times \text{tan. } 38° \, 30' \, 35'' \\ &\quad - .26485 \times \text{tan.}^3 \, 38° \, 30' \, 35'' \\ &\quad + \&\text{c.} \\ &= 38° \, 31' \, 23''.5, \text{ nearly,} \end{aligned}$$

and the latitude would $= 51° \, 28' \, 36''.5$, a quantity less, by three seconds, than the latitude found by Bradley's formula, in which A (the coefficient) is $57''$.

We have already seen a similar instance (p. 221.). If we *increase* the value of the coefficient (A) of the first term of the formula of refraction, or increase the mean refraction at 45° of zenith distance, we *diminish* the resulting value of the latitude. In the preceding instance of observations made at Mountjoy, the latitude of which is (see p. 226,) about 41° 21′ 40″, we diminish the latitude as much as we increase A, and *vice versâ*. And of this circumstance (which is worthy of attention in the theory of refractions) M. Delambre furnishes us with an additional confirmation. The *true*, or actual, refraction is reduced to the *mean*, on the score of temperature, (we shall soon more fully explain this part

of the subject) by multiplying the former by $\frac{350+t}{400}$, in Bradley's Theory, t denoting the number of degrees in Fahrenheit's scale above zero. By this multiplier for temperature, the observations made at Mountjoy were *reduced*, and the formula of p. 227, obtained. But Astronomers are not all agreed upon the correctness of the above multiplier. The French use a different one: according to them, the correcting multiplier is more nearly $\frac{450+t}{500}$. If the actual observations then are reduced by this last fraction, or by any other not the same as Bradley's, the resulting coefficients of the formula of refraction will be different. M. Delambre informs us that he did reduce the observations by Mayer's Tables, and the formula of refraction became

$$r = 63''.302 \tan. Z - 0''.34396 \tan.^3 Z + 0''.0033923 \tan.^5 Z,$$

very different from the former one of p. 227, but, apparently, equally well adapted to the observed zenith distances.

In this case, however, the latitude was *diminished* by 2'', that is, nearly, by the difference between 63''.302, and 61''.176.

In deducing the coefficients of the formula of refraction, and the latitude, from the eight observed zenith distances of four circumpolar stars, the formula was made to consist of three terms. If a fourth term, $D \tan.^7 Z$, had been introduced, the problem could not have been resolved: since there would have been five unknown quantities, (A, B, C, D and H) and but four equations. But, if we assume A to be of some value between 57'' and 61'', we may, from equations, similar to those of p. 226, and resulting from the observations, deduce the latitude and the coefficients of the second, third, and fourth terms. But, in each assumption, that will happen, which has before been noted to happen. As A is assumed of greater value H, the latitude, will, and by equal degrees, result of less. Thus,

Values of A	Corresponding Values of H.
57''.13	41° 21' 44''.1
58	41 21 43.2
59	41 21 42.2
60	41 21 41.2.

The inference drawn, by M. Delambre, from this and other instances, is, *that the construction of a Table of Refractions by observations is a truly indeterminate problem*.

In determining the polar and equatoreal refractions from the sum of refractions, Bradley assumed the refraction to vary as the tangent of the zenith distance. But, he afterwards expressed the law of its variation, more correctly, by the formula,

$$r = 57''.\tan.(Z - 3r).$$

We are ignorant of the means by which he arrived at this formula: whether they were empirical or theoretical. But the formula is compact and elegant, and not difficult of application. It gives the mean refraction at 45° of zenith distance equal to

$$57''.\tan.(45^{0} - 3r).$$

Suppose, in order to approximate to the value, that, at first, $3r$ is neglected: then

$$r = 57''\tan.45^{0} = 57'';$$

$$\therefore r \text{ at } 45^{0} = 57''\tan.(45^{\circ} - 2'\ 51'') = 56''.9, \text{ nearly.}$$

Again, and similarly, in order to find the mean refractions at 60° of zenith distance,

1^{st}, $r = 57''.\tan.60^{\circ} = 57'' \times 1.732 = 1'\ 38''.7$;

$\therefore 3r = 4'\ 56''.1$,

and secondly, $(r) = 57''.\tan.59^{\circ}\ 55'\ 3''.9 = 1'\ 38''.4$.

Again, and similarly, if the altitudes were 27° 39′ 17″, 62° 13′ 6″,

First, Zenith distance $= 62^{\circ}\ 20'\ 43''$.

$r = 57''.\tan.62^{0}\ 20'\ 43'' = 57'' \times 1.908 = 1'\ 48''.75$,

$3r = 5'\ 26''.25 = 5'\ 26''$, nearly.

Secondly, $(r) = 57''.\tan.62^{0}\ 15'\ 17'' = 1'\ 48''.3$,

Again, when zenith distance $= 27^{\circ}\ 46'\ 54''$.

First, $r = 57''.\tan.27^{0}\ 46'\ 54'' = 30''$, nearly,

$3r = 1'\ 30''$.

Secondly, $(r) = 57''.\tan.27^{\circ}\ 45'\ 24'' = 29''.99$.

By these means, that is, either by Bradley's formula or by Brinkley's, or by that of the French mathematicians, the *mean* refraction may be deduced. The results will not agree: according to Bradley, the *mean* refraction at 45° = 56″.9

according to the French......... = 57.5

* according to Brinkley.......... = 57.72.

But the *true* or actual refraction differs from the mean, if the temperature and weight of the air are not the same, when the observation is made, as they are *supposed* to be in the mean state of the air. Such mean state is denoted or represented by fifty degrees of Fahrenheit, and 29.60 inches of the common barometer. If the temperature be at its mean state, but the air less dense than at its mean state, or the height of the barometer be less than 29.6, if it be, for instance, 29.35, then the actual refraction is less than the mean, and in order to reduce it to the latter state we must multiply the former by $\frac{29.6}{29.35}$; and, generally, if h be the height of the barometer, by $\frac{29.6}{h}$. If the barometer stand at its mean height, but the temperature be greater or less than 50, the actual refraction will be less or greater than the mean, and the correction, by which the former is to be reduced to the latter, is had by multiplying the former by $\frac{350+t}{400}$, according to Bradley, and by $\frac{450+t}{500}$ according to the French. If, therefore, (which will almost always happen) neither the barometer nor thermometer be at their mean states,

$$\text{the actual refraction} \times \frac{29.6}{h} \times \frac{350+t}{400} = \text{mean refraction},$$

* The latitudes of Observatories determined from observations of circumpolar stars will vary according to the Tables of refractions by which the observations are reduced. Thus,

	By French Tables.	Bradley's.
Latitude of Dublin........	53° 23′ 13″.5	53° 23′ 14″.2
Latitude of Greenwich.	51 28 38	51 28 39.5

and, if, according to Bradley, the mean refraction (r) be denoted by

$$56''.9 \times \tan.(Z - 3r),$$

we have

$$\text{the actual refraction } (r) = \frac{400}{350+t} \times \frac{h}{29.6} \times 56''.9 . \tan.(Z-3r),$$

or, if we use the French corrections, and take one of M. Delambre's formulæ,

$$r = \frac{500}{450+t} \times \frac{h}{29.6} \times \left\{\begin{matrix} 60''.\tan. Z - 0''.14207 \tan.^3 Z \\ -0''.0045053 \tan.^5 Z + \&c. \end{matrix}\right\}$$

We must now examine the grounds on which the preceding corrections have been made.

With regard to the first correction, that of the barometer, it is founded on this assumption (which is confirmed, very nearly, by experiment) of the refraction increasing and diminishing, and proportionally, with the increased and diminished densities of the air. Of which latter, the greater and less heights of the barometer are the indications and measures. Hence, if dr, dh, represent the corresponding variations of the refraction, and of the height of the column of mercury in the barometer,

$$r + dr = r \times \frac{h + dh}{h} = r \times \left(1 + \frac{dh}{h}\right).$$

The other correction, that for the thermometer, is obtained on principles less simple and sure. The temperature increasing increases the volume of air, which varies inversely as the density, and the greater the density, the greater the refraction. What is required to be known then, is, the relation between the increases of temperature, and of the volume of air: or, in a more scientific form, how much will the volume of air be increased by an increase of 1° of temperature? Let m be an indeterminate coefficient: then if the volume of air at the mean temperature be V, it may be represented by $V \times (1 + m \times 1^\circ)$, when the thermometer indicates an increase of 1° of temperature, and $V \times (1 + m k^\circ)$ when k^0 is the increase of temperature: if r be the refraction in the first case, r' in the second, we have

$$\frac{r'}{r} = \frac{V}{V.(1 + mk^\circ)} = \frac{1}{1 + mk^\circ};$$

and in order to determine m, we must have recourse to experiment.

Now in order to determine m from actual observations, we must get rid of r, and use another equation similar to the former: let it be

$$r'' = \frac{r}{1 + m \times l^0},$$

in which the same star is observed and at the same place: for then, we shall have m by the common process of elimination. Thus, by the first and second equations,

$$r = r'(1 + m \times k^0) = r''(1 + m \times l^0);$$

$$\therefore\ m = \frac{r'' - r'}{r' k^0 - r'' l^0},$$

in which equation, r'', r', k^0, l^0, are known from actual observation.

To determine m, with correctness, select stars having low altitudes, and compare those altitudes observed under the circumstances of great differences of temperature. M. Delambre has selected such altitudes from Lemonnier's *Histoire Celeste*, p. 32: in which

$$r' = 10'\ 40'', \quad k^0 = 54^0,$$

$$r'' = 9'\ 20'', \quad l^0 = -\ 4^0.5;$$

$$\therefore m = \frac{80''}{33120} = .002415, \text{ nearly.}$$

Now 50^0 is the temperature at which *mean* refraction is held to take place. *The multiplier*, therefore, for reducing the true or actual refraction to the mean is

$$1 + .002415 \times (t - 50), \text{ or, } .87924 + .002415t,$$

or, $\frac{879240 + 2415t}{1000000}$, which nearly equals to $\frac{364 + t}{414}$; a fraction not differing much from Bradley's (see p. 230,) which is $\frac{350 + t}{400}$.

If we examine the preceding method, it will be found liable to considerable uncertainty. In order to procure large differences of refraction, those stars were selected which are at considerable distances from the zenith. Now, of such stars the refractions are very *irregular:* by which it is to be understood, that the refractions are not always the same, whilst those circumstances, that are supposed to cause refraction, do remain the same. As a proof too of the uncertainty of the method, there are considerable differences of opinion respecting the value of m. We subjoin its values according to different authors.

	Value of m.
Bradley	.002444
Lemonnier	.002415
Mayer	.002012
Lacaille	.001644
Bonne	.001777
Laplace	.002186

The uncertainty respecting the correction of refraction for difference of temperature, is rather an embarrassing circumstance, when minute inequalities are to be detected, or when a question arises concerning the exact mean places of stars *.

* In the preceding instance the correction for temperature was astronomically determined. But it has been determined, independently of the observations of stars, and by direct experiments. Thus, a column of air called 1, at 32° of Fahrenheit, becomes 1.375, at 212°. If the expansion be held to be equable, at a temperature t, the column will be equal to

$$1 + \frac{.375}{180} \times (t - 32), \text{ or } 1 + .002083 \times (t - 32);$$

$\therefore$ at 50 (when *mean* refraction is held to happen) it will equal to 1.0375, nearly. But the refraction, if it vary as the density, will vary inversely as the volume of the same column of air: hence,

$$\text{the true refraction equals the mean} \times \frac{10375}{933343 + 2083t},$$

which latter fraction is nearly equal to $\frac{500}{450+t}$ (see *Irish Trans.* 1815, Dr. Brinkley.)

We will take an instance to elucidate the preceding statement. Let the observed star be *Procyon* and its zenith distance (Greenwich being the place of observation) $45^0\ 46'\ 53''$, and suppose, at the time of observation, Fahrenheit's Thermometer to be 70.

The mean refraction by Bradley's formula, [$56''.9$ tan. $(Z - 3r)$] equals $58''.44$: and if we use Bradley's Correction for Temperature (see p. 230,) we have the actual refraction equal to

$$58''.44 \times \frac{400}{350+70} = 58''.44 \times \frac{40}{42} = 55''.66, \text{ nearly.}$$

If we use the French Correction, then the actual refraction is equal to

$$58''.44 \times \frac{500}{450+70} = 58''.44 \times \frac{100}{104} = 56''.2, \text{ nearly.}$$

The true zenith distance then of Procyon by Bradley's Correction would be........................ $45^0\ 47'\ 48''.66$

and by the French Tables................ $45\ \ 47\ \ 49.2$

so that, under the circumstances of the observation, (the material circumstance being the height of the thermometer) there would be a difference in the north polar distance of the star of $0''.54$.

We have taken the temperature at 70^0 which is not enormous in the month of July*, about the hour of noon, when Procyon would pass the meridian. Suppose, now, the same star to be observed, half a year after, in January, when it would pass the meridian about midnight, and that the thermometer is at 30^0: in this case the true refraction, by Bradley $= 58''.44 \times \frac{400}{380} = 61''.5$,

by the French.................. $= 58''.44 \times \frac{500}{480} = 60''.87$.

* In July the Sun's right ascension is from 7^h to 8^h and Procyon's right ascension being about $7^h\ 30^m$, the star passes the meridian about noon-tide. In January the Sun's right ascension is from 19^h to 20^h, and consequently, Procyon will be on the meridian about midnight.

In this case, the contrary to what happened in the former takes place. The correction for reducing the apparent zenith distance to the true is larger by Bradley's than by the French Tables. If, therefore, the star's place were deduced by observations like the preceding, there would be uncertainty to the amount, in each case, of more than half a second respecting the star's mean polar distance *.

If we had used the French *mean* refraction at 45°, instead of the English, the mean refraction for Procyon instead of 58″.44 would have been 59″.4.

It appears, from what has preceded, that there is, at present, considerable doubt respecting that correction of refraction, which is due to a variation of temperature. With regard to the correction due to an increased or diminished density of the air, as indicated by the height of the mercury in the barometer, there is, amongst Astronomers, no difference of opinion. If h be the height of the barometer, the true refraction is less or greater than the *mean* refraction (which is held to take place when h is 29.6 inches) as h is less or greater than 29.6 inches †, and in that proportion; the principle is, the variation of the refraction as the density of the air.

The states of the barometer and thermometer must be noted down at the time of each observation. But, Astronomers hold it needless to consult the *hygrometer*. According to M. Laplace, Gay Lussac, and Biot, refraction is not influenced by the relative moisture of the atmosphere.

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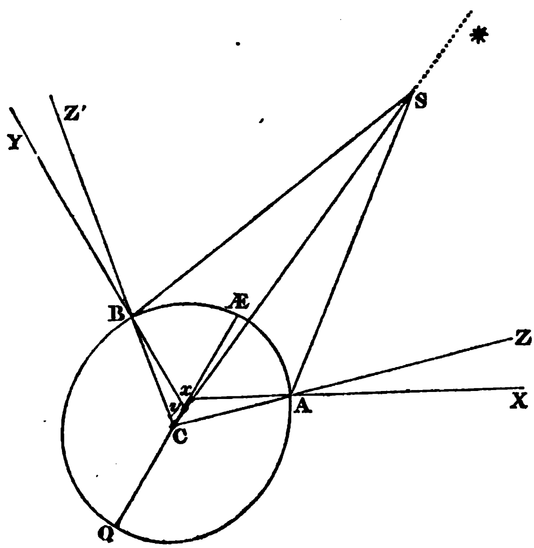
* This illustration is taken from Dr. Brinkley's Paper on Parallax (*Irish Trans.* 1815), in which he shews the effect of the uncertainty of the correction for temperature on the index error of the mural circle.

† A very small correction must be applied to the height of the barometer when the temperature is other than 50 its mean state. If the temperature be above 50, part of the height of the mercury in the barometer is owing to the expansion of the mercury: that part, therefore, must be subducted. If the expansion be .0001 inch for one degree of Fahrenheit, for $t-50$ degrees, it will be $(t-50)\times .0001$, hence the *correcting* fraction (see p. 233.) for the barometer, instead of being $\frac{h}{29.60}$, will be $\frac{h\times[1-(t-50).0001]}{29.6}$, or $\frac{h}{29.6}(1.005-.0001\,t)$.

There remains, however, much to be done on the subject of refraction. We may, amongst other guesses, conjecture that, from some defect in its theory, arises the difference in the values of the obliquity of the ecliptic, as they result from observations of the winter and summer solstices. Maskelyne, with Bradley's refractions, made the obliquity from the winter, less by 8″ than from the summer solstice. Delambre makes it less by 4″. The formula of refraction may be so altered, as to make the two values of the obliquity to agree. But, then, the altered formula, applied to the observations of circumpolar stars, would produce anomalous results; it would, for instance, produce different values of the co-latitude.

There are other methods, than those already mentioned, for determining the quantities of refraction. The method of the Abbé Lacaille is ingenious and founded on good principles. It does not happen to every Astronomer to be able to practise a similar method. Lacaille observed certain stars at Paris, and the same stars at the Cape of Good Hope, and from such observations deduced the quantities of refraction corresponding to the different altitudes of the same stars at the respective places of



observation. In order to illustrate his method, let Z be the

zenith of the Paris Observatory, Z' that of the Cape, and let the true zenith distances of a star S be esteemed to be ZAS, $Z'BS$, respectively, then

$$\begin{aligned} ZAS &= ACS + CSA \\ Z'BS &= BCS + CSB \end{aligned}$$

$$\therefore\ ZAS + ZBS = ACB + BSA = ACB,$$

if the angle of parallax (see p. 42,) BSA be neglected.

If then, Z, Z' be the apparent or observed zenith distances of a star, r, r', the corresponding refractions, we have

$$ACB = Z + Z' + r + r':$$

A second star, the zenith distances of which are V, V', and the refractions ρ, ρ', will give a similar equation, viz.

$$ACB = V + V' + \rho + \rho',$$

and, a third star will give a third similar equation, a fourth star, a fourth equation, and so on: if we suppose the refraction to vary as the zenith distance (to equal A tan. Z), we have, by equating the two first equations,

$$A\,(\tan. Z + \tan. Z' - \tan. V - \tan. V') = V + V' - Z - Z',$$

from which equation, Z, Z', V, V', the *observed* zenith distances, being known, A may be determined, and thence the angle ACB.

But if instead of $r = A \tan. Z$, which imperfectly expresses the law of refraction, we assume

$$r = A \tan. Z + B\,.\tan.^3 Z,$$

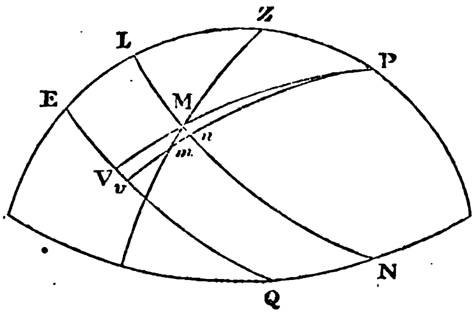
we must have three values of ACB, to determine A and B, and ACB: and if we add to the last formula a third term $C \tan.^5 Z$, we must, in order to deduce A, B, C, and ACB have four values of ACB, or the zenith distances of four stars observed both at the Cape of Good Hope, and at Paris.

According to the results of Lacaille obtained by the preceding method, the mean refraction at 45^0 is much greater than $56''.9$, which is Bradley's value: the coefficient A would equal $66''$.

The preceding method of Lacaille is one that cannot often be resorted to. We will explain another method for determining

the refraction which does not require the observer to change the place of his observations, but which will admit of his changing it.

Let M be the star's apparent place, raised, by refraction, above m its true place.



Suppose, by previous observations, to be known,

PZ, the co-latitude............ $= 90 - H$

Pm, the star's north polar distance.. $= \delta$

and, by immediate observations,

ZM, the star's zenith distance.......... $= Z$,

ZPm, the hour angle................ $= P$,

then (see *Trigonometry*, p. 139,)

$$\cos. (Z + r) (= \cos. Zm) = \cos. P . \cos. H . \sin. \delta + \sin. H . \cos. \delta,$$

from which formula, $Z + r$ becomes known, and thence

$$r = (Z + r) - Z.$$

This is, what may be called, the bare scientific process, which, however, in practice, becomes invested with circumstances that require great attention. For instance, the observations of the zenith distances may be made at a place, that is not the scite of an Observatory, and the latitude of which may be uncertain to the amount of two or three seconds. Indeed, if we are uncertain about the quantities of refraction due to the zenith distances of stars, we must be uncertain with respect to the co-latitude, which (see p. 129,) is half the sum of the greatest and

least zenith distances of a circumpolar star. Let p, p', be the apparent zenith distances of such a star, ρ, ρ', the corresponding refractions, then

$$90° - H = \tfrac{1}{2}(p + p' + \rho + \rho');$$

but dH representing the variation of the latitude from refraction,

$$90° - (H + dH) = \tfrac{1}{2}(p + p');$$

$$\therefore\ dH = \tfrac{1}{2}(\rho + \rho').$$

Hence, the error in determining the latitude, is half the sum of the refractions due to the two zenith distances of the circumpolar star used in determining it. If *Polaris* be that circumpolar star, $\frac{1}{2}(\rho + \rho') = \rho$, nearly. In order then to determine the error of the quantity of refraction resulting from the formula of p. 238, take its differential and substitute ρ instead of dH; if this be done,

$$-dr\,.\,\sin.(Z + r) = -\rho(\cos.H.\cos.\delta - \cos.P.\sin.H.\sin.\delta)$$

whence dr may be computed.

In this deduction δ has been supposed constant, or not subject to error. But, if the process of p. 237, be viewed as an original one, in which the observer (and Lacaille was so circumstanced, nearly,) had to determine not only the quantities of refraction, but the latitude of his Observatory and the declinations of his stars, it is plain that the resulting values of the refraction would be erroneous, from the errors of the latter quantities.

Now,

the apparent polar distance, or, $\delta + d\delta = \frac{1}{2}(p - p')$

the real polar distance, or $\delta = \frac{1}{2}(p - p' + \rho - \rho')$;

$$\therefore\ d\delta = \tfrac{1}{2}(\rho' - \rho),$$

which, if Polaris be the circumpolar star, is a very small quantity.

In the above method, then, the refraction, an unknown quantity, is to be determined from quantities which themselves involve the refraction: a kind of dilemma, in which the Astronomer repeatedly finds himself, but which the same kind of

artifice, almost always, enables him to loose himself from. He first neglects the indeterminate quantity where it appears in its involved state and finds an approximate value. This first value is then substituted, in every part of the original equation, and a second value is obtained, which second value serves, as a stepping stone, to ascend to a value nearer to the truth. The third, fourth, &c. values (although it is scarcely ever necessary to proceed so far) may be either taken as the true values, or may be made alike subservient to truer values. Thus, in the instance before us, the quantity of refraction is to be computed from the formula of p. 238, the values of H and δ being those which result from observation. The resulting value of the refraction will then serve to correct H and δ, and, their corrected values being substituted in the formula, a second value of the refraction is to be deduced, with which H and δ are again to be corrected, &c. &c.

In the preceding method we must use a clock to determine the hour angle P; but there is an instrument, called *an Altitude and Azimuth Instrument*, which will enable us to determine the refraction without the aid of a clock. Now this instrument determines, at once, both the altitude and azimuth of the star: the latter truly, the former as it is made greater by refraction. If we use the former figure and symbols, and make, besides, A to represent the azimuth, we have

$$\sin. PmZ \;(\sin. B) = \sin. A \,. \frac{\cos. H}{\sin. \delta};$$

thence B becomes known.

Again, by Naper's Analogies, (*Trig*. p. 169,)

$$\tan. \frac{Zm}{2} = \tan. \left(\frac{Z+r}{2}\right) = \tan. \tfrac{1}{2}\,(90^0 - H + \delta)\,\frac{\cos. \frac{1}{2}(A+B)}{\cos. \frac{1}{2}(A-B)}.$$

EXAMPLE.

Latitude of place of observation.	51^0	$31'$	$0''$
Star's observed altitude	18	13	5
. . . . azimuth.	74	53	30
. . . . N. P. D	66	32	0

B found.

sin. $74^\circ\ 53'\ 30'' = 9.9847229$
cos. $51\ \ 31\ \ 0 = 9\ 7939907$

19.7787136

sin. $66\ \ 32\ \ 0 \ldots . 9.9625076$

$9.8162060 = $ log. sin. $40^\circ.54'\ 56'' = B.$

Again,

$\frac{1}{2}(90 - H + \delta) = 52^\circ\ 30'\ 30'' \ldots\ldots\ldots\ldots$ tan. $= 10.1151503$
$\frac{1}{2}(A + B) \ldots = 57\ \ 54\ \ 13 \ldots\ldots\ldots\ldots$ cos. $= 9.7253768$
$\frac{1}{2}(A - B) \ldots = 15\ \ 59\ \ 17 \ldots$ arith. comp. cos. $= 10.0193759$

29.8599030

Rejecting 20, we have

$$\text{log. tan.} \frac{Z + r}{2} = 9.8599030,$$

$$\text{thence } \frac{Z + r}{2} = 35^\circ\ 54'\ 53''.5$$

$$Z + r = 71\ \ 49\ \ 47$$

$$\text{but } Z = 71\ \ 46\ \ 55$$

therefore, *r*, the refraction $= 0\ \ 2\ \ 52$

In the preceding instance the zenith distance is about 72°; up to that distance, and beyond it by about ten degrees, the formulæ and their deduced Tables, *satisfy,* (to borrow a French mode of expression) the observations. That is, the half sum of the greatest and least *corrected* zenith distances of a circumpolar star, is, very nearly, the same quantity, whether the greatest zenith distance of the star, be forty or eighty degrees. If we go beyond eighty, the refractions are irregular.

Dr. Brinkley has shewn those of Capella * to be so. Some

* The co-latitude of the Dublin Observatory being 36° 36′ 46″.7, and the north polar distance of Capella being greater than 44°, the greatest zenith of that star exceeds 80°.

Tables (the French, for instance,) represent the refractions near to the horizon, more nearly than others: but, hitherto, there has been invented no formula that restricts the irregularity of refraction that begins to take place about 80° of zenith distance. Laplace's formula does not extend to distances beyond 74°. At 82° 30′, the formulæ of Bradley and Simpson are erroneous, to the amount of 8″.

But, if we advert to the results which M. Delambre has given us of observations of stars near the horizon, it is hopeless to expect to reduce all refractions under one law. Those bordering on ninety degrees of zenith distance seem freed from all restraint. They disagree amongst themselves, and are, in this way, *irregular;* namely, they are not the same, when other circumstances, the altitude, and the heights of the barometer and thermometer are the same. It is certain, then, that the theory of refraction is imperfect: not solely because it does not restrict all its cases within the same law, but because it has no tests of, or means of measuring certain circumstances, on which, at great zenith distances, the refraction must depend. This is a perplexity, from which mathematical skill alone can never extricate us.

If the theory, however, be imperfect, the results of its formulæ, or its Tables, are easy of application; and we now subjoin one or two specimens of those Tables and instances of their uses. The specimens and the instances are both taken from the Volumes of the Greenwich Observations.

From the Table I. of mean refractions, computed to every ten minutes of zenith distance,

Zenith Distance.	Refraction.	Diff. for 10′.
77° 10′	4′ 6″.15	
		3″.25
77 20	4 9.40	
		3 .31
77 30	4 12.71	

From TABLE II.

Apparent Zen. Dist.	Log. Refraction.	Differences.
28° 50′	1.49665	304
29 0	1.49969	
&c.	&c.	
47 0	1.78533	256
47 10	1.78789	
&c.	&c.	
77 20	2.39690	572
77 30	2.40262	

From TABLE III.

	Height of the Barometer in English Inches.			
Thermom.	29.6	29.7	29.8	29.9
54	9.99568	9.99715	9, &c.	9, &c.
55	0.99460	0.99607	&c.	
56	0.99353	0.99500		
57	0.99247	0.99394		9.99685
58	0.99140	0.99287		&c.
59	0.99034	0.99181		
60	0.98928	0.99075		

Extracts from the Greenwich Observations.

June, 1812.	Barometer.	Thermometer. Out.	In.	Star.	Zenith Distance.
14	29.71	57	60	Antares.	77° 24′ 34″.7
16	29.60	54	57	Antares.	77° 24′ 33″.9
22	29.76	56	54	α Arietis.	28° 54′ 10″.2

1816.	Barometer.	Thermom.		N. P. D.
Sept. 12.	29.95	57	⊙'s U.L.	85° 34′ 28″.1

Suppose it were required, in the first place, to find the *mean* refraction of Antares on June 14, by Table I. This Table gives the refraction for 77° 20′, and 77° 30′: but the zenith distance of Antares is between these two zenith distances: it must, therefore, be found by interpolation, or by proportion, just as we find the logarithmic sine of an arc expressed in degrees, minutes and seconds, from Tables not extended beyond minutes.

Thus, see Table I, diff. for 1′ . . . . = .331

∴ for 4 . . . . = 1.324
0 30 165
4 22

4 34 . . . . 1.51, nearly.

But refraction for. . . . 77° 20′ 0″. . . . = 4′ 9″.4

∴ for . . . . 77° 24′ 34″ . . . = 4′ 10″.9, nearly.

Hence the zenith distance of Antares on the 14th, corrected for *mean* refraction, is

77° 28′ 45″.6.

But it is scarcely ever necessary to correct an apparent zenith

distance by the *mean* refraction. It is the *true* refraction that must be added to the observed zenith distance: and that refraction must be computed by Tables II. and III.

The Rule is: *Take from Table II, the logarithm* (A) *corresponding to the apparent zenith distance, and add it to a logarithm* (*B*) *of Table III, answering to the proposed heights of the barometer and thermometer. The sum* (*rejecting* 10) *is the logarithm of the true refraction.*

It will be necessary in this, as in the former, computation, to deduce, by proportion, logarithms intermediate to those expressed in the Tables.

To compute the refraction on the 14th.

By Table II, diff. for 1′.......... = 57.2

∴ for 4............ 228.8

0 30″.......... 28.6

4.7........ 4.5

4 34.7........ 261.9

but logarithm for 77° 20′ 0″ is 2.39690;

∴ logarithm for 77° 24′ 34″.7 is 2.39951.9....(*A*);

next,

log. barometer 29.7, thermometer 57...... 9.99394

thermometer 60...... 9.99075

2)19.98469

9.99234.5

correction for .01 of barometer.......... 14

9.99248.5 (*B*)

2.39951.9 (*A*)

(log. of 246.6)..................... 2.39200.4

Hence the true refraction is .... 0° 4′ 6″.6

and since the apparent zenith distance is.. 77 24 34.7

the true zenith distance is.............. 77 28 41.3

Again, in order to compute the refraction on the 16th,

log. barometer 29.6, thermometer 54 .... =	9.99568
thermometer 57 .... =	9.99247
	2)19.98815
	9.99407.5 (*B*)
(*A* will be a little less than in the former instance)	2.39950 (*A*)
(log. of 247.5)	2.39357.5

Hence the true refraction is	0°	4′	7″.5
and since the apparent zenith distance is ..	77	24	33.9
the true zenith distance is	77	28	41.4

In these two instances the mean of the two thermometers has been taken to represent the temperature. In the next instance (that of June 22,) we will compute the refraction from the *In* thermometer.

From Table II, diff. for	1′ 0″	is	30.4
∴ for	4 0	is	121.6
	0 10		5.06
	0 0.2		0.1
	4 10.2		126.76

but log. for 28° 50′ 0″ is 1.49665;

∴ log. for 28° 54′ 10″.2 is 1.49791.76 (*A*).

Again,

log. barometer 29.7, thermometer 54	9.99715 (*B*)
correction for .06	87
(log. of 31° 32′)	1.49593.76

Hence, the true refraction is	0°	0′	31″.3
and since the apparent zenith distance is	28	54	10.2
the true zenith distance is	28	54	41.5.

In these three instances, the apparent zenith distances, observed (see pp. 65, 66, &c.) by a mural quadrant, are expressed. In the next instance we must deduce the zenith distance, the north polar distance of the Sun's upper limb being observed by the mural circle, (see pp. 110, &c.)

North polar distance, Sun's upper limb =	85° 34′ 28″.1
(pp. 112, &c.) index error..............	+ 2.5
	85 34 30.6
co-latitude..........................	38 31 21.5
apparent zenith distance Sun's upper limb	47 3 9.1

Computation for refraction,

From Table II, diff. for	1′ 0″ is	25.6
∴ for	3 0	76.8
	0 9	3.96
	3 9	80.76

but log. for 47° 0′ 0″ is 1.78533

∴ for 47° 3′ 9″ is 1.78613.76 (*A*).

But, from Table II,

log. barometer 29.9, thermometer 57, is	9.99685
correction for .05	72 (*B*)
(log. of 60.75)......................	1.78360.76

If, therefore, we add this refraction (= 1′ 0″.75) to the zenith distance of the Sun's upper limb, and add the Sun's semi-diameter, we shall have the zenith distance of the Sun's centre.

Apparent zenith distance of the Sun's upper limb	47° 3′ 9″.1
refraction	0 1 0.75
Sun's semi-diameter	0 15 56.1
zenith distance of the Sun's centre	47 20 5.95
(see p. 209,) parallax......................	6.5
true zenith distance of the Sun's centre	47 19 59.45
and true altitude	42 40 0.55

The explanation of the theory of refraction, the deduction of its formulæ, the construction of Tables and their application, are the main objects of the present Chapter. There is begun in it, what will be continued, a series of investigations of those corrections by which the star's apparent place may be reduced to its mean place. The first in this series, the correction for refraction, is a correction for an inequality unlike, in its nature, to all other inequalities. It can never, even during short intervals in the same day, be presumed to be the same. It varies, every hour, with the temperature, and requires the unceasing attention of the observer to his thermometer and barometer.

But although what has been principally aimed at is, the divesting of instrumental zenith distances of the errors of refraction, yet the principle, or the ascertained effects, of that inequality may be applied (as to collateral objects) to the explanation of certain ordinary phenomena. Such are the elliptical forms of the orbs of the Sun and of the full Moon when near to the horizon; or their then *curtate* vertical diameters. The appearance of the Sun above the horizon, previously to the computed time of its rising, &c.

The first phenomenon arises from the rapid variation of the refraction when the observed body is near to the horizon. For instance, the upper limb of the Sun in the horizon is elevated by refraction, but the lower limb is much more than proportionally elevated. Let

zenith distance of the Sun's upper limb be......	90°	0	0
if the refraction be..........................	0	28	29
apparent zenith distance of the Sun's upper limb	89	31	31

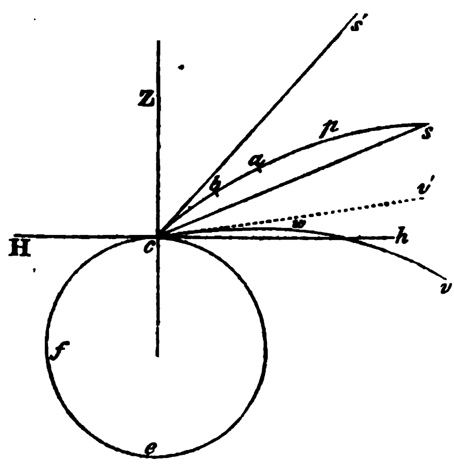
Suppose the Sun's diameter to be 32′;

then, zenith distance of the Sun's lower limb is	90°	32′	0″
but the refraction........................	0	32	46
∴ apparent zenith distance of the Sun's lower limb	89	59	14
subtract..................................	89	31	31
Sun's apparent diameter..................	0	27	43

The vertical diameter is, therefore, in this case, shortened 4′ 17″ by the effect of refraction, whilst the horizontal diameter is scarcely at all affected.

The upper and lower boundaries of the Sun's disk, in the preceding case, will be nearly elliptical: for, conceive a vertical circle to pass through the Sun contiguous to that which passes through his centre. That part of the vertical circle, which is intercepted between the Sun's horizontal diameter and either the upper or lower boundary of his disk, is nearly parallel to the vertical semi-diameter. It may, then, be conceived as an ordinate of the boundary curve. It would have been, were there no refraction, an ordinate of a circle (the Sun's orb being circular). It is less than this latter ordinate in the same proportion, nearly, as the *curtate* vertical semi-diameter of the Sun is less than his horizontal semi-diameter: and the above is the property of an ordinate to an ellipse.

The Sun's disk, or a star, may also appear above the horizon when it is, in fact, or, astronomically, below it. For instance,



the star v, the course of the light of which is vwc, will be seen in the direction cv'. On like principles, it is possible to see both the Sun and the Moon above the horizon at the time of a central

eclipse. For suppose, at such a conjuncture, the Sun to be just above the horizon; the Moon, being diametrically opposite, must, indeed, be beneath the horizon, but may be so little beneath, as, by refraction, to appear above. This phenomenon is recorded to have happened at Paris on July 19, 1750.

The next correction is due to an inequality called *Aberration.* It is difficult to prescribe the *natural* order of the *inequalities*, and, perhaps, we have already departed from it, in not first treating of *Precession.* In fact, if the historical were the natural order, we have already done so. The latter inequality was known to the antients, and its quantity, not very exactly indeed, assigned: whereas, it was not until the time of Tycho Brahé and Dominic Cassini that the effects of refraction on observations were computed and allowed for. The researches of preceding Astronomers did not extend beyond some speculations concerning its cause.

The historical order (the order of their successive discoveries) of the inequalities, is, Precession, Refraction, Aberration, Nutation. As for the inequality of Parallax (we are now speaking of those inequalities that affect the fixed stars) we are doubtful what place we ought to assign it. One hundred and fifty years ago, Flamstead thought he had discovered it, whereas its existence is now doubted of at Greenwich. It cannot, therefore, be even now said to be discovered. For the historical place of a discovery must be dated from the time at which it is, beyond controversy, established, and not from that at which it may have been either vaguely surmised, or erroneously affirmed, to exist. But, dismissing this enquiry, we have no difficulty in assigning a place to parallax in a scientific arrangement. It will be immediately after aberration: because, the formulæ of the latter inequality, become, with a very slight alteration, the formulæ of parallax. We shall not, indeed, use those formulæ in correcting observations, because the effects of parallax on the right ascensions and declinations of stars, if any, are, certainly, very inconsiderable. It is necessary, however, to be possessed of its appropriate formulæ; to know, in fact, the laws of its variation, that, should the comparison of reduced observations present us any anomalies, we may be able to ascertain whether, and to what degree, such anomalies are attributable to parallax.

According, then, to the plan of the present Treatise, parallax will be treated of immediately after aberration; Next, *Precession*, the *Inequality of Precession*, *Nutation*. The north polar distances and right ascensions of stars, corrected for these inequalities (which with refraction are, at present, the only accredited inequalities) become, or are to be held as, the *mean* north polar distances and *mean* right ascensions. Should these mean quantities, computed for two, or more, epochs, and then compared by being reduced to the same, or a common epoch, be found to differ, the causes of the differences would become subjects of enquiry: and till such causes are detected, might be designated by the title of *Proper Motions*.

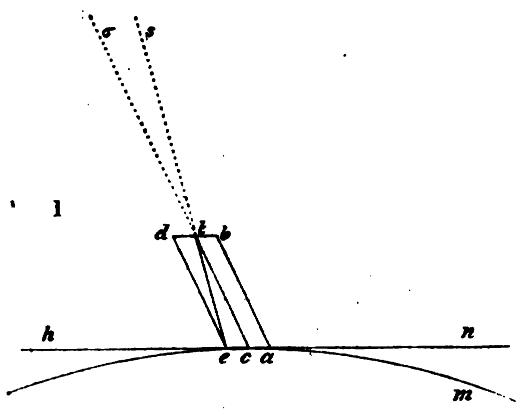
The formal propositions of a scientific Treatise have many advantages, but are not exempt from this objection: namely, that the Student is too suddenly carried into the middle of the subject, and too abruptly introduced into a system. He finds himself, with little preparation, amongst arrangements that are the results of many trials, many failures, and much thought. This evil will be felt in the following subject. The principle on which aberration depends is not an obvious one. Its effects do not admit of easy proof or familiar illustration. They cannot be exhibited separately, but are mixed up with embarrassing circumstances. But, in truth, it does not happen in this, otherwise than it does in other subjects. If the Student would thoroughly understand the doctrine of aberration, he must look to the history of its rise and first promulgation. Its propositions and precepts he must view, not as the first and natural suggestions which arose in the mind of its author, but as ideas carefully methodised and arranged for imparting instruction in the most convenient and concise form.

CHAP. XI.

ABERRATION.

Its Principle.—Illustration of it.—Roemer's Discovery of the Progressive Motion of Light.—The general Effect of Aberration is the Apparent Translation of a Star's Place towards the Path of the Earth's Motion. The partial Effects of Aberration on the Right Ascension and Declination of a Star: on its Latitude and Longitude.—The Effects of Aberration on a Star situated on the Solstitial Colure at the Seasons of the Equinoxes and Solstices. Formulæ for the Aberration, in Right Ascension; in Declination; in Latitude; in Longitude.—Application and Use of such Formulæ.

SUPPOSE σ to be the place of a star, and the eye of the observer, who is at rest, to be at c, then, (if there were no refraction) the



star would be seen in the direction ct; and this would be the case, whether the light were instantaneously transmitted from σ to the eye at c, or gradually descended to it in the line σc.

But let us now suppose the spectator to be in motion in the direction of the line ce: then, in the case of the *instantaneous* transmission of light, the eye at c would still view the star in the direction $c\sigma$, but in the second case, namely, that in which light

is supposed to *take time* in coming from σ to c, the fact would be different, as we shall now shew.

Let tc be conceived to be the axis of a tube, of which ab, ed are the parallel sides : then, if, whilst the light were descending down the axis tc, the tube were carried in a direction parallel to itself, from a towards e, the *hinder* part ab of the tube would continually approach the light in the successive points of its descent, and might, were its velocity sufficient, impinge on it; but, in any case, that is, whatever should be the velocity of the tube's motion, the light, on arriving at the line nae, would no longer be found at c the extremity of the axis of the tube.

Hence, if a star were at σ, it could not be seen in the direction of the axis of the tube, if the tube were in motion. We must, then, consider where the star, instead of being at σ, ought to be, in order to be seen in the above-mentioned direction: where, in short, its *true* place ought to be that σ may be its *apparent* place.

It cannot be such that the direction of a ray proceeding from it shall be parallel to tc: the direction must be *inclined* to tc and *towards* de, as, for instance, the line te is. If the ray of light be so inclined, and describe te whilst the axis ct is moved, parallel to itself, into the position ed, it will, in every point of its descent through the tube, be found in the axis tc of the tube. The eye, therefore, will judge the direction of the ray to be that of $ct\sigma$: or, under the above circumstances, a star at s will seem to be at σ*:

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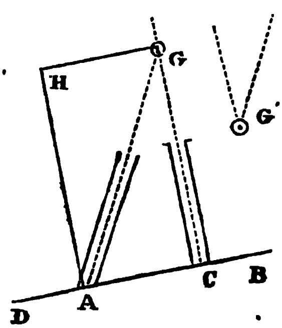
* Clairaut's Illustration is founded on the above principles. Sup-



pose G to be one of many drops falling in the direction GA. How

ought

The motion, therefore, of the spectator, combined with the motion of light, causes the star s to appear at σ; and, the difference of the two places, of which the angle $st\sigma$ is the measure, is the *aberration.*

This consequence must follow if light, instead of being instantaneously transmitted, be successively propagated. Whatever be the time of the light's transmission from t to e, no matter, how small, the above phenomenon, or circumstance, must take place in degree: whether the degree be large enough to become sensible by our instruments remains to be considered.

The fact of the *propagation* or *progression* of light was discovered by Roemer, and by means of the eclipses of Jupiter's satellites. The time of the emersion of one of the satellites (the first for instance) from the shadow of Jupiter's body is determined from a vast number of observations; the Earth, at the times of such observations, being variously situated with respect to Jupiter. The deduced time of the emersion of such satellite is the *mean* time of its happening. But Roemer found that such mean time did not always accord with a single observed time. It was sometimes greater, at other times less. The former was found to happen when the Earth was at a distance from Jupiter less than its mean distance; the latter when at a distance greater. These

---

ought a tube to be held by a person walking from C towards A and D that the drops shall descend down the tube? It cannot be held in the direction of AG: for then, if it were transferred from C to A, the drop would come into contact with the *hinder* side of the tube. That side of the tube, therefore, must be withdrawn from the direction of the falling drop: and the quantity through which it must be withdrawn, must depend on the relative velocities of the falling drop, and of the moving tube; and may be determined by drawing GH parallel to CA, and by completing the parallelogram $GHAC$. CG is the direction in which the tube ought to be held: GA, AC being the relative velocities of the drop of rain and of the tube.

The principle also may be established by supposing two impacts to be made on the eye at A: one from the light and measured by GA: the other from the Earth's motion measured by AC and in the direction from D to A. The resulting effect would be AH.

circumstances, then, are perfectly compatible with, and explicable by, the principle that time is absolved whilst the reflected light from the satellite, when it issues from the shadow, is transmitted to the Earth. For, the time will be longer, the more distant the Earth is from the Sun.

This is the first point which establishes, or renders probable, the principle of the *progression* of light. The second point, which is now to be considered, is the velocity of that progression: is it within such limits of magnitude that the aberration can become sensible by our instruments?

By a number of comparisons of the computed mean time at which an emersion of Jupiter's satellite ought to happen, with the observed times when the Earth was in positions most remote from, and most near to, Jupiter, it is found that the reflected light is about $16^m\ 26^s$ in traversing the Earth's orbit. If, therefore, r be the radius of the Earth's orbit, the velocity of light $= \dfrac{r}{8^m\ 13^s}$. If $365^d.25638$ be the Earth's period, the velocity of the Earth $= \dfrac{2r \times 3.14159}{365^d.25638}$; consequently, $\dfrac{\text{velocity } \oplus}{\text{velocity of light}}$ $= \dfrac{2r \times 3.14159 \times 493^s}{365^d.25638}$; which, expressed in seconds of space, is equal to $20''.246$ *.

If, therefore, tc be perpendicular to ae, the value of the angle of aberration (the angle $st\sigma$) is $20''.246$; which is a quantity easily cognisable by the best instruments.

But, if the place of the same star were always affected with the same aberration, it would be impossible to detect it, whatever were its value. We must, therefore, consider whether the change of the Earth's position will produce any change in the angle of aberration.

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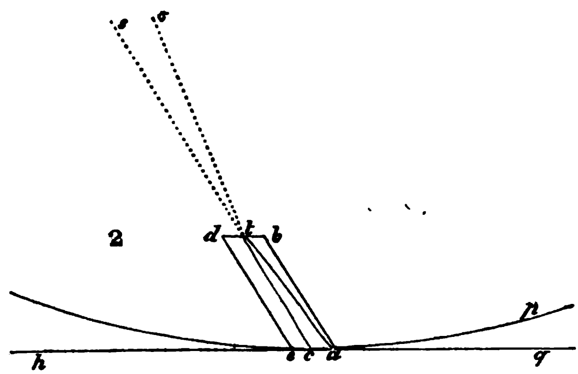
* Log. 493	= 2.6928469	log. 365.2563	= 2.5625976
log. 3.14159 ...	= .4971495	log. 3600	= 3.5563025
log. arc (= rad.)	= 5.3144254	log. 12	= 1.0791812
	8.5044218		7.1980813
	7.1980813		
(log. 20.246)	1.3063405		

In the former Figure, the line *nce* was intended to be a tangent to the Earth's orbit, (of which *mace* is a part) at the point *c*. Let now, in the present Figure, *ecap* represent an opposite portion of the Earth's orbit, *ecq* being a tangent to the point *c*, and the Earth (since it must have described, after leaving its first position, more than 90^0) now moving from *c* towards *a*.



In this case, and precisely for the reasons already alleged, (see p. 253,) the star must be at *s* in order that its light may descend down the axis of the tube and be seen in the direction *ctσ* *. In this case, then, from the combination of the motion of light with the motion of the Earth in its orbit, the apparent place of the star *s* will be *σ*, and the angle of aberration will be *stσ*. If the star *s* be the same as before, the angle *stσ* may be of the *same value* as it was in the former case: but there will be this difference in the two cases. In the former case the angle made by *se* with the tangent *neh* is *seh*, and the angle of the apparent direction of the star with *neh* is *σch*, which equals to

$$\angle seh - \angle st\sigma.$$

In the second case

$$\sigma ch = \angle sah + \angle st\sigma.$$

The *real* angle, then, of the star's direction with the line of the Earth's way, is diminished by *aberration* in the first case, and augmented in the second, and may be, in certain corresponding situations of the Earth, as much diminished as augmented.

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* *ctσ* is meant to be a straight line as *ats* is.

We have thus, then, the certain means of detecting the aberration by observing a star in opposite positions of the Earth's orbit, or in different seasons of the year.

Let us now consider the nature of the angles *seh*, *σch*, and their relations to the common Astronomical angles of declination, right ascension, latitude and longitude.

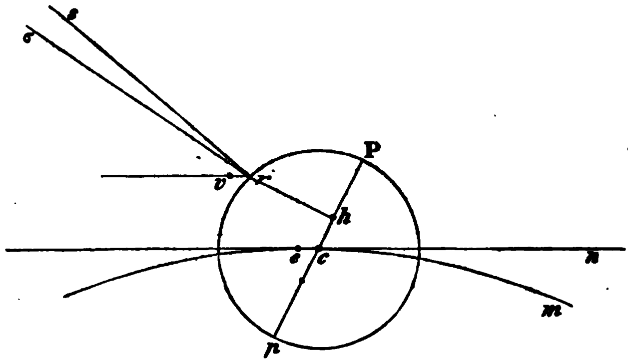
If the plane passing through *ae* and *es* (Fig. 1.) be conceived to be perpendicular to the plane of the Earth's orbit (of which *mace* is a part) the angle *seh* would be the star's latitude. In such a supposition, then, the star's latitude would be diminished by aberration. In the other Figure (Fig. 2.) and on a like supposition of the tube and star, the star's latitude would be augmented by aberration.

The plane passing through *ae* and *cσ* may be, as in figs. 3, and 4, parallel to the plane of the ecliptic; in which case, the effect of aberration will take place on the star's longitude: augmenting it in the position of Fig. 3, diminishing it in that of Fig. 4. These are particular effects of aberration. Its general effect, without reference to declination or latitude, is to translate the star's place towards the direction of the Earth's motion.

In the preceding illustrations, the eye of the spectator has been supposed to coincide with the centre of the Earth, and to move as that centre is moved. But this is a mere supposition. We must, therefore, now consider what modifications of the phenomena already described will be produced, the spectator being placed, as he ought to be, on the Earth's surface, the Earth revolving round its axis.

Let *Prp* be the Earth, *Pp* its axis, *r* a point on its surface: draw *rv* parallel and equal to *ce*: then, if there were no rotation, the point *r* would be translated through the space *rv*, in the same time that *c*, the Earth's centre, is translated through *ce*. The same effect, therefore, arising from the combined motion of the light and of the Earth, would happen to a spectator at *r*, as, we have shewn, would happen to a spectator at *c*: that is, the apparent place of a star *s* would be at *σ*, and the angle of aberration would be *srσ*.

But, during the translation of r through rv, the point r (or the spectator) describes, in consequence of the Earth's rotation,



an arc of a circle to the plane of which Pp, the Earth's axis, is perpendicular. The space, therefore, really described by the point, is the result of two motions, the one just mentioned, and rv due to the motion of the Earth's centre, in the direction of a tangent to the point c of the Earth's orbit. The former motion will variously affect the aberration: sometimes, scarcely at all, as would be the case, if the spectator were moving along rv, and rv should be in a plane passing through sr, Pp, perpendicular to the ecliptic. Its greatest effect, however, in increasing the aberration is very inconsiderable; the arc due to it being $0''.3084$*,

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* Let (see figure in opposite page) C be the centre of the Earth, a a spectator on its surface, and suppose the point a to describe a space ab in $8^m\ 13^s$, that is, in the time of the transmission of light from the Sun (s) to the Earth: then as being perpendicular to ab,

$$\frac{ab}{as} = \text{angle of aberration} = 20''.25, \text{ nearly,}$$

$$\text{but } as = cs, \text{ nearly,} = \frac{ac}{\sin. \odot\text{'s horizontal parallax}} = r \times 57^\circ.2957795;$$

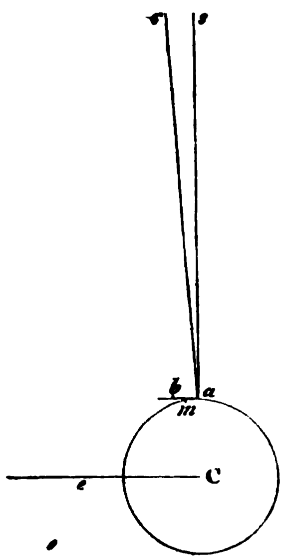
$$\therefore\ 8''.6\ ab = \frac{r}{8''.6} \times 20''.25 \times 57^\circ.2957795.$$

Again,

whilst the arc described by a point of the Earth's surface, and in consequence of the Earth's motion in her orbit is $20''.25$.

With the slight modification, then, which has just been explained, the aberration of light would happen to a spectator on the Earth's surface as it would to a spectator placed in the Earth's centre, and moving solely with the Earth's annual motion. This enables us to make a great step in the doctrine of aberration. Still, however, we must consider the spectator on the Earth's surface and the mode by which the effects of aberration will be made manifest to him. His observations are those of right ascension and declination: quantities which have no existence when the spectator is in the Earth's centre.

Again, let am be the space described by a, during $8^m\ 13^s$, and in consequence of the Earth's rotation: then



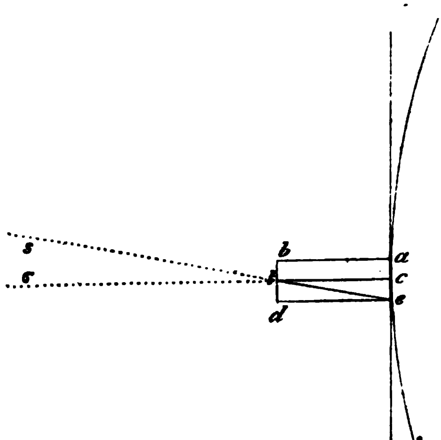
$$\frac{am}{r} = \frac{360° \times 8^m\ 13^s}{24^h} = 2°\ 3'\ 18'';$$

$$\therefore \frac{am}{ab} = \frac{2°\ 3'\ 18''}{57°.2957795} \times \frac{8''.6}{20''.25};$$

$$\therefore \text{ if } ab \text{ be made } = 20''.25,\ am = \frac{2.055}{57°.2957795} \times 8''.6 = 0''.3084.$$

It is not difficult to shew that, in certain positions of the Earth, the aberration will affect, solely, the declination of a particular star and, in other positions, the right ascension. For instance, suppose c (fig. p. 258.) to be the position of the Earth's centre at the vernal equinox, Prp the meridian of the spectator, and let the time be such, that a line drawn from the Sun to c is perpendicular to the plane of the meridian. The time, therefore, must be six in the morning; for, in six hours the meridian Prp will be brought opposite to the Sun. If s be a star situated in the solstitial colure, the plane of the meridian produced, will pass through s, or, sv will lie in that plane. In this position, then, the spectator's motion, represented by rv, being in the plane of the meridian, the aberration will take place, and exclusively, in the same plane: s will be thereby depressed to σ, and the star's north polar distance (P being the north pole) will be increased.

In like manner, if the Earth were at the opposite equinox, the motion of the Earth being directly from the star, the aberration



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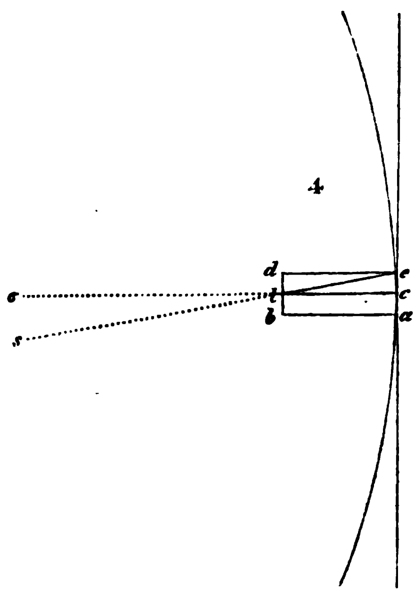
would take place entirely in the plane of the meridian, but its effect would be to *elevate* the star towards P the north pole, or to lessen the star's north polar distance. The former effect took place at six in the morning; this must take place at six in the evening.

If we suppose the Earth in a position intermediate to the two last, and at the summer solstice, then, a line drawn from the Sun to the Earth's centre will lie entirely in the plane of the meridian when the star (the star which is on the solstitial colure) is on the meridian. In this position the direction of the Earth's motion, being *a e*, is at the time of the star's passing the meridian, *perpendicular* to the meridian. The *aberration*, therefore, can then have no effect in the direction of the meridian, or cannot affect the star's declination. It will affect the right ascension, and solely that. The star, situated in the solstitial colure, will in the position of fig. 3, be on that part of the meridian which is opposite to the Sun. The time of the star's passage over the meridian, therefore, will be midnight.

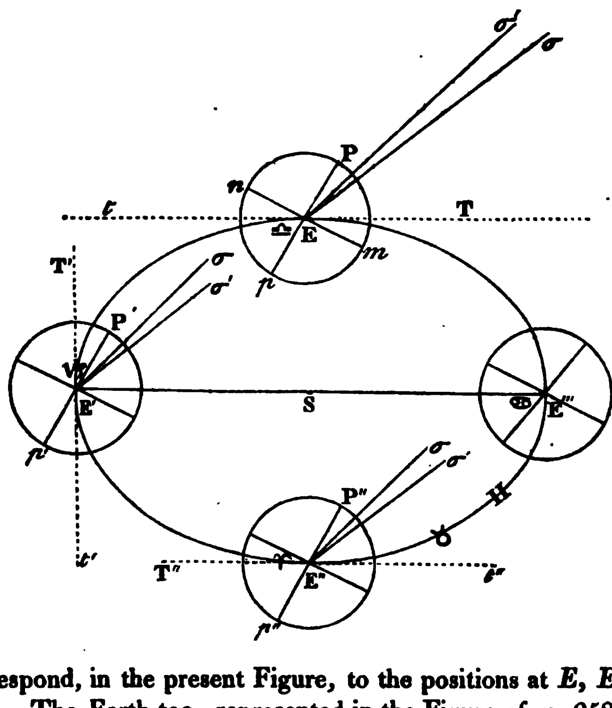
In Fig. 4, the Earth is in a position opposite to the last position, and is at the winter solstice. The motion of the Earth



being now from *a* towards *e*, the true place of the star being at *s*, *σ* will be the apparent place, and, as before, the *translation* of place will be in a direction perpendicular to the plane of the

meridian; in other words, the aberration will solely affect the star's right ascension which it will diminish. The time of the star's passing the meridian, in this position of the Earth, will be noon.

The four positions of the Earth, at the vernal and autumnal equinoxes, and at the solstices, which we have been considering separately, are represented, under one view, in the following Figure. The positions of the Earth in the Figure 1, 2, 3, 4,



correspond, in the present Figure, to the positions at E, E'', E', E'''. The Earth too, represented in the Figure of p. 258, corresponds to its representation in the present Figure at E. There is, however, this difference in the two cases. The rotation of the Earth being from r towards h, the right ascension of the star in the Figures of pp. 252 &c. is 270^0 or 18^h; whereas, in the present Figure, the right ascension is 90^0 or 6^h. The Earth, therefore, moving from E towards t, the star's place σ is apparently transferred to σ', or, its north polar distance is *diminished*

by aberration. The contrary to this happens in the position E''. In the position E' the right ascension of the star is *diminished*, in that of E''', *increased* by aberration. The times too differ from the former times, when the star's right ascension is 18^h. The Earth being at E, the hour of the star's passing the meridian is six in the evening; at E', noon: at E'', six in the morning; at E''', midnight.

But it is the star, with a right ascension of 270°, or of 18^h, that is situated, nearly, as γ Draconis is: which latter is the principal star in the history of Bradley's discovery of the *Aberration of Light*. The right ascension of γ Draconis, at the time of Bradley's Observations (1750) was about 267° 42′. The star, therefore, was, nearly, in the solstitial colure, and situated as the star s is, in the Figures 1, 2, 3, 4, &c. of pp. 252, &c. In the position E, then, which is that of the Earth at the vernal equinox, or about March 20th, γ Draconis must have been on the meridian about six in the morning (see p. 260,) and being *depressed towards* Et, or from the north pole P, must have passed the meridian to the *south* of its true place (see *Phil. Trans.* No. 406, p. 640.) At the autumnal equinox, or about September 20th, γ Draconis must have been on the meridian about six in the evening, and (see p. 260,) being elevated towards P, must have passed the meridian to the *north* of its true place: and in these two positions (of E and E'') the effect of aberration will take place, almost entirely, in the plane of the meridian; diminishing the star's declination in the first position, augmenting it in the second.

In the position at E', when the Sun was at the summer solstice, or about June 22, γ Draconis must have passed the meridian about midnight and *later* than it would have passed, had there been no aberration. The Earth being at the winter solstice, γ Draconis must have passed the meridian about noon, and *sooner* than it would have done, had there been no aberration. In these two last positions, the effect of aberration would be consumed, almost entirely, in retarding and accelerating, respectively, the times of the star's transit; or, in other words, in increasing and diminishing its right ascension.

It is easy to shew that the apparent translation of the star's place *towards* the direction of the Earth's motion (which translation is the general and constant effect of aberration) is, in some positions of the Earth, equivalent to, or amounts to the same as a retardation, and, in other positions, to an acceleration, of the star's transit. Thus, we have seen (see p. 258,) that the effect of aberration will be the same to a spectator placed in the centre of the Earth and moving with it, as to spectator placed on the Earth's surface. In this latter case, the tube, or telescope *abde*, moving with the motion of the Earth's centre, and, also, turning round, by virtue of the Earth's rotation, will be directed towards *s* before it occupies the position in the Figure: but, that is the position in which *s* is seen, and apparently seen at σ; *s*, therefore, is not seen till *after* that the telescope has been directed towards it; or, is seen not so soon as it would have been had there been no motion in the Earth, or, had there been an *instantaneous* transmission of light; in other words, the time of its passing across the middle wire of the telescope is retarded, or its right ascension is increased.

In the opposite position which the Earth occupies in the Figure 4, the spectator's motion, from that of the Earth in her orbit, is from *a* towards *e*, but the axis of the telescope, by reason of the Earth's rotation, will be in the direction *ets* *after* it has been in that of $ct\sigma$. But it must be (see p. 261,) in this latter position in order that *s* may be seen. *S*, therefore, is seen sooner * than it would be were there no aberration: or, its right ascension is diminished by the effects of aberration.

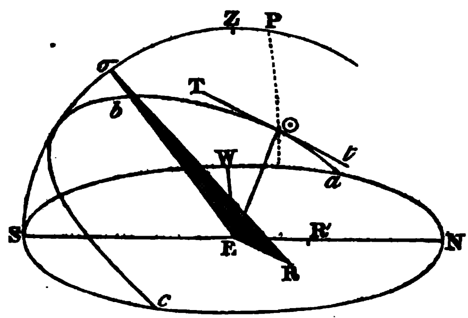
The illustration of the principle of aberration (and no other Astronomical subject stands more in need of illustration) has been principally shewn by means of a star, situated in the solstitial colure, and having a right ascension of eighteen hours. The reason of this has been assigned; γ Draconis, the chief star in Bradley's researches, is, nearly, so circumstanced, but it is not

* By being *seen sooner*, we mean the star, if observed by a transit, or other, telescope, furnished with a system of cross wires, would sooner occupy the centre of those wires (see pp. 74, &c.)

exactly so circumstanced. It is distant about 2°, or 8^{m} of sidereal time from the solstitial colure. When it passes the meridian, therefore, about six in the morning, it will pass to the *south* of its mean place, but its aberration will not be entirely in declination *. There will be, in this situation of the Earth, a small

* The aberration of a star, passing the meridian at six in the morning, and not situated in the solstitial colure, will be partly in declination and partly in right ascension. This is a fair inference from what has already been proved (see p. 260,) namely, that the aberration of a star passing at the above hour and situated in the solstitial colure, is *wholly* in declination. It is not difficult, however, to prove the same thing formally and independently; thus,

Conceive $cSWa$ to be the horizon, and $cb\odot a$ the ecliptic elevated above it; also S to be the south, W the west, P the pole of the equator, and $\odot$ the Sun at six in the evening, above the horizon, and consequently



to the north of the point W. Draw $\odot T$ a tangent to the ecliptic, and ER representing the Earth's way parallel to it, and in the plane of the ecliptic $cb\odot a$: then if σ be the star, the aberration (see p. 257,) will take place in a plane passing through $E\sigma$, ER.

Now, if the star were in the solstitial colure, and on the meridian, the Sun, at six o'clock, would be in the horizon, and the ecliptic, instead of being as it is in the Figure, would pass through W: in that case also, WE would be perpendicular to a line ER', and since it is perpendicular to $E\sigma$, it would be so also to a plane passing through $E\sigma$, ER': but, EW is perpendicular to the plane of the meridian; consequently, in this case, the plane of the meridian, would coincide with that passing through $E\sigma$, ER', in which the aberration takes place, and, accordingly, as it has been

aberration in right ascension. In other positions of the Earth, the aberration of γ Draconis, as well as the aberrations of other stars, will, generally speaking, be partly in right ascension, and partly in declination. Those must be, it is evident, *particular* positions of the Earth (to be determined by calculations) in which the aberration of a star shall take place entirely in the plane of the meridian, or in a direction perpendicular to that plane.

Having now gone through the above preliminary illustrations of the inequality of *aberration*, we will enter into the investigations of the *formulæ*, by which, at any assigned time, the aberrations of a particular star, whether they be in latitude and longitude, or in declination and right ascension, may be determined.

This process is purely mathematical. The first step is to compute the aberration, such as takes place in a plane passing through the *Earth's way* (as it may be called) and the star.

This, however, is a quantity not seen or noted, except in particular cases, by Astronomical instruments. It must, therefore, be reduced, and expressed as an *error* affecting the right ascension and declination; or the longitude and latitude. The latter reduction, or the *aberration of a star in longitude and latitude*, is of inferior importance. It is occasionally useful in Astronomical calculations; in those, for instance, which belong to the 'occultations of stars by the Moon.' The expressions, however, of the aberrations in right ascension and declination are important expressions. They enable us, at once, to correct

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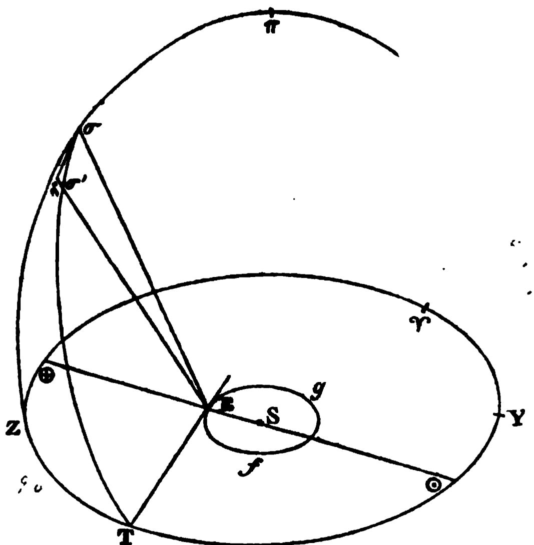
been before shewn (p. 260.) the aberration would take place wholly in the meridian. If, however, ☉ be to the north of *W*, *E* ☉ will not be perpendicular to the plane of the meridian, and the plane passing through $E\sigma$, ER, instead of coinciding with the plane of the meridian passing through $E\sigma$, ER', will be withdrawn from it towards the east. But, the aberration takes place in such plane, and any line representing its effect, may be resolved into two others, one perpendicular to the plane of $SE\sigma$, representing the aberration in right ascension, the other in that plane and representing the aberration in declination.

The aberration therefore of a star, not in the solstitial colure, which passes the meridian at six o'clock, is not wholly in declination.

the observations made with the mural quadrant and transit instrument, and to reduce, as much as they ought to be reduced, on account of *aberration*, a star's apparent to its mean place.

General Expression for Aberration.

Let S be the Sun, E the Earth; Efg its orbit; $ZT♈$ that orbit extended to the fixed stars, and in which the signs are supposed to lie; ET a tangent to the Earth's orbit at E; ⊙ the



place of S amongst the fixed stars, or in the ecliptic as seen from E the Earth; ⊕ the place of E the Earth in the ecliptic, as seen from the Sun S; σ a fixed star; σT the arc of a circle, (of which the centre is E) passing through σ and T: then, by what has preceded, the aberration of a star σ takes place in a plane σET, passing through σE and ET; and, the Earth moving according to the order Efg, and towards T, the aberration may be represented by $\sigma E \sigma'$.

The circle σT, in the Figure, is not a great circle; it would be one, if E coincided with S. Now this latter condition may be conceived to take place: for, the annual parallax of the Earth's orbit is insensible; in other words, the radius SE of its

orbit, with regard to SZ, or ST, (the radius of the imaginary concave in which the stars are conceived to be placed) may, by reason of its smallness, be neglected.

If E then be considered as coincident with S, the arc σT measures the angle σET: hence, since *

$\sin. \sigma E\sigma' : \sin. \sigma ET$ :: velocity of the Earth : velocity of light;

and, since the velocities of the Earth and of light may be considered as constant;

$$\sin. \sigma E\sigma', \text{ or } \sigma E\sigma' \ (\sigma E\sigma' \text{ being very small}) \propto \sin. \sigma T,$$

or, the aberration $\propto \sin. \sigma T$: consequently, the aberration is the greatest, when $\sin. \sigma T$ is, that is, when σT equals a quadrant, or when σ is in π the pole of the ecliptic.

By observation, the greatest effect of aberration is about $20''.25$. Hence, generally,

$$\textit{The aberration} = 20''.25 \sin. \sigma T.$$

The Earth's orbit being nearly circular, SE is nearly perpendicular to ET: and $\oplus T$ is a quadrant, or T is 90° degrees before the Earth's place seen from the Sun: and if ♈ represents the first point of *Aries*, the longitude of T is ♈ T; and the longitude of the Sun, which, by a spectator on the Earth's surface, is referred to $\odot$, is ♈ $\odot$ = ♈ $T+90^\circ$.

We have now obtained what may be called a general expression for the aberration: an expression for the aberration which takes place in the circle σT, and which, except in particular cases, does not affect, with its whole quantity, the observations of right ascension and declination. The *resolved* parts, therefore, of the general effect of aberration become the proper objects of enquiry: and, with the view of investigating, most conveniently, such resolved parts, we shall first determine those positions of the point T (see the Figure of p. 267,) in which the resolved parts, the aberrations in right ascension and declination, &c. are nothing.

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* In Fig. p. 252,

$\sin. cte : \sin. tce$ ($=\sin. tec$, nearly,) :: $ce : tc$,

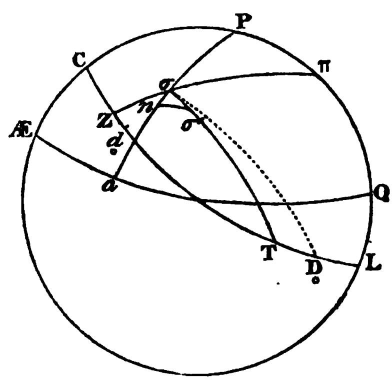
or $\sin. st\sigma : \sin. tec$ :: vel. $\oplus$: velocity of light.

General Construction for the Point T, *when any resolved part of the Aberration is equal to nothing.*

Let, as before, σ be the star, π the pole of the ecliptic CTL, P the pole of the equator, $ÆQ$, and σT the arc of the circle, in the plane of which, aberration takes place. Then, if σT coincide with πZ, or with Pa, there is, respectively, no aberration perpendicular to πZ, or none perpendicular to Pa: in other words, there is no aberration in longitude, or none in right ascension. If σT be perpendicular to πZ or to Pa, there is no aberration in the plane of πZ, or none in that of Pa; in other words, there is either no aberration in latitude, or none in declination. And the determination, on these principles, of the several positions of the point T when the respective aberrations are equal to nothing, is preparatory to the investigation of the formulæ that expound, generally, the laws of the aberration.

Investigation of the Position of the Point T, *when the Aberration in North Polar Distance is equal to* 0.

Draw σD_0 perpendicular to $P\sigma a$ at the point σ: then D_0 is the place of T when the aberration in declination, or in north polar distance, is equal to 0. In the present Figure, the star is in the second quadrant, and the angle $D_0\sigma Z$ is greater than 90°;



consequently, $D_0 Z$ is greater than 90°. If, therefore, $D_0 d_0$ be taken equal to a quadrant, d_0 is between Z and D_0. In order to

compute $D_0 Z$ and $d_0 Z$, we have, since the spherical triangle $D_0 \sigma Z$ is right-angled at Z,

$$1 \times \sin.\ \sigma Z = \cot.\ D_0 \sigma Z\ .\ \tan.\ D_0 Z$$
$$= \cot.(90^\circ + P)\ .\ \tan.\ (90^\circ + d_0 Z),$$

P being the angle of position, $P \sigma \pi$,

Hence, by *Trigonometry*, pp. 10, 35.

$$* \tan.\ d_0 Z\ (= -\cot.\ D_0 Z) = \frac{\tan.\ P}{\sin.\ \text{star's latitude}} \ldots (1).$$

From which expression the positions of the points d_0, D_0 may be determined.

If we place the star in any one of the three other quadrants we shall obtain the same expression for tan. $d_0 Z$.

Formula for the Aberration (A) *in North Polar Distance.*

Draw $\sigma' n$ perpendicular to $P \sigma a$, and σn expresses the aberration (A) in north polar distance: in order to compute it, we have, supposing $\sigma \sigma' = 20''.25\ .\ \sin.\ \sigma T$, (see p. 268,)

$$\sigma n = \sigma \sigma'\ .\ \cos.\ n \sigma \sigma' = 20''.25\ .\ \sin.\ \sigma T\ .\ \cos.\ n \sigma \sigma'$$
$$= 20''.25\ .\ \sin.\ \sigma T\ .\ \sin.\ D_0 \sigma T$$
$$= 20''.25\ .\ \sin.\ D_0 T\ .\ \sin.\ T D_0 \sigma\ (\textit{Trig.}\ \text{p. } 141.)$$

But, since $D_0 \sigma Z$ is a right-angled triangle, we have by Naper's rule,

$$1 \times \cos.\ D_0 \sigma Z = \sin.\ T D_0 \sigma\ .\ \cos.\ D_0 Z,$$

and, consequently, $1 \times \sin.\ P = \sin.\ T D_0 \sigma\ .\ \sin.\ d_0 Z.$

Hence, substituting in the above value of σn, we have

$$A\ (= \sigma n) = 20''.25\ .\ \sin.\ D_0 T \times \frac{\sin.\ P}{\sin.\ d_0 Z}\ .$$

During a short period (a year, for instance,) P, and the points d_0, Z, may be considered to be constant; $D_0 T$, therefore,

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* Bradley's Rule: see *Phil. Trans.* No. 406, p. 650: see also *Mem. de l'Acad.* 1732, p. 213: Clairaut: also T. Simpson's Essays, p. 16.

is the only quantity, in the above value of A, that is variable; and A is the greatest, when sin. $D_0T=1$, or when $D_0T=90^0$. Let M be that greatest value of A: then

$$M = 20''.25 . \frac{\text{sin. } P}{\text{sin. } d_0 Z} \ldots\ldots\ldots (2),$$

$$\text{and } {}^{*}A = M . \text{sin. } D_0 T \ldots\ldots\ldots\ldots (3).$$

Hence, in order to compute the aberration for any assigned time, we must compute from (1) the position of d_0: secondly, the value of M from (2), and thirdly, A from (3), in which expression the position of D_0 being determined, and that of T, from the assigned time, D_0T will be known.

We will shew how D_0T may be more conveniently expressed. Let $\odot$, $\oplus$, represent, respectively, the longitudes of the Sun and Earth, then

$$\begin{aligned} D_0 T &= \text{long. } D_0 - \text{long. } T \\ &= \text{long. } * + D_0 Z - 90^\circ - \oplus \\ &= \text{long. } * + D_0 Z + 90^\circ - \odot; \end{aligned}$$

$$\therefore A = M . \text{cos. (long. } * + D_0 Z - \odot),$$

$$\text{and, if we make long. } * + D_0 Z = N\dagger,$$

$$A = M \text{ cos. } (N - \odot), \text{ or, } = M . \text{cos. } (\odot - N).$$

The only variable quantity in the above expression of the aberration of the same star is $\odot$, the Sun's longitude. We shall

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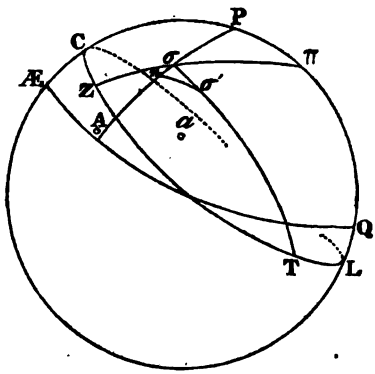
* See Bradley, *Phil. Trans.* No. 406, p. 650: Clairaut, *Mem. de l'Acad.* 1737, p. 213: T. Simpson's Essays, p. 16. D_0, in the above construction, is the node of a great circle drawn perpendicularly to the circle of declination at the place of the star. The maximum of aberration happens, therefore, when the Sun is in the above-mentioned, or in the opposite, node. For, $D_0T=90^0$; therefore $D\oplus$ (see the Fig. of p. 267,) $=90^\circ+90^\circ=180^\circ=\oplus\odot$. This is the conclusion which Delambre, by a different way, has arrived at in Tom. III. p. 120. of his Astronomy.

† In the first and fourth quadrants $D_0Z = 90^0 - d_0Z$.
In the second and third........ $D_0Z = 90 + d_0Z$.

again advert to this circumstance, as it affords the means of easily expressing in Tables the results of the preceding formulæ.

Investigation of the Position of the Point T, *when the Aberration in Right Ascension is equal to* 0.

If σT coincide with $P\sigma$, A_0 is the corresponding position of



the point T: and, if we make $A_0 a_0 = 90^0$, a_0 will be the corresponding position of the Earth.

Now we have, by Naper's rule,

$$1 \times \sin. \sigma Z = \tan. A_0 Z . \cot. P \text{ (since } P = \angle Z\sigma A_0);$$

$$\therefore \tan. A_0 Z = \frac{\sin. \sigma Z}{\cot. P};$$

$$\text{or } \cot. A_0 Z = \frac{\cot. P}{\sin. *\text{'s lat.}} \ldots\ldots (4).$$

Formula for the Aberration in Right Ascension.

Draw $\sigma' n$ perpendicularly to $P\sigma A_0$; the aberration in right ascension (a) is measured by the angle $nP\sigma'$: and

$$a = \frac{n\sigma'}{\sin. P\sigma} = \frac{\sigma\sigma' . \sin. n\sigma\sigma'}{\sin. *\text{'s N.P.D.}};$$

but $\sigma\sigma' = 20''.25 \,.\, \sin.\, \sigma T$, and $\sin.\, \sigma T \,.\, \sin.\, n\sigma\sigma' = \sin.\, A_0 T \,.\, \sin.\, TA_0\sigma$; and by Naper,

$$1 \times \cos.\, Z\sigma A_0, \text{ or, } 1 \times \cos.\, P = \cos.\, A_0 Z \,.\, \sin.\, ZA_0\sigma$$
$$= \cos.\, A_0 Z \,.\, \sin.\, TA_0\sigma.$$

Hence, by substitution,

$$a = 20''.25 \,.\, \sin.\, A_0 T \,.\, \frac{\cos.\, P}{\sin.\, *\text{'s N. P. D} \,.\, \cos.\, A_0 Z},$$

in which expression, since, under the circumstances before stated, (see p. 270,), A_0T is the sole variable quantity, a must become a maximum (m) when $\sin.\, A_0T = 1$, or $A_0T = 90^0$, or 270^0; accordingly,

$$m = 20''.25 \,.\, \frac{\cos.\, P}{\sin.\, *\text{'s N. P. D.} \cos.\, A_0 Z} \ldots\ldots (5).$$

and * $a = m \,.\, \sin.\, A_0 T \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6).$

This last expression admits of remarks and a transformation like those made on the expression (3), thus,

$$A_0 T = \text{long. } T - \text{long. } A_0;$$
$$\text{but long. } T = \odot - 90^0,$$
$$\text{and long. } A_0 = \text{long. } * \pm A_0 Z,$$

the upper sign to be used in the second and third quadrant, the lower in the first and fourth. But in the second and third quadrants,

---

* The preceding method of deducing the expressions for the aberrarations in north polar distance and right ascension, very nearly resembles a method given by Lalande at pp. 199, &c. tom. III. *Astronomy*, Ed. 2d. His formulæ, too, are similar; instead of (5) his expression is

$$m = 20''.25 \,.\, \frac{\cos.\, 23^\circ\, 28'}{\cos.\text{ dec. } \cos.\text{ dec. } A_0};$$

$$\text{which, since } \frac{\cos.\, P}{\cos.\, A_0 Z} = \frac{\cos.\, 23^\circ\, 28'}{\cos.\text{ dec. } A_0};$$

is the same, in substance, as (5).*

$$A_0 Z = 90^\circ - a_0 Z,$$

in first and fourth.... $= a_0 Z - 90^\circ$;

$$\therefore A_0 T = \odot + a_0 Z - \text{long.} \ast,$$

and

$$\begin{aligned} a = &\ m . \sin. (\odot + a_0 Z - \text{long.} \ast - 180^\circ) \\ = &- m . \sin. (\odot + a_0 Z - \text{long.} \ast) \\ = &- m . \sin. (\odot - n), \end{aligned}$$

if we make $n = \text{long.} \ast - a_0 Z$.

In order then to compute the aberration (a) in right ascension, we must, as in the former case, (see p. 271.) previously compute the position of the point a_0, and the maximum (m). But then these two values being computed, the aberration for any time in the year may be found by a simple process.

The subject is not without its intricacy: we will endeavour to unfold it by the aid of instances. The stars selected for illustration will be γ Pegasi, α Arietis, Polaris, η Ursæ majoris, γ Draconis, and α Aquarii; and the first steps will be made in computing the maxima of aberration, and the positions of the points D_0 and A_0*.

---

* In the Examples that succeed, we have not been solicitous to reduce the longitudes, latitudes, and angles of position to the same epoch, because there may be considerable variations in the values of those quantities, without any changes, or, at most, with very slight ones, in the resulting values of M, m, N, and n. Polaris is the only star in which it is necessary that the epoch appertaining to the values of the longitudes, latitudes, &c. should not be more than ten years distant from that epoch at which the values of M, m, N, and n, are required.

EXAMPLE I. γ *Pegasi.*

For 1800, longitude........	=	6^{0} $26'$ $19''$
latitude..........	=	12 35 41
angle of position ..	=	24 4 44
north polar distance	=	75 54 4

which values are taken from the *Connoisance des Tems.* for 1804.

$d_0 Z$ computed (p. 270,)		$A_0 Z$ computed (p. 272,)	
10 +		10 +	
log. tan. 24^{0} $4'$ $44''$....	19.65017	......log. cot.	20.34983
log. sin. 12 35 41.....	9.33856	....	9.33856
(tan. $d_0 Z$)........	10.31161	(cot. $A_0 Z$)	11.01127
$d_0 Z$ =	63^{0} $59'$ $21''$	$A_0 Z$ =	5° $33'$ $55''$
$\therefore D_0 Z$ =	26 0 39	$\therefore a_0 Z$ =	95 33 55
but $*$ long. =	6 26 19	....	6 26 19
$\therefore N$ =	32 26 58	$\therefore -n$ =	89 7 36

Secondly,

M		m	
		log. r......	10
log. $20''.25$......	1.30642	....	1.30642
log. sin. 24° $4'$ $44''$	9.61063	.. log. cos. 24° $4'$ $44''$	9.96046
	10 91705		21.26688 (a)
log. sin. $d_0 Z$....	9.95362	log. sin. 75° $54'$ $4''$	9.98671
(log. M)	0.96343	log. cos. 5 33 55	9.99795
			19.98466 (b)
M =	$9''.19$	(log. m)	1.28222
		hence m =	$19''.15$.

Hence, (see p. 271,)

$$A = M \,.\, \cos. (\odot - N) = M \,.\, \cos. (\odot - 32^\circ\ 26'\ 58''),$$

$$\text{and } a = - m \sin. (\odot - n) = - m \,.\, \sin. (\odot + 89\ \ 7\ \ 36),$$

from which two expressions, the aberrations in right ascension and north polar distance may be determined for every day in the year.

The first of the two preceding expressions involves the cosine of the *difference* of two arcs: the second the sine of the *sum* of two arcs, but negatively expressed. If we wish to express (and it is convenient they should be so expressed) both aberrations by the *positive* sines of the sums of two arcs, we must transform the preceding formulæ after the following manner:

$$\begin{aligned} M \cos. (\odot - 32^\circ\ 26'\ 58'') &= M \cos. (\odot + 57^\circ\ 33'\ 2'' - 90^\circ) \\ &= M \sin. (\odot + 57\ \ 33\ \ 2) \end{aligned}$$

Again,

$$\begin{aligned} - m \sin. (\odot - n) &= - m \,.\, \sin. (\odot + 89^\circ\ 7'\ 36'') \\ &= - m \,.\, \sin. (\odot + 6^s + 89^\circ\ 7'\ 36'' - 6^s) \\ &= \ \ m \,.\, \sin. (\odot + 8^s\ 29^\circ\ 7'\ 36''), \end{aligned}$$

and if we express the above two formulæ logarithmically, we have

$$\log.\ A = \log.\ \sin. (\odot + 1^s\ 27^\circ\ 33'\ \ 2'') + .96343,$$

$$\log.\ a = \log.\ \sin. (\odot + 8\ \ 29\ \ \ 7\ \ 44) + 1.28222,$$

and, if we wish to express the aberration in right ascension in time, we must subtract from 1.28222 (log. m) the log. 15 (= 1.17609) in which case, 0.10613 will be the logarithm of the maximum.

EXAMPLE II. *α Arietis.*

For the latitude, longitude, angle of position of this star in 1815, see pp. 160, &c.

	d_0Z		A_0Z
	10 +		10 +
log. tan. 20° 39′ 52″	19.57653	cot.	20.42346
log. sin. 9 57 37	9.23796		9.23796
(tan. d_0Z)	10.33857	(cot. A_0Z)	11.18550

$d_0Z = 65°\ 21'\ 50''$ $\qquad A_0Z = 3°\ 43'\ 56''$

$\therefore\ D_0Z = 24\ \ 38\ \ 10$ $\qquad \therefore\ a_0Z = 93\ \ 43\ \ 56$

$*$'s long. $= 35\ \ 4\ \ 41$ .... $35\ \ 4\ \ 41$

$N = 59\ \ 42\ \ 51$ $\qquad -n = 58\ \ 39\ \ 15$

Again,

M		m	
		log. r	10
log. 20.25	1.30642		1.30642
log. sin. 9° 57′ 37″	9.54764	log. cos. 20° 39′ 52″	9.97112
	10.85406		21.27754 (a)
log. sin. 65° 21′ 50″	9.95855	log. sin. 67° 25′ 1″	9.96535
(log. M)	.89551	log. cos. 3 43 56	9.99908
$M = 7''.86$			19.96443 (b)
		(log. m)	1.31311 $(a-b)$

$$m = 20''.56.$$

Hence,

aberration in N. P. D. $= 7''.86 \times \cos.(\odot - 1^s\ 29°\ 42'\ 51'')$

aberration in Æ $= -\ 20''.56\ \sin.(\odot + 1^s\ 28°\ 39'\ 15'')$,

or, expressed by the sines of the sums of arcs,

$$A = 7''.86\ .\ \sin.(\odot + 1^s\ \ 0°\ 17'\ \ 9'')$$
$$a = 20''.56\ .\ \sin.(\odot + 7\ \ 28\ \ 39\ \ 15).$$

EXAMPLE III. *Polaris*, (see pp. 167, 175, 178.)

d_0Z		A_0Z	
10+		10+	
log. tan. $72^\circ\ 59'\ 39''$....	20.51450....	log. cot. =	19.48549
log. sin. 66 4 42.....	9.96099	....	9.96099
(log. tan. d_0Z)........	10.55351	(log. cot. A_0Z)	9.52450

$$d_0Z = 74^\circ\ 22'\ 50'' \qquad A_0Z = 71^\circ\ 30'\ 2''$$

$$\therefore D_0Z = 15\ \ 37\ \ 10 \qquad \therefore a_0Z = 161\ \ 30\ \ 2$$

$$\text{but } *\text{'s long.} = 85\ \ 46\ \ 16 \ldots\ldots\ldots\ldots\ 85\ \ 46\ \ 16$$

$$\therefore N \ldots\ldots = 101\ \ 23\ \ 26 \qquad n = -75\ \ 43\ \ 46$$

Again,

M		m	
		log. r......	10
log. 20.25.......	1.30642	....	1.30642
log. sin. $72^\circ\ 59'\ 39''$	9.98058	log. cos. $72^\circ\ 59'\ 39''$	9.46608
	11.28700		20.77250 (a)
log. sin. $74^\circ\ 22'\ 50''$	9.98460	log sin. $1^\circ\ 45'\ 34''$	8.48722
(log. M)........	1.30240	log. cos. 71 30 2	9.50146
$M = 20''.06$			17.98868 (b)
		(log. m)	2.78382 ($a-b$)

$$m = 607''.9 = 10'\ \ 7''.9.$$

Hence,

$$A = 20''.06 \,.\, \cos.(\odot - N) = 20''.06 \,.\, \cos.(\odot - 3^s\ 11^\circ\ 23'\ 26''),$$

$$a = -607''.9 \,.\, \sin.(\odot - n) = -607''.9\ \sin.(\odot + 2\ \ 15\ \ 43\ \ 46),$$

or, as before, (see pp. 276, 277,)

$$A = 20''.06 \,.\, \sin.(\odot + 11^s\ 18^\circ\ 36'\ 34''),$$

$$a = 607''.9 \,.\, \sin.(\odot + 8\ \ 15\ \ 43\ \ 46).$$

EXAMPLE IV. η *Ursæ majoris.*

Suppose the latitude, longitude, and angle of position of this star (to be computed by the formulæ of pp. 159, 168, 175) to be, in the year 1725 (the time of Bradley's Observations) as follow:

latitude..................	$54^0\ 23'\ 53''$
longitude.................	173 3 15
angle of position (P)	38 37 26
the N. P. D. is	39 18 5

$d_0 Z$ $\qquad\qquad$ $A_0 Z$

10+		10+	
log. tan. 38° 37′ 26″...	19.90253..	cot. 38° 37′ 26″	20.09746
log. sin. 54 23 53...	9.91013	....	9.91013
(log. tan. $d_0 Z$).......	9.99240	(log. cot. A_0Z)	10.18733

$d_0 Z = 1^s\ 14^\circ\ 29'\ 55''$ $\qquad$ $A_0 Z = 1^s\ 3^\circ\ 0'\ 33''$

$\therefore D_0 Z = 4\ 14\ 29\ 55$ $\qquad$ $a_0 Z = 1\ 26\ 59\ 27$

but ✱'s long. $= 5\ 23\ 3\ 15$ .... $5\ 23\ 3\ 15$

$N = 10\ 7\ 33\ 10$ $\qquad$ $n = 3\ 26\ 3\ 48$

M $\qquad\qquad$ m

		log. r......	10
log. 20.25........	1.30642	....	1.30642
log. sin. 38° 37′ 26″	9.79532	log. cos. 38° 37′ 26″	9.89279
	11.10174		21.19921 (a)
log. sin. 44 29 25	9.84565	log. sin. 56 59 27	9.92354
(log. M)........	1.25609	log. sin. 39 18 5	9.80167
$M = 18''.03$			19.72521 (b)
		(log. m)	1.47400 ($a-b$)

$m = 29''.78.$

Hence,

$$A = 18''.03 . \cos. (\odot - 10^s\ 7^0\ 33'\ 10''),$$
$$\text{or} = 18''.03 . \sin. (\odot + 4\ 22\ 26\ 50),$$
$$a - 29''.78 . \sin. (\odot - 3\ 26\ 3\ 48),$$
$$\text{or} = 29''.78 . \sin. (\odot + 2\ 3\ 56\ 12).$$

Example V. γ *Draconis.*

(See the latitude, longitude, angle of position of this star for 1815, at pp. 166, 174, 177.)

$d_0 Z$		$A_0 Z$	
10 +		10 +	
log. tan. $2^\circ\ 56'\ 53''$	18.71180	log cot. $2^\circ\ 56'\ 53''$	21.28820
log. sin. 74 56 51	9.98484		9.98484
(log. tan. $d_0 Z$)	8.72696	(log. cot. $A_0 Z$)	11.30336
$d_0 Z =$	$0^s\ 3^\circ\ 3'\ 9''$	$A_0 Z =$	$0^s\ 2^\circ\ 50'\ 50''$
$\therefore D_0 Z =$	3 3 3 9	$a_0 Z =$	0 87 9 10
but $*$'s long. $=$	8 25 14 36		8 25 14 36
$N =$	11 28 17 45	$n =$	5 28 5 26

M		m	
		log. r	10
log. 20.25	1.30642		1.30642
log. sin. $2^\circ\ 56'\ 53''$	8.71122	log. cos. $2^\circ\ 56'\ 53''$	9.99942
	10.01764		21.30584 (*a*)
log. sin. $3^\circ\ 3'\ 9''$	8.72633	log. cos. $2^\circ\ 50'\ 50''$	9.99946
(log. M)	1.29131	sin. 38 29 5	9.79400
$M = 19''.557$			19.79346 (*b*)
		(log. m)	1.51238

$$m = 32''.53.$$

Hence,

$$A = 19'',557 \,.\, \cos.(\odot - 11^s\ 28^\circ\ 17'\ 45''),$$
$$\text{or} = 19''.557 \,.\, \sin.(\odot + 3\ \ 1\ \ 42\ \ 15),$$
$$a = -32''.53 \,.\, \sin.(\odot - 5\ \ 28\ \ 5\ \ 26),$$
$$\text{or} = 32''.53 \,.\, \sin.(\odot + 0\ \ 1\ \ 54\ \ 34).$$

EXAMPLE VI. *a Aquarii.*

The Latitude &c. of this Star for the year 1800 are as follow:

latitude..............	0^s	10^0	$41'$	$34''$,
longitude	11	0	35	45,
(P) angle of position	0	20	17	48,
N. P. D.............	0	91	16	58.

$d_0 Z$ $\qquad\qquad\qquad\qquad$ $A_0 Z$

10 + $\qquad\qquad\qquad\qquad$ 10 +

log. tan. $20^0\ 17'\ 48''$.. 19.56802.. log. cot. $20^0\ 17'\ 48''$ 20.43198

log. sin. 10 41 34 .. 9.26777................ 9.26777

(log. tan. $d_0 Z$)...... 10.30025 $\qquad$ (log. cot. $A_0 Z$) 11.16421

$d_0 Z = 2^s\ 3^o\ 23'\ 37''$ $\qquad$ $A_0 Z = 0^s\ 3^o\ 55'\ 50''$

$\therefore\ D_0 Z = 0\ 26\ 36\ 23$ $\qquad$ $a_0 Z = 3\ 3\ 55\ 50$

but $*$'s long. = 11 0 33 45.............. 11 0 33 45

$\therefore\ N = 11\ 27\ 10\ 8$ $\qquad$ $n = 7\ 26\ 37\ 55$

M $\qquad\qquad\qquad\qquad$ m

log. r...... 10

log. $20''.25$ ...... 1.30642................ 1.30642

log. sin. $20^o\ 17'\ 48''$ 9.54018 log. cos. $20^o\ 17'\ 48''$ 9.97216

10.84660 $\qquad$ 21.27858 (a)

log. sin. $63^o\ 23'\ 37''$ 9.95137 log. cos. $3^o\ 55'\ 50''$ 9.99897

(log. M)........ 0.89523 log. cos. 1 16 58 9.99989

19.99886 (b)

$M = 7''.856$ $\qquad$ (log. m) 1.27972 ($a-b$)

$m = 19''.04$.

Hence,

$A = 7''.856 . \cos. (\odot - 11^s\ 27^o\ 10'\ 8'')$,

or, $= 7''.856 . \sin. (\odot + 3\ 2\ 49\ 52)$,

$a = -19''.04 . \sin. (\odot - 7\ 26\ 37\ 55)$,

or, $= 19''.04 . \sin. (\odot + 10\ 3\ 22\ 5)$.

From these expressions* A and a may be deduced, for any assigned time, by one operation. The assigned time gives ⊙, the Sun's longitude: and we may deduce A and a, either by multiplying the coefficients, that express the maxima of aberration, into the natural sines of the sums of the arcs, or by a logarithmic process: for instance, suppose the aberrations of γ Draconis were required for December 3;

$$\begin{array}{lrrrl} \text{By the Nautical Almanack, } \odot\ldots = & 8^s & 10^\circ & 34' & \\ \text{arc to be added (see p. 280.)} = & 3 & 1 & 42 & \text{(negl}^g\text{. the seconds)} \\ \hline & 11 & 12 & 16 & \end{array}$$

$$\text{natural sine of } 11^s\ 12^\circ\ 16' = -.3045;$$

$$\therefore A = 19''.55 \times -.3045 = -5''.95,$$

a quantity, with its affixed sign, to be added to the *mean* north polar distance in order to obtain the apparent north polar distance.

Again, to find, by the logarithmic process, the aberration of η Ursæ majoris on the same day,

$$\begin{array}{lrrrl} \odot\ldots\ldots\ldots\ldots\ldots\ldots = & 8^s & 10^\circ & 34' & \\ \text{arc to be added (see p. 279.)} = & 4 & 22 & 27 & \\ \hline & 13 & 3 & 1 & \ldots\ \text{log. sin.} = 9.73630 \\ & & & & \text{log. } M = 1.25609 \\ & & & & (10+\text{log.})\ A\ \ 10.99239 \end{array}$$

$\therefore A = 9''.82$ to be added to the *mean* north polar distance in order to obtain the apparent north polar distance.

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* These values of A, a, are to be added, as it has been already remarked, to the *mean* north polar distances and *mean* right ascensions, in order to obtain the *apparent*. If we wish for expressions still *additive*, and for the reverse operation, we must increase, or diminish, according to the case, the arcs, which are the *arguments*, by 6^s: thus in the last case,

$$A' = 7''.856 \,.\, \text{sin.}\,(\odot + 9^s\ 2^\circ\ 49'\ 52''),$$
$$a' = 19''.04 \,.\, \text{sin.}\,(\odot + 4\ \ 3\ \ 22\ \ 5)$$

A', and a', being quantities to be added to the *apparent* in order to obtain the mean north polar distances and right ascension.

With equal facility and brevity may the aberrations of other stars be deduced for any assigned period: but still more conveniently by means of a Table: the columns of which should contain log. M, log. m, N, n, or, the sines of the sums of arcs being used, quantities (N', n') analogous to N, n. We will shew a specimen of such a Table by means of the results we have already obtained.

Stars.	Log. M.	N'	Log. m.	n'
γ Pegasi	.96343	1^s 27° $33'$	1.28222	8^s 29° $7'$
α Arietis.	.89551	1 0 17	1.31311	7 28 39
Polaris.	1.30240	11 18 36	2.78382	8 15 44
η Ursæ majoris .	1.25609	4 22 27	1.47400	2 3 56
γ Draconis. . . .	1.29131	3 1 42	1.51238	0 1 54
α Aquarii	.89523	3 2 50	1.27972	10 3 22

With such a Table the two rules for finding the aberration in north polar distance and right ascension would be

$$\log. A = \log. \sin. (\odot + N') + \log. M,$$
$$\log. a = \log. \sin. (\odot + n') + \log. m.$$

In computing M, N', &c. we have used different periods: that of 1815, for γ Pegasi; of 1725 for η Ursæ majoris, &c. In a Table properly constructed, the numbers ought to be computed for the same epoch. Still, the present Table will give, if we exclude Polaris, results nearly true: for, although the angles of position and the longitudes of stars, or, to go farther, the right ascensions and declinations of stars and the obliquity of the ecliptic are continually varying, yet they may considerably vary without much affecting the values of the aberrations in right ascension and north polar distance. Thus, of the last star in the above Tables, a change of 1° in its right ascension will not produce a change exceeding $0''.34$ in its aberration in right ascension. A special Table then of aberrations will *last* 50 or 60 years, for

stars that have not great declinations. A Table for Polaris will require to be renewed every eight or ten years. The resulting values of A and a are to be added to the *mean* north polar distances, and *mean* right ascensions, respectively, in order to obtain the apparent north polar distances and right ascensions.

In the Volume of the Greenwich Observations for 1812, &c. there is inserted, at p. 250, a Table for ninety-six stars, similar to the preceding one. It is founded, however, upon the *first* of the formulæ which have been investigated for expressing the aberrations in north polar distance: upon this

$$A = M . \cos. (\odot - N),$$

and since the Table is to be used for reducing *apparent* north polar distances to *mean*, it gives results with signs different from those that belong to the Table of p. 283. The two Tables, however, are essentially the same *.

In the precept, (see p. 282,) for using the Table we are directed to take $\odot$, the Sun's longitude, from the Nautical Almanack. This part of the rule, however, stands in need of some modification: for, if we look (see pp. 269, &c.) to the investigation of the formulæ, it is, clearly, a condition of such investigation that the Sun's longitude should be that, which it ought to be, at the time of the star's passage over the meridian †. The Sun's longitude, therefore, taken from the Nautical Almanack is not truly expressed, except (which is a particular case) the Sun

* The Tables may be deduced, the one from the other. For, in the Tables for the aberrations in north polar distance, the sum of the respective *numbers* (such as N', N) always equals 9^s or 21^s. Thus, if the numbers under the column N', in a Table so constructed, should be for the stars, *a* Cassiopeæ, *a* Ceti, *a* Persei, *a* Coronæ Bor. *a* Herculis, respectively,

$$\left.\begin{array}{rrr} 0^s & 10^\circ & 38' \\ 2 & 23 & 25 \\ 11 & 4 & 26 \\ 3 & 22 & 4 \\ 3 & 5 & 36 \end{array}\right\} \begin{array}{l}\text{The numbers in a} \\ \text{Table constructed like} \\ \text{the Greenwich Table I,} \\ \text{would be} \ldots\ldots\ldots\ldots\end{array} \left\{\begin{array}{rrr} 8^s & 29^\circ & 22' \\ 6 & 6 & 35 \\ 9 & 25 & 34 \\ 5 & 7 & 56 \\ 5 & 24 & 24 \end{array}\right.$$

† The observations of right ascension and north polar distance are supposed to be made on the meridian.

and star are on the meridian together: for instance, the right ascension of η Ursæ majoris is $13^h\ 40^m\ 7^s$, and on June 20, 1812, the Sun's right ascension was $= 5^h\ 55^m\ 14^s$; consequently, the star was on the meridian about $7^h\ 45^m$ after *noon*, at which latter time, the Sun's longitude, by the Nautical Almanack, was $2^s\ 28^\circ\ 54'\ 32''$: this longitude, therefore, must be increased (if $59'$ be the Sun's increase of longitude in twenty-four hours) by $\frac{7^h\ 45^m}{24^h} \times 59'$, or, by about $19'\ 24''$. This quantity, then, in the above instance, and like proportional quantities, in other instances, must, in forming the *arguments* $(\odot + N')$, &c. be added, *as corrections* to the Sun's longitude. A Table, in the Greenwich Observations, immediately following that we have already noticed, contains, for each star, the correction due to it for every tenth day of the year.

The labour of an Astronomer, in reducing his observations, is so great, that the construction of *convenient* Tables is a matter of considerable importance. The Tables, which we have described, hold a middle place between *special* and *general* Tables. *Special* Tables express, in numbers, the aberrations of certain stars for every tenth day, or for every ten degrees of the Sun's longitude. Such Tables are the most convenient and the most sure in practice. They have, over other Tables, that kind of advantage which *Taylor's Logarithms* have over Sherwin's. But they are inconvenient from the largeness of their Volume *. *General* Tables of aberration are, indeed, small in size, but cannot be used without considerable computation. Besides the labour of using them there are the chances (which are excluded from Special Tables) of mistakes. From the right ascension, and declination of the star and the day we may deduce from these Tables the star's aberration; but not without six or seven small *processes* of computation.

---

* M. Zach has, in his Tabulæ Speciales Aberrationis et Nutationis: Gotha, 1806, given the aberrations of six hundred zodiacal stars. These are contained in two thick Volumes. Their learned Author remarks that like Tables, for Piazzi's Catalogue of nine thousand stars, would require fourteen octavo Volumes of five hundred pages each.

By the Table which has been described and constructed, we arrive, with little risk of a mistake, at the result after twice using the Trigonometrical and Logarithmic Tables: once to *take out* a logarithmic sine: again, to *take out* the number coresponding to a resulting logarithm. But, perhaps, it will be better to shew the convenience of the Table by one or two illustrations.

Example I.

Required the aberration in right ascension and north polar distance of Polaris* on July 23, 1800.

$$\begin{array}{rl}
 & \qquad A \\
\odot = & 4^s\ 0^\circ\ 27' \\
N = & 11\ 18\ 36 \\ \hline
 & 15\ 19\ \ 3 \ldots\ldots\ldots\ \text{sin.} = 9.97554 \\
 & \qquad\qquad\qquad \text{log.}\ M = \underline{1.30240} \\
 & \qquad\qquad (10 + \text{log.}\ A) = 11.27794 \\
 & \therefore A = 18''.96
\end{array}$$

$$\begin{array}{rl}
 & \qquad a \\
\odot = & 4^s\ 0^0\ 27' \\
n = & 8\ 15\ 44 \\ \hline
 & 12\ 16\ 11 \ldots\ldots\ldots\ \text{sin.} = 9.44515 \\
 & \qquad\qquad\qquad \text{log.}\ m = \underline{2.78382} \\
 & \qquad\qquad (10 + \text{log.}\ m)\ \ 12.22897 \\
 & \therefore m = 169''.42 = 2'\ 49''.42.
\end{array}$$

---

* It was mentioned in page 283, that Tables of Aberration will serve, during fifty years, for stars, the polar distances of which are not very small. But Polaris, the north polar distance of which is less than 2°, requires to have a new Table of aberration constructed for it every ten years. β Ursæ minoris is in a like predicament. We subjoin what, according to M. Zach, would be the *numbers* and logarithms of maxima for *Polaris* for the years 1790, 1800, 1810, 1820.

	Log. M	N'	Log. m in time.	n'
1790	1.3035	$11^s\ 19^\circ\ 7'$	1.5945	$8^s\ 16^\circ\ 18'$
1800	1.3034	11 18 39	1.6081	8 15 46
1810	1.3033	11 18 9	1.6215	8 15 14
1820	1.3032	11 17 33	1.6361	8 14 35

EXAMPLE II.

Required the aberration in north polar distance of η Ursæ majoris on Feb. 20th: and its aberration in right ascension on Dec. 3, 1726.

$$
\begin{array}{rrrr}
\text{Feb. 20th, } \odot = & 11^s & 1^\circ & 6' \\
\text{(see p. 284,) correct}^n. & 0 & 0 & 40 \\
\hline
 & 11 & 1 & 46 \\
N' \ldots\ldots\ldots\ldots & 4 & 22 & 27 \\
\hline
 & 15 & 24 & 13 \ldots\ldots\ldots \sin. = 9.95999 \\
\end{array}
$$

$$\log. M = 1.25609$$

$$(10 + \log. A) \quad 11.21608$$

$$\therefore A = 16''.44.$$

Again,

$$
\begin{array}{rrrr}
 & a & & \\
\text{Dec. 3, } \odot = & 8^s & 11^\circ & 30' \\
n' = & 2 & 3 & 56 \\
\hline
 & 10 & 15 & 26 \ldots\ldots\ldots\ldots \sin. = 9.84617 \\
\end{array}
$$

$$\log. m = 1.47400$$

$$(10 + \log. a) \quad 11.32017$$

$$a = -\ 20''.9.$$

EXAMPLE III.

Required the aberrations in north polar distance of a Arietis on Feb. 16, and May 22, 1812.

$$
\begin{array}{rrrr}
\text{Feb. 16, } \odot = & 10^s & 26^\circ & 44' \\
\text{(see p. 284,) corr.} = & 0 & 0 & 9 \\
\hline
 & 10 & 26 & 53 \\
N' \ldots\ldots\ldots\ldots\ldots & 1 & 0 & 17 \\
\hline
 & 11 & 27 & 10 \ldots \sin. = 8.69400 \\
\end{array}
$$

$$\log. M = 0.89551$$

$$9.58951 \therefore A = -.388.$$

Again,

$$\begin{array}{lrrrr}
\text{May 22, } \odot & = 2^s & 1^\circ & 10' \\
\text{correction} & = 0 & 0 & 55 \\
\hline
& 2 & 2 & 5 \\
N' \ldots\ldots\ldots\ldots & 1 & 0 & 7 \\
\hline
& 3 & 2 & 12 \ldots\ldots\ldots\ldots \text{sin.} = 9.99968
\end{array}$$

$$\log. M = \underline{0.89551}$$

$$(10 + \log. A) \ 10.89519$$

$$\therefore A = + 7''.85.$$

Before we proceed any farther with the mathematical processes that belong to this subject, we will illustrate the formulæ already obtained, and shew how completely they explain the phenomena observed by Bradley.

By p. 280, it appears

the aberr^n. in N. P. D. of γ Draconis $= 19''.55 \sin. (\odot + 3^s\ 1^\circ\ 42'')$.

The aberration, therefore, is a maximum, equal to $19''.55$, and negative, when $\odot + 3^s\ 1^\circ\ 42'$ is equal to 9^s, and also a maximum, equal to $19''.55$, and positive, when

$$\odot + 3^s\ 1^\circ\ 42' \text{ is equal to } 15^s;$$

that is, the star is, from the effect of aberration,

most *northerly*, when $\odot = 5^s\ 28^\circ\ 18'$,
most *southerly*, when $\odot = 11\ \ 28\ \ 18$.

The Sun has the former longitude about Sept. 22, the latter about March 19.

Now Bradley says (*Phil. Trans.* No. 406. p. 640.) 'About the beginning of March *(Old Stile)* the star was found to be more *southerly* than at the time of the first observation. It now, indeed, seemed to have arrived at *its utmost limit* southward.'

Again, the aberration in north polar distance, will be nothing,

either when $\ldots\ldots\ldots\ldots \odot + 3^s\ 1^\circ\ 42' = 6^s$,
or, when $\ldots\ldots\ldots\ldots \odot + 3\ \ 1\ \ 42 = 12$;

that is, either on June 20th, when $\odot = 2^s\ 28^\circ\ 18'$
or, on Dec. 20th, when ...... $\odot = 8\ \ 28\ \ 18.$

Now Bradley says (*Phil. Trans.* No. 406. p. 639.) 'on the 5th, 11th and 12th, there appeared no material alteration in the place of the star.' Which agrees with our results, since Bradley's dates are according to the Old Stile. Again, we read (p. 640.) 'about the beginning of June (Old Stile) it (the star γ Draconis) passed at the same distance from the zenith as it had done in December.'

The formula belonging to η Ursæ Majoris furnishes us with like illustrations. We have (see p. 279,)

$$A \text{ in N. P. D.} = 18''.03 \,.\, \sin.\,(\odot + 4^s\ 22^\circ\ 27')$$

(expressing the *argument** in the nearest minutes), consequently, A is a maximum, equal to $18''.03$, and negative, when $\odot + 4^s\ 22^0\ 27' = 9^s$: it is, also, a maximum, equal to $18''.03$, but positive, when $\odot + 4^s\ 22^\circ\ 27' = 15^s$; the first case happens when.......... $\odot = 4^s\ 7^\circ\ 33'$, about July 31,
the latter, when .... $\odot = 10\ \ 7\ \ 33$, about Jan. 27.

On this latter day, then, the star is most remote from the north pole, or *farthest* south: and Bradley says (p. 658.) 'it was farthest south about the 17th of January,' the reckoning being according to the Old Stile.

There are several other inferences to be easily drawn from the formulæ of aberration; for instance, the aberration of γ Draconis in north polar distance is a maximum, either when

$$\odot = 5^s\ 28^\circ\ 18', \text{ or, } = 11^s\ 28^\circ\ 18'.$$

Now, (see p. 280,)

$$\text{the aberration } (a) \text{ in } \text{Æ} = 32''.53 \,.\, \sin.\,(\odot + 1^\circ\ 54'\ 34'')$$
$$\therefore\ = 32''.53 \,.\, \sin.\,(6^s\ 0^0\ 22'\ 34''),$$
$$\text{or } = 32''.53 . \sin.\,(12\ \ 0\ \ 22\ \ 34);$$

* The argument is the arc $(\odot + 4^s\ 22^\circ\ 27')$.

when A in north polar distance is a maximum: but these values of a, although very small, are not nothing. When the aberration in north polar distance, therefore, is a maximum, it does not follow that the aberration in right ascension is nothing; such would be the case, if the star were situated exactly in the solstitial colure. But γ Draconis is not exactly so situated. If we take η Ursæ majoris, we shall find that its aberration in right ascension, is considerable when its aberration in north polar distance is a maximum; for, when this latter happens (see p. 279,) $\odot = 4^s\ 7^\circ\ 33'$: at that time, therefore,

$$a = 29''.78 \,.\, \sin.\,(\odot + 2^s\ 3^\circ\ 56')$$
$$= 29''.78 \,.\, \sin.\,(6^s\ 11^\circ\ 29')$$
$$= -29''.78 \,.\, \sin.\,(11^\circ\ 29') = -5''.93.$$

The time at which any particular star passes the meridian, when its aberration is either a maximum or nothing, is also easily determined from the preceding formulæ. For instance, when γ Draconis passes the meridian most to the north, the Sun's longitude (see p. 289,) is $5^s\ 28^\circ\ 18'$ (on Sept. 22): its right ascension, at that time equal $11^h\ 55^m$. But (see p. 165,) the star's right ascension $= 17^h\ 50^m$; and the difference between the right ascensions ($17^h\ 50^m - 11^h\ 55^m$), or $5^h\ 55^m$, is, nearly, the time at which the star passes the meridian after the Sun. The star then passes the meridian, very nearly, at six in the evening: it would pass (when its aberration in north polar distance is greatest) exactly were it situated in the solstitial colure.

We arrive at like and consistent conclusions, if we investigate the aberration when γ Draconis *did pass* at six in the evening, or at six in the morning: suppose we take the latter time, then

$$24^h + 17^h\ 50^m - 18^h = \odot\text{'s } Æ;$$
$$\therefore\ \odot\text{'s } Æ = 23^h\ 50^m, \text{ and } \odot = 11^s\ 27^\circ\ 17';$$

consequently, (see p. 280,)

$$A = 19''.55 \,.\, \sin.\,(11^s\ 27^\circ\ 17' + 3^s\ 1^\circ\ 42')$$
$$= 19''.55 \,.\, \sin.\,(14\ \ 28\ \ 59)$$
$$= 19''.55 \,.\, \sin.\,(\ 2\ \ 28\ \ 59),$$

which is evidently less than $19''.55$ the maximum of aberration.

Again, when the aberration in right ascension of η Ursæ majoris is a maximum, $\odot = 0^s\ 26^\circ\ 4'$, or, $6^s\ 26^\circ\ 4'$; but, when the Sun's longitude ($\odot$) is $0^s\ 26^\circ\ 4'$

$\odot$'s Æ.	$= 1^h\ 36^m\ 44^s$
and since $*$'s Æ	$= 13\ \ 35\ \ \ 0$
the approximate time of passing the meridian is . .	$11\ \ 58\ \ 16$

or η Ursæ majoris, when its aberration in right ascension is the greatest, passes the meridian about two minutes before midnight.

When the same star passes the meridian at six in the evening,

$$13^m\ 35^m - \odot\text{'s Æ} = 6^h;\ \text{nearly,}$$

$$\text{and consequently, } \odot\text{'s Æ} = 6^h\ 25^m,$$

$$\odot\ \text{(the Sun's longitude) is about } 3^s\ 5^\circ\ 48';$$

$$\therefore \text{ then } \mathcal{A} = 18''.03\ (\sin.\ 7^s\ 28^\circ\ 15') = -18''.03 \sin.\ (1^s\ 28^\circ\ 15'),$$

$$\text{and } a = 29''.78\ (\sin.\ 5^s\ 9^\circ\ 44') = 29''.78 \sin.\ (20^\circ\ 16').$$

There is, in the preceding instances, abundant evidence of the truth of Bradley's observation (p. 644.) 'I have since discovered, that the maxima of these stars do not happen exactly when they come to my instrument at those hours.'

We will continue, a little longer, the illustration and explanation of Bradley's original methods.

In the preceding pages the coefficient $20''.25$ has been used instead of $20''$, which is Bradley's value. It is the aberration (see p. 268,) which a star, situated in the pole of the ecliptic, will constantly have in the plane of the circular arc σT. But Bradley did not determine its value either by observations of a star in, or near to, the pole of the ecliptic. Had there been a large star so situated it would not, for other reasons, have suited Bradley's purpose. It would have been too remote from the zenith of his Observatory, to have been observed by his Zenith Sector, and if it could have been observed, its refractions would have, in some degree, perplexed the deduction of results. The stars that Bradley did observe were all within a few degrees of his

zenith. The greatest aberrations in north polar distance of such stars were observed, and the coefficient (20″), we are speaking of, deduced in the following manner:

Thus, instead of 20″.25, suppose we represent the coefficient of the expression of p. 177, 271. by an indeterminate quantity x, then

$$M = x \cdot \frac{\sin. P}{\sin. d_0 Z}.$$

M was determined by observation, $d_0 Z$ and P (see pp. 270,) by computation; and thence x was deduced. Thus, suppose γ Draconis to have been the observed star, and the interval between its most northward point of aberration, and it most southward, to have been 39″: then 39″ is twice the value of M;

$$\therefore 39'' = 2x \cdot \frac{\sin. P}{\sin. d_0 Z};$$

$$\text{and } 2x = 39'' \cdot \frac{\sin. d_0 Z}{\sin. P}.$$

At the time of Bradley's observation, suppose (see p. 177,) the values of $d_0 Z$ and P to have been (and these were nearly their values) 3° 52′, 3° 44′, respectively: then, computing, by logarithms, the value of the above expression, we have

log. 39″	1.59106
log. sin. 3° 52′	8.82888
	10.41994
log. sin. 3° 44′	8.81366
(log. 40.05)	1.60628

$$\therefore 2x = 40''.05, \text{ and } x = 20''.025.$$

This was the result from one of Bradley's stars. The other stars, seven in number, gave, by similar computations, results a little different. The following Table contains those results, not, indeed, exactly those which Bradley obtained, but those which

M. Zach, on repeating Bradley's computations, affirms to be the true values.

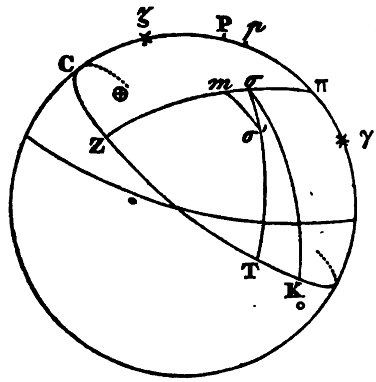
Stars.	Zenith Distance. in 1760.	Distance from Pole of Ecliptic.	Values of M.	Values of $2x$.
γ Draconis ...	0° 2′ 58″.5	15° 3′ 0″	39	40″.378
η Ursæ majoris.	0 57 30.2	35 36 0	36	40.423
α Cassiopeæ...	3 44 28.5	43 24 0	34	41.085
β Draconis ...	1 0 40.5	14 42 0	39	40.236
α Persei	2 29 29.5	59 54 0	23	40.201
τ Persei	0 17 3.5	55 39 0	25	38.820
Capella......	5 44 21.5 *	69 51 0	16	39.658
35 Camelopard Hevelii......	0 4 11.7	62 56 23	19	38.281

The mean of the first five stars gives 40″.464 for the value of $2x$, and, consequently, 20″.232 for the maximum of aberration, or, as Bradley expresses it (*Phil. Trans.* No. 406. p. 654.) for the radius 'of the little circle described by a star in the pole of the ecliptic.'

The history of this discovery, one of the most curious and interesting in Astronomical science, resembles the histories of many other discoveries. It was not soon found out, nor immediately suggested. Many fruitless trials and erroneous conjectures preceded it. Bradley devised several hypotheses for the

* In the second column the zenith distances of the observed stars are inserted. The range of Bradley's Zenith Sector was about 6°¼ on each side of the zenith. It is obvious that stars, remote from the zenith, would have been unfit for the detection of so small an inequality as that of the aberration. The refraction of stars, having zenith distances greater than 36°, would have exceeded the whole quantity of the aberration, and by being mixed up with it would have rendered difficult the disentangling of the latter.

explanation of the phenomenon he had discovered. A *Nutation* of the Earth's axis, or an inclination of its position, naturally suggested itself, (see *Phil. Trans.* No. 406, p. 641). In September γ *Draconis* was more northerly, that is, nearer to



the north pole, than it had been in the preceding June: might not then the pole P have shifted its place from P to p? if it had so shifted, then this must happen: the north polar distance of a star ζ, situated also in the solstitial colure, but in an opposite part of it, that is, differing from it in its right ascension by 180^0, would, instead of being $P\zeta$, be increased to $p\zeta$, and precisely by the quantity Pp. Now what was the fact? The north polar distance of ζ, or $P\zeta$, was found to be increased, ***but not by the quantity*** Pp, that, by which the north polar distance of γ had been diminished, but, by about half that quantity. This, therefore, was quite decisive against the hypothesis of a nutation of the axis, or of a shifting of the pole from P to p.

But, on Bradley's last hypothesis, that which has been propounded as the true one, is the phenomenon, just mentioned, explicable? The star ζ was one in the constellation of *Camelopardalus*, with a north polar distance equal to that of γ *Draconis;* its co-latitude, therefore was equal to the obliquity of the ecliptic $+$north polar distance, that is, it was about 62°, and its latitude accordingly, would be 28°. Therefore since the latitude of γ *Draconis* (see p. 57.) is 74°: and the maximum $(N) = 20'' \times$ sin. star's latitude: hence,

$N(\gamma\ Draconis) : N(\zeta\ Camelopardali) :: \sin. 74^\circ : \sin. 28^\circ$
$:: 9612 : 4694$
$:: 2.04,\ \&c. : 1,$

which result agrees with the observed phenomenon; and accordingly, Bradley's theory explains it.

For the understanding of the subject of the present Chapter, and for the application of its formulæ, there is, perhaps, enough already done. We wish, however, to say a word or two on certain formulæ from which general Tables of aberration are constructed. The Tables, of which the construction has been given in the preceding pages, have been constructed by the intervention, or aid, of certain angles, called Angles of Position, and of the longitudes and latitudes of stars. Now these quantities depend and (see pp. 153, 168, &c.) are, in fact, derived (the obliquity of the ecliptic being given) from the right ascensions and declinations of stars. We ought, therefore, to consider whether there may not be some simple or some convenient mode of expressing the inequalities of aberration, in terms of the star's right ascension, declination, and of the Sun's longitude. For, catalogues of these latter quantities are easily resorted to, being usually inserted in Astronomical Treatises, and in National Ephemerides. Whereas catalogues of the latitudes and longitudes of stars and of their angles of position are rarely to be met with.

We will now, then, proceed to deduce, from the formula we have already established, those other formulæ which it is, at the least, an object of curiosity, to enquire after.

The aberration in right ascension (a) depends (see p. 272,) on these two formulæ,

$$\tan. a_0 Z = \frac{\cot. P}{\sin. \lambda};$$

$$a = 20''.25 . \frac{\cos. P}{\sin. \delta \cos. A_0 Z} . \sin. A_0 T,$$

λ is the star's latitude, let L denote its longitude, then

$$A_0 T = \odot - 180^\circ - L + a_0 Z,$$

and consequently, $\sin. A_0 T = \sin. (L - \odot - a_0 Z)$,

and since $a_0 Z = 90^0 \pm A_0 Z$, $\sin. a_0 Z = \cos. A_0 Z$;

$$\therefore\ a = 20''.25 . \frac{\cos. P}{\sin. \delta} . \{\sin. (L - \odot) \cot. a_0 Z - \cos. (L - \odot)\}$$

$$= \frac{20''.25 \cos. P}{\sin. \delta} . \left(\sin. (L - \odot) \frac{\sin. \lambda}{\cot. P} - \cos. (L - \odot)\right)$$

which latter is, in fact, Cagnoli's expression given in p. 441, of his Trigonometry.

Again, by expanding the sine and cosine of the binomial arc,

$$a = -\frac{20''.25}{\sin. \delta} . \left\{ \begin{array}{l} \cos. \odot (\cos. L \cos. P - \sin. L \sin. P \sin. \lambda) \\ + \sin. \odot (\cos. L \sin. P \sin. \lambda + \sin. L . \cos. P) \end{array} \right\}$$

$=$ (by forms 11 and 10 of p. 182.)

$$-\frac{20''.25}{\sin. \delta} (\sin. \odot \sin. Æ + \cos. \odot \cos. Æ . \cos. I),$$

which agrees with the first part of Delambre's expression given at p. 111. tom. III. of his Astronomy. We may express the latter form differently, by substituting, instead of

$$\cos. I (= \cos. 23^\circ\ 27'\ 56'')$$

its numerical value.

Thus, $20''.25 . \cos. I = 18''.575$;

$$\therefore\ a = -\frac{1}{\sin. \delta} (20''.25 . \sin. \odot \sin. Æ + 18''.575 \cos. \odot \cos. Æ)$$

$$= -\frac{1}{\sin. \delta} . \left\{ \begin{array}{l} 10''.125 \cos. (\odot - Æ) - 10''.125 \cos. (\odot + Æ) \\ + 9''.287 \cos. (\odot - Æ) + 9.287 \cos. (\odot + Æ) \end{array} \right\}$$

$$\therefore\ a = \frac{0''.838 . \cos. (\odot + Æ) - 19''.412 . \cos. (\odot - Æ)}{\sin. \delta},$$

which is the same expression which Delambre has given, in p. 115. tom. III, Cagnoli, in p. 443, and Vince, in p. 236; of their respective Treatises.

We may express differently the preceding formulæ: thus, since

$$\sin. \odot \sin. Æ = \tfrac{1}{2}\left\{\begin{array}{l}\sin. \odot \sin. Æ\,(1 + \cos. I) \\ + \sin. \odot \sin. Æ\,(1 - \cos. I)\end{array}\right\},$$

$$\cos. \odot \cos. Æ \cos. I = \tfrac{1}{2}\left\{\begin{array}{l}\cos. \odot \cos. Æ\,(1 + \cos. I) \\ - \cos. \odot \cos. Æ\,(1 - \cos. I)\end{array}\right\},$$

we have

$$a = - \frac{10''.125}{\sin. \delta}\left\{\begin{array}{l}\cos. (\odot - Æ)(1 + \cos. I) \\ - \cos. (\odot + Æ)\,(1 - \cos. I)\end{array}\right\},$$

which is Delambre's formula given in the *Connoissance des Tems* for 1788, p. 239, and in that for 1810, p. 460.

Instead of $1 + \cos. I$, $1 - \cos. I$, in the above expression, we may substitute $2 . \cos.^2 \frac{I}{2}$, $2 \sin.^2 \frac{I}{2}$, and then

$$a = \frac{20''.25}{\sin. \delta} . \left\{\cos. (\odot + Æ) . \sin.^2 \frac{I}{2} - \cos. (\odot - Æ) . \cos.^2 \frac{I}{2}\right\},$$

which is the expression of Delambre: (see his *Astronomy*, tom. III. p. 115. also Suanberg's *Exposition*, &c. p. 115.)

By like transformations, the formula previously obtained, (see p. 271,) for the aberration in north polar distance, may be transformed into that which Delambre has used for the constructing of his general Tables of aberration.

The subject of aberration has proved fruitful in the invention of formulæ and their dependent Tables. There is no great difficulty in multiplying such formulæ, or rather, as we wish to view the matter, in variously modifying the formulæ that have been originally obtained, (see p. 271, &c.) We will give one more instance,

$$a = - \frac{20''.25}{\sin. \delta}(\cos. \odot \cos. Æ \cos. I + \sin. \odot \sin. Æ),$$

$$a = - \frac{20''.25}{\sin. \delta} . \cos. \odot \cos. I \left\{\cos. Æ + \frac{\tan. \odot . \sin. Æ}{\cos. I}\right\}.$$

Let $\dfrac{\tan. \odot}{\cos. I} = \tan. (\odot + x)$;

$$\text{then } a = -\frac{20''.25}{\sin.\delta}\cdot\frac{\cos.\odot\cos.I}{\cos.(\odot+x)}\cdot\left\{\begin{array}{l}\cos.Æ\cos.(\odot+x)\\+\sin.Æ\sin.(\odot+x)\end{array}\right\}$$

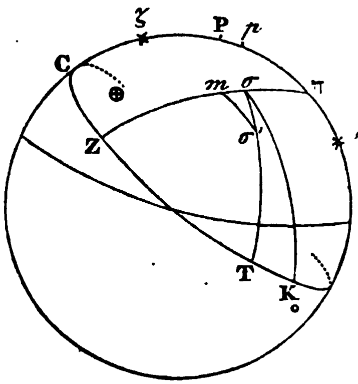
$$= -\frac{20''.25}{\sin.\delta}\cdot\frac{\cos.\odot\cos.I}{\cos.(\odot+x)}\cdot\{\cos.(\odot+x-Æ)\},$$

which formula is the foundation of the construction of M. Gauss's Tables of aberration in right ascension.

The formulæ for the aberrations in north polar distance and right ascension are the most important formulæ, since they enable the observer to correct the observations made with the mural quadrant, transit telescope, and zenith sector. The latitudes and longitudes of stars are, as it has been more than once said, angular quantities not observed but deducible from observations. It rarely happens then that it becomes necessary to correct these quantities for aberration. Still there are astronomical calculations in which the aberrations in latitude and longitude are required to be known. For that reason, we will now proceed to deduce, and on the plan already acted on (see pp. 269, &c.) the formulæ for such aberrations.

Investigation of the Position of the Point T *when the Aberration in Latitude* = 0.

Draw σK_0 perpendicular to $\pi\sigma$, a secondary to the ecliptic;



then σK_0 is the position of σT, and K_0 of T, when the aberration in latitude is = 0.

Now $K_0 Z$ is perpendicular to πZ; and since $K_0\sigma$ is drawn so, K_0 (see *Trig*. p. 128,) is the pole of the circle πZ; $\therefore$ $K_0 Z$ is a quadrant; $\therefore$ since K_0 is 90° before the corresponding place of the Earth, the Earth is at Z, or is in *syzygy* with the star.

Formula for the Aberration in Latitude.

Draw $\sigma' m$ perpendicular to πZ; then σm $(=K)$ is the aberration in latitude,

$$\begin{aligned} \text{and } \sigma m, \text{ or } K &= \sigma\sigma' . \cos. m\sigma\sigma' \\ &= 20''.25 \sin. \sigma T . \sin. T\sigma K_0 \\ &= 20.25 \sin. K_0 T . \sin. TK_0\sigma. \end{aligned}$$

But, since K_0 is the pole of πZ, the angle $TK_0\sigma$ is measured by σZ, the star's latitude. Hence,

$$K = 20''.25 . \sin. K_0 T \times \sin. \text{ star's latitude.}$$

Hence, K, the aberration, is a maximum (N) when $K_0 T$ is equal 90°; that is, when T is in Z or 180° distant from it; or when the Earth is in *quadratures* (see p. 135,) with the star: the formulæ become then

$$N = 20''.25 . \sin. \text{ star's latitude} \ldots . (7),$$
$$K = N . \sin. K_0 T \ldots\ldots\ldots\ldots (8).$$

Investigation of the Position of the Point T *when the Aberration in Longitude* = 0. (See Fig. in p. 298.)

This must happen, when σT coincides with σZ: or, when T falls in Z; that is, since T is 90° before the corresponding place (⊕) of the Earth, when the Earth is in *quadratures* with the star.

Formula for the Aberration in Longitude.

$$\text{The aberration } (k) = \angle m\pi\sigma' = \frac{m\sigma'}{\sin. \pi\sigma} = \frac{\sigma\sigma' . \sin. Z\sigma T}{\cos. Z\sigma}$$

$$= 20''.25 . \frac{\sin. \sigma T . \sin. Z\sigma T}{\cos. Z\sigma}.$$

But, since $Z\sigma T$ is a right-angled spherical triangle, by Naper's rule, we have

$$1 \times \sin. ZT = \sin. \sigma T' \times \sin. Z\sigma T;$$

$$\therefore\ k = 20''.25 . \frac{\sin. ZT}{\cos. \text{ star's latitude}} = 20''.25 . \frac{\cos. \oplus Z}{\cos. \text{ star's latitude}} .$$

Hence k is a maximum (n) when cos. $\oplus Z$ is the greatest, that is when $\oplus Z$ either $=0$, or $180°$: in other words, when the Earth, or Sun, is in syzygy with the star :

$$\text{hence the maximum, or } n = \frac{20''.25}{\cos. \text{ star's latitude}} \ldots\ldots (9),$$

$$\text{and } k = n . \cos. \oplus Z \ldots\ldots\ldots\ldots\ldots\ldots (10).$$

We might have avoided this direct process and deduced the aberrations in latitude and longitude from those in right ascension and declination. In its technical enunciation, the enquiry would have been to find the *errors* in latitude and longitude from the given *errors* in right ascension and declination: which errors might have been found by two ways: either after Cotes's manner, as Cagnoli has done, or by deducing the values of dL, $d\lambda$ from some of the formulæ given in p. 182.

It is plain, in investigating the formulæ of aberration, that we might have pursued a method the reverse of that which has been now described: that is, the first steps of investigation might have been directed to the finding out (and they are, in fact, the most easily found) the aberrations in longitude and latitude: thence we might have proceeded, by a route strictly mathematical, and without any clue furnished by the nature of the enquiry, to the aberrations in north polar distance and right ascension. Such, generally, has been the course of investigation. Clairaut, Thomas Simpson, Cagnoli, and Suanberg have followed it. The last-mentioned Author, in his Treatise *, has derived his formulæ, from the differentials or the fluxions of equations 1, 2, of

* Exposition des Operations faites en Lapponie, pour la determination d'un arc du meridien, &c. Stockholm, 1805.

p. 182. He goes, however, farther than the generality of authors*, and adds, to his formulæ, certain minute corrections which are due to the eccentricity of the Earth's orbit.

But, as it has been already remarked, there is a great variety in the ways of deducing the formulæ of aberration. The formulæ of aberration in right ascension and north polar distance, being the most important, have been investigated by the most direct and shortest methods. Of such investigations, the curve of the star's aberration, described during a year, was not a condition. It was not enquired, since it was not essential to enquire, whether the curve were circular or elliptical. The laws of the several aberrations (which laws are expressed by their appropriate formulæ) are indeed connected with the form of the curve, inasmuch as two results derived from a common source are connected. If the one were varied the other would: and, in consequence of this sort of connexion, it is easy to see that, from one established or proved, the other might be deduced as a Corollary or consequence. From the nature, or law, then, of the curve apparently described, during a year, by a star, in consequence of the principle of *aberration of light*, the respective formulæ expressing the aberration in its several directions may be supposed to be derived. And, in fact, the original proposition in the present theory was 'that the apparent path described by a star, in consequence of aberration, was a circle the plane of which was parallel to the ecliptic.'

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* Delambre has done the same thing (see his *Astronomy*, pp. 110, &c.) We have not entered into these investigations, of which, perhaps, the chief use is the shewing that the corrections sought are so small, that they may safely be neglected. If π be the longitude of the perigee the term to be added to the aberration in right ascension, (see p. 297.) is

$$\frac{-0''.34}{\sin.\delta}(\cos.I.\cos.\text{Æ}.\cos.\pi+\sin.\text{Æ}\sin.\pi).$$

The aberration on north polar distance will be

$$20''.25(\cos.\text{Æ}\sin.\odot-\cos.I.\sin.\text{Æ}\cos.\odot)$$
$$+20''.25.\sin.I\cos.\odot\cos.\delta-0''.34\sin.I\cos.\pi\cos.\delta$$
$$+0''.34\{\sin.\delta(\cos.I\sin.\text{Æ}\cos.\pi-\cos.\text{Æ}\sin.\pi)\}.$$

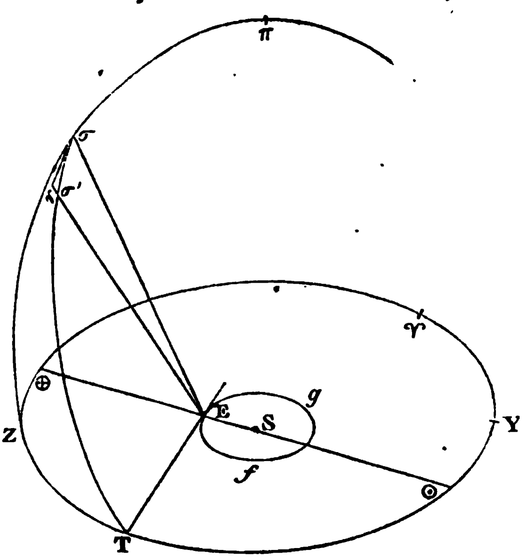
This certainly was a happy beginning of a beautiful Theory. But the origin of the formulæ was thrown a little farther back than when they were made to commence from the effects of aberration on the latitudes and longitudes of stars. These latter quantities are not, except in particular cases, objects of observation. But the *circle* of aberration, is still less so; it is a mere fiction whatever be the star in the Heavens we select for our observation. It cannot be called a phenomenon, because it would be only such were the circumstances of observation different from what they really are. The curve *would* be a circle, if we saw the aberrations in directions parallel to the plane of the ecliptic. And this point we will now proceed to establish.

By p. 267, the aberration always takes place in a plane passing through σE, ET. But, T, in the course of a year, is carried through the circle ZTY; therefore, if we conceive $E\sigma$ to remain parallel to itself, (which it may be conceived to do, by reason of the relative smallness of ES) $E\sigma'$, will in a year generate round $E\sigma$ a conical surface.

Draw $r\sigma$ parallel to ET; then, by p. 268,

$r\sigma : E\sigma ::$ velocity of the Earth : velocity of light.

Now, the latter velocity is assumed to be constant, and if the first



be, then $r\sigma$ is so also; that is, during the revolution $r\sigma$ will

describe a circle, parallel to the plane in which ET is, or parallel to the plane of the ecliptic. This circle may be considered as the base of the conical surface described by Er.

Since $E\sigma$ is not necessarily perpendicular to the plane of the ecliptic, and consequently not so to the plane of the circle described by $r\sigma$, the generated surface belongs to that species of cone which is called oblique.

The above is, as we have stated, a merely Geometrical Theorem: the spectator sees no circle. The star always appears to him in the direction of $E\sigma'$, and he constantly refers σ' to the imaginary concave surface of the heavens to which $E\sigma$ is perpendicular: consequently, since the intersection of the oblique cone by the concave surface, or by a tangent plane at σ, is an ellipse*, the star, during the year, will constantly appear to be in the circumference of such curve.

In one case, indeed, if a star were situated in the pole of the ecliptic, the star's apparent path will be circular; for, then, $E\sigma$ will be perpendicular to the plane of the ecliptic, and the conical surface generated by $E\sigma'$, will belong to a *right* cone, or a cone of revolution.

This is sufficiently plain, if σr be constant, or if the Earth's velocity be constant. But, if we suppose, which is the case in nature, the Earth's velocity to vary, what then will be the imaginary curve which σr describes, or, what will appear to be the curve of aberration of a star situated in π the pole of the ecliptic? It is a curious result, that, in this, as well as in the preceding simple case, the curve is a circle.

Let E be the Earth, in her elliptical orbit; S the Sun in one focus, and let H be the other focus, HZ a perpendicular to TEt, a tangent at E. Draw from σ the star, σh parallel to Bb,

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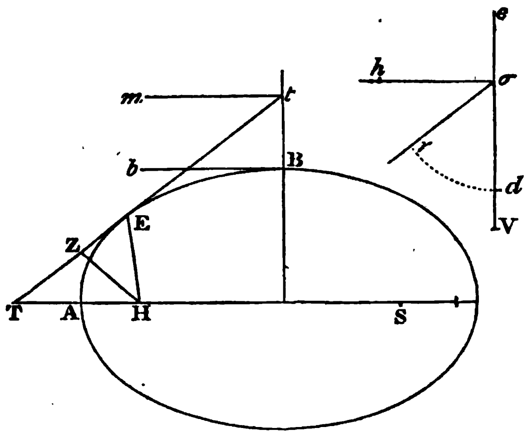
* The intersection of an oblique cone by a plane not parallel to the circular base of the cone, and not a *sub-contrary* section, is an ellipse.

aud σr to TEt; and take σr proportional to the Earth's velocity



at E.

Since the $\angle h\sigma r = \angle mtT = \angle ZTH$; $\therefore$ the complement of $h\sigma r$, or $\angle r\sigma V = \angle THZ$, the complement of ZTH, in other terms, σr, HZ make equal angles with σV, HT. Moreover, the Earth's velocity varies inversely as a perpendicular from S on the tangent TEt, or, by Conics, directly as HZ: but, σr varies as the Earth's velocity, and therefore as HZ. Hence, σr varying as HZ, and revolving towards σV with the same angular velocity as that with which HZ revolves towards HT, r and Z must describe similar curves: but (Vince's *Conics*, p. 17. Edit. 1781.) Z describes a circle, consequently r does.

At the point A, HZ is the least, and the angle $THZ = 0$; therefore in the line σV, the aberration σd is the least, and consequently there perpendicular to the circle. In the opposite part of the line, σe is the greatest and also perpendicular to the circle.

Hence the centre of the circle is in the line de, and its distance from σ is equal to $\frac{\sigma e - \sigma d}{2}$.

Such are the propositions which, as it has been remarked, are, in some Treatises, first established, and then become the

foundation of the formulæ of aberration. According to the view which we have taken of the theory, they are not essential to it, of no use to the practical Astronomer, and are speculative and mathematical. Their excellence, however, as such, has been the cause of their present introduction.

We must now (for this is one of the objects of an Elementary Treatise) say a few words on the application and uses of the formulæ of Aberration.

When we speak of the comparison of the zenith distances, or of the polar distances, of the same star at different epochs, we cannot mean to speak of their observed distances. For, such expressions would be altogether vague and ambiguous. We mean to speak of distances *cleared* of inequalities, or alike affected by the same inequality. And we cannot better illustrate this point than by considering one of the methods of determining the differences of the latitudes of places.

The difference of the distances of the same star from the zeniths of two places, is the difference of the latitudes of those places, if the star be either north or south of both zeniths (see p. 12.) If north of one, and south of the other, then the sum of the distances is the difference of the latitudes. If the star be observed on the same day by two observers, then, since the aberration would equally affect each observation, no correction, beyond that of refraction, would be necessary. The zenith distances might be immediately added or subtracted. But, which generally is the case, if we make an observation in one place, and avail ourselves of an observation made previously in another, then this latter will need correction. In the interval between the two observations, or, in the interval between the actual observation, and the epoch at which the star's place is registered in Tables, the star, with respect to the pole, and consequently to the zenith, will have changed its mean place: it must, therefore, by the means of Tables, be *brought up* from its tabulated place, to its mean place at the time of observation. But, at that time, from the effect of aberration, the observed star is either seen to the north or the south of its true place. The quantity of deviation therefore, or the aberration in declination, must be

either added to, or subtracted from, the place of the observed star; or, subtracted from or added to the place of the tabulated star. The latter is the usual mode, by which, accordingly, the *apparent* and not the *mean* zenith distances of stars are compared. The following instance will illustrate the preceding explanation:

May 10, 1802, Blenheim Observ. apparent zenith distance (north) of γ *Draconis*	0°	19′	44″.59
1802. Greenwich *mean* zen. dist. (south).	0	2	16.65
Aberration to May 10	0	0	12.58
May 10, 1802, *Apparent* zenith distance of γ *Draconis* at Greenwich	0°	2′	4″.07
∴ sum. of zen. dist. or difference of latitudes*. .	0	21	48.66
and since latitude of Greenwich Observatory . .	51	28	39.5
latitude of Blenheim	51	50	28.16

In a similar way, may the difference of the latitudes of places be determined, if, instead of a recorded observation and one actually made, we use two recorded observations. Thus, we may determine the difference of the latitudes of Cambridge and Greenwich, by means of a zenith distance of γ *Draconis* made, in the former place, June 3, 1790, and of a zenith distance of the same star made in the latter, Jan. 5, 1797. The two observations, by applying, with other corrections, that of aberration, may be *reduced* either to June 3, 1790, or to Jan. 5, 1797, or both may be reduced to some other; for instance, Jan. 1, 1790, or Jan. 1, 1800.

With regard to the formulæ of aberration in right ascension, we will now shew their use in regulating astronomical clocks. The foundation of all our methods of making time the measure of right ascensions, is the supposition of the Earth's equable rotation round its axis. If that rotation alone regulated the intervals between the successive transits of stars over the meridian, all such transits would

* The *aberration* is additive to the north polar distance; therefore since γ *Draconis* is north of the zenith, *subtractive* of such zenith distance.

be equal, and performed in twenty-four hours of sidereal time. In such a case, nothing would be more simple than the mode of regulating a clock. We should have merely to observe the star on the middle wire of the transit telescope, and to note the contemporaneous position of the index of the clock. But other circumstances influence the intervals between the transits of stars. Of such circumstances the inequality of aberration is one. It (see p. 264,) sometimes causes a star to appear on the middle wire, sooner than it otherwise would have done, at other times later. The intervals between the transits of stars, then, which would be equal from the Earth's rotation, can no longer be so. But if the observer should know by *how much* they are unequal, he could, as truly, although not so simply, regulate his clock as in the first supposition. And this knowledge is afforded him by the formulæ of aberration, or their derived Tables.

Our attention, in the preceding pages, has been directed to the fixed stars: but, it is plain, the places of the Sun and of the planets must be affected with aberration. Thus, during the passage of the Sun's light to the Earth (in $8^m\ 13^s$) the Sun itself describes $20''.25$ in its orbit. The Sun, therefore, in consequence of the *progressive motion* of light, is seen $20''.25$ behind its true place. The true place being that in which the Sun at the instant at which it is seen. The same result will follow from the expression for the aberration in longitude, which is

$$\frac{20''.25}{\text{cos. } \ast\text{'s latitude}} \times \text{cos. } \oplus Z,$$

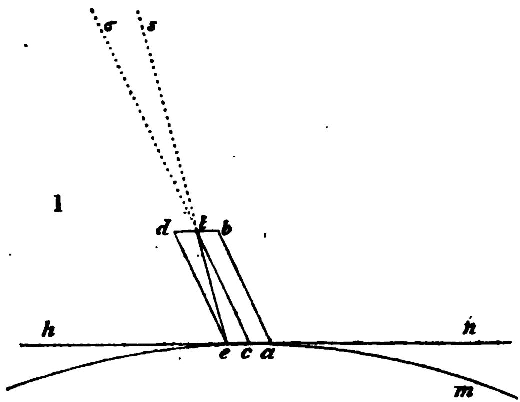
in the case of the Sun, the denominator $=$ cos. $0 = 1$,

$$\text{cos. } \oplus Z = \text{cos. } 180° = -1\,;$$

therefore the aberration $= -20''.25$.

A planet's place is differently affected by the aberration of light. In the case of a fixed star, we have shewn (see p. 253, &c.) that a star's place s would be apparently transferred to σ. Now suppose s to be a planet, and whilst its light is descending to the Earth that it moves from s to σ; then, the true and apparent places of the star will coincide; there will be no aberration; or, the star, in consequence of the aberration of light, will be neither

before nor behind its true place: the *true* place, as we have said,



being that which it ought to have at the moment of its being seen.

In the above supposition, which is a particular one, the Earth and planet moving, the same way, through equal spaces, the planet has no geocentric motion; or, a spectator on the Earth refers, at the beginning and end of the interval of the time, the planet to the same point in the Heavens.

Let the next supposition be that of the planet's describing a space less than $s\sigma$, whilst its light is transmitted to the Earth: the true place then of the planet would be between s and σ, whilst, in consequence of the aberration of light, its apparent place was at σ. The deviation arising then from these two causes would be an angle less than $st\sigma$, and formed by two lines drawn, respectively, from that intermediate position, of which we have spoken, and from σ. It would be equal to an aberration caused by the Earth moving with a velocity which is the difference of its own and the planet's, and is, as before, the planet's *geocentric* motion; or, the angle which, to a spectator on the Earth, is apparently described by the planet.

If the planet moving the same way as the Earth, should move faster than the Earth, or if, during the transmission of its light to the spectator, it should have moved from s to a point beyond σ, then, the planet will appear *behind* its true place.

If the planet (s) should be moving *from* σ, or, in a direction contrary to that of the spectator's motion, then by reason of the aberration of light, the planet's place would seem to be at σ, but the planet itself would be at some point to the right of s; and, consequently, the whole aberration, or deviation, would be some angle greater than $s t \sigma$, and would be equal to the aberration which would arise, did the planet remain at rest, whilst the Earth moves with the sum of its own and of the planet's motion. It is also equal to the planet's geocentric motion, or to the angle which a spectator on the Earth's surface imagines the planet to describe.

The expressions, then, which are essential to be known, in constructing the formulæ of aberration for the planets, are the geocentric motions of the planets; which are quantities not, as yet, investigated.

The *coefficients* of the geocentric motions are easily investigated.

Let M be a planet's horary motion, then $\frac{M}{3600}$ is its motion during one second. If 1 represent the Sun's distance from the Earth, d the planet's distance from the Earth, then, since light takes $8^m\ 13^s.2$ of time, to pass over the radius (1) of the Earth's orbit, its time of describing d will be $8^m\ 13^s.2 \times d$, consequently,

$$1^s : 8^m\ 13^s.2 \times d :: \frac{M}{3600} : Md \times \frac{493.2}{3600},$$

$$1^s : 493^s.2 \times d :: \frac{M}{3600} : \frac{Md \times 4932}{36000}\ (=Md \times .137);$$

if M, therefore, be the horary motion (whether it be in longitude, latitude, declination, or right ascension) the corresponding aberration will be

$$0.137\ M d.$$

To the above inequalities of refraction and aberration that of parallax succeeds; of which there are two kinds; one arising from, or being, the difference of the place of a star seen from

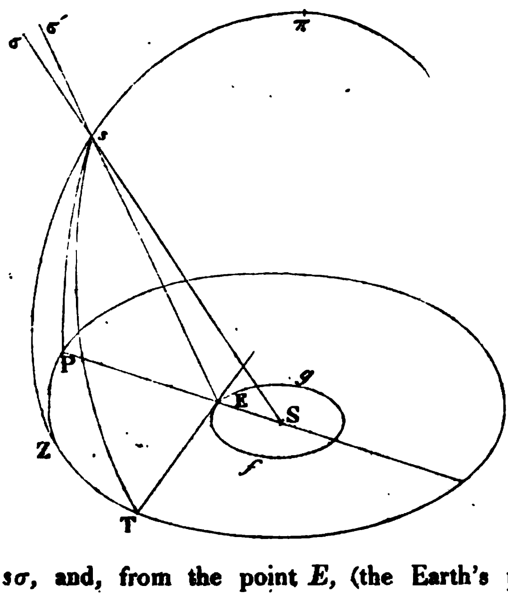
different points of the Earth's orbit; the other, the difference of a star's place seen from different parts of the Earth. The formulæ of the first kind applied to the fixed stars, would enable us, were their *parallaxes* sensible, to correct their apparent zenith distances, &c. just as we have already corrected such distances on account of refraction and aberration. But, in fact, this reduction is never made. The maximum of parallax, if parallax exist, does not exceed two seconds. Still, for reasons already stated (see p. 250.) it is useful to know its laws: which, in the beginning of the next Chapter, will be laid down and mathematically expressed in formulæ. Such formulæ are made to succeed those of aberration, because (this, indeed, is not a reason drawn from the natural order or connexion of the subjects) they may, by the most simple process, be derived from them.

CHAP. XII.

ON PARALLAX.

Its Formulæ similar to the Formulæ of Aberration.—Values of the Parallaxes of particular Stars deduced from the Tables of Aberration.—Table of Parallaxes similar to the Table of Aberrations.—Expressions for the Parallax in Right Ascensions; North Polar Distance; in Longitude; in Latitude.—Attempts made to establish the Existence of Parallax.—Parallax of a Planet.—Method of determining it.—Its Use and Exemplification.

THE description of the present Figure is nearly the same as that of the Figure of p. 267. sP is the arc of a circle of which E is the centre: s is a star seen, from the point S, in the direc-



tion $Ss\sigma$, and, from the point E, (the Earth's place), in the

direction $Es\sigma'$. The *difference* of these two places of the star, or, the angle SsE is the *parallax;*

$$\text{since, } Es : ES :: \sin. ESs : \sin. SsE,$$

$$\sin. SsE; \text{ or, nearly, } SsE = \frac{ES}{Es} . \sin. ESs.$$

Now $\frac{ES}{Es}$, the star being the same, is, nearly, a constant quantity: it would be exactly so, if the eccentricity of the Earth's orbit, which is small, were nothing. Again,

$$\angle ESs = \angle PEs - \angle EsS = \angle PEs, \text{ nearly,}$$

(in extreme cases it cannot be supposed to differ from it by $1''$). Hence,

$$\text{the parallax } (P) = \frac{ES}{Es} . \sin. PEs$$

$$= \frac{ES}{Es} . \sin. Ps;$$

P, therefore, is a maximum (B) when $Ps = 90°$, that is, when the star is in π the pole of the ecliptic: consequently,

$$B = \frac{ES}{Es},$$

$$\text{and } P = B . \sin. Ps.$$

The Earth's orbit being, nearly, circular, ET is, nearly, perpendicular to SEP, and PT (SE being extremely small relatively to SP) is, nearly, a quadrant. Now (see p. 268,) the aberration varies as the sine of sT, and, as we have just seen, the parallax varies as the sine of sP; sP will become sT after E has described $90°$: consequently, the formula which expressed the variation of the aberration, three months previously, will now express that of the parallax: or, since the aberrations in opposite points of the Earth's orbit are equal, although in different directions, the formula for the aberration, three months hence, will be the formula for the parallax at the present time.

We will now consider which of these two formulæ ought to be taken, so that the signs may be the same in each.

The star S viewed from E is seen at σ': from S at σ; therefore, by the effect of parallax, the star, in the above positions, is elevated *above* the ecliptic. By the effect of aberration (see pp. 252, &c.) a star, the Earth being at g and moving according to gEf, is *depressed* towards the ecliptic, and elevated when the Earth is at f: consequently, it is the expression for the aberration three months *after* the present time that we must take to represent the parallax.

The only point that remains to be considered is the coefficient. In the formula for aberration, the coefficient is $20''.25$: that for parallax has been represented by B. Find, therefore, by the formulæ, or by the derived Tables, the aberration (A) which would take place if the Sun's longitude were increased by three signs, and the parallax

$$(P) = \frac{B}{20''.25} \times A$$

For instance, to find the parallaxes in north polar distance of α Cygni on June 21, August 1; and November 11, add three signs to the Sun's longitudes on these days, and take out from the Tables (or compute) the corresponding aberrations: which will be, nearly, the aberrations on Sept. 22, Nov. 1, and Feb. 8 : and which aberrations (corrections to the observed distances), will be respectively,

$$-15''.65, \qquad -18'', \qquad +6''.56,$$

the parallaxes will be, (supposing B the *semi-annual* parallax to be one second)

$$\frac{-15''.65}{20.25} = -0''.78, \quad \frac{-18''}{20.25} = -0''.89, \quad \frac{6''.56}{20.25} = 0''.32.$$

By such means we obtain the values of the parallax from the Tables of aberration : but we may easily, from the principles that have been laid down, deduce the formulæ of parallax. Thus, according to the method explained in p. 274, &c. the star

being α Cygni, the number N' is $2^s\ 0^\circ\ 53'$, and the log. $M = 1.2613$; $\therefore$ log. A = log. sin. ($\odot + 2^s\ 0^\circ\ 53'$) + 1.2613: to convert this into a formula for parallax, we must, (see p. 313,) add 3^s to $\odot$, and, should $1''$ be the semi-annual parallax, deduct 1.30642 (the logarithm of $20''.25$) from 1.2613 the logarithm of the maximum. Hence,

$$\log. P \text{ (in N. P. D.)} = \log. \sin. (\odot + 5^s\ 0^\circ\ 53') + \bar{1}.9549.$$

Let us apply this formulæ to deduce the preceding results,

June 21,	$\odot$ =	2^s	29°	$51'$		
		5	0	53		
		8	0	44	.... log. sin. =	9.9407
					log. max. =	$\bar{1}$.9549
					(log. .786)	19.8956

$\therefore$ parallax = $-\ 0''.78$,

the sine of $8^s\ 0^\circ\ 44'$ being negative.

Again,

August 1,	$\odot$ =	4^s	8°	$59'$		
		5	0	53		
		9	9	52	.... log. sin. =	9.9935
						$\bar{1}$.9549
					(log. .88)	19.9484

$\therefore$ parallax = $-\ 0''.88$.

Again,

Nov. 11,	$\odot$ =	7^s	18°	$59'$		
		5	0	53		
		12	19	52	.... log. sin. =	9.5312
						$\bar{1}$.9549
					(log. .306)	19.4861.

$\therefore$ parallax = $0''.306$.

Generally, if

$$\log. A = \log. \sin. (\odot + N') + \log. M,$$

$$\log. P (\text{in N.P.D.}) = \log. \sin. (\odot + 90^\circ + N') + \log. M - 1.3064,$$

$1''$ being the semi-annual parallax.

We might, then, were it worth the while, from a Table of Aberrations, (see p. 283,) form a Table of Parallaxes: for instance,

Stars.	Number for N. P. D.	Log. maximum for N. P. D.	Number for Right Ascen.	Log. maximum for Right Ascen.
α Lyræ . .	$2^s\ 24^\circ\ 42'$	$\bar{1}.9452$	$2^s\ 23^\circ\ 5'$	0.1066
α Cygni. .	5 0 53	$\bar{1}.9549$	1 23 50	0.1334
β Aurigæ.	0 7 17	$\bar{1}.5652$	9 3 28	0.1497
α Aquilæ.	5 13 9	$\bar{1}.7175$	2 6 34	$\bar{1}.9982$

From this Table, and by the above formula, we may immediately compute the parallaxes of the above stars, for any day in the year.

With a facility like that which the preceding transformations admit of, and on the same principle, we may transform any other formulæ of aberration into formulæ of parallax: for instance, let us take Delambre's formulæ, (see p. 301.)

$$A (\text{in N.P.D.}) = 20''.25 . \cos. \delta (\cos. Æ . \sin. \odot - \cos. I . \sin. Æ \cos. \odot)$$
$$+ 20''.25 \sin. I . \cos. \odot ;$$

therefore, adding by the rule (see p. 313,) 3^s to $\odot$,

$$\text{par}^x. (\text{in N.P.D.}) = 1''. \cos. \delta (\cos. I \sin. Æ \sin. \odot + \cos. Æ \cos. \odot)$$
$$- 1''. \sin. I . \sin. \odot$$

Again,

$$\text{Aberr}^n. \text{ in } Æ = - \frac{20''.25}{\sin. \delta} (\cos. I . \cos. Æ . \cos. \odot + \sin. Æ \sin. \odot);$$

$$\therefore \text{par}^x. (\text{in } Æ) = \frac{1''}{\sin. \delta} (\cos. I . \cos. Æ . \sin. \odot - \sin. Æ \cos. \odot).$$

Again, L and λ representing, respectively, the longitude and latitude of a star,

$$\text{the aberration in longitude} = -\frac{20''.25}{\cos.\lambda}\cos.(L - \odot);$$

$$\therefore \text{ the parallax in longitude} = -\frac{1''}{\cos.\lambda}\sin.(L - \odot).$$

Again,

$$\text{Aberration in latitude} = 20''.25 . \sin.\lambda . \sin.(L - \odot);$$

$$\therefore \text{ parallax in latitude} = - 1''.\sin.\lambda . \cos.(L - \odot).$$

Hence, when the aberrations in longitude and latitude are the greatest, the parallaxes in longitude and latitude are the least: and conversely.

Since the variations of the parallax and aberration are expressed by the same formulæ, the *curves* (should any question arise concerning them) of aberration and parallax are similar. That is, (see p. 303.) the curve apparently described by a star, in consequence of parallax, is an ellipse, of which the minor axis is $2''.\sin.\lambda$, $2''$ representing the major axis, or, generally, if 2π should represent the major axis, $2\pi\sin.\lambda$ would represent the minor.

These ellipses, like those that represent the aberration, are easily traced out. M. Lalande in his *Astronomy*, vol. III, has traced out the ellipses of parallaxes of Sirius and Arcturus: now

the latitude of Sirius (1805).......... =	0^s	39°	$33'$	$40''$
of Arcturus	0	30	52	17
longitude of Sirius	3	11	23	0
of Arcturus	6	21	30	0.

Hence, (see l. 7,)

$$\text{paral. in lat. of Sirius} = -1''.\sin.(39^\circ 33' 40'') \times \cos.(3^s\, 11^\circ 23' - \odot)$$

$$\therefore \text{ the parallax is a maximum either when } \odot = 3^s\ 11^\circ\ 23'$$
$$\text{or when } \odot = 9\ \ 11\ \ 23$$

the two maxima of parallax happens then, about July 3, and January 3, and are, respectively, equal to $-0''.6367$, $+0''.6367$:

draw, therefore, a line parallel to the ecliptic equal to $2''$, and, through its middle point, draw, on each side of it, a line equal to .6367 which will be the semi-minor axis. The whole line will be the minor axis; at the extremity farthest from the ecliptic the star will appear to be on January 3, at the extremity nearest to the ecliptic on July 3.

In order to find when Sirius will appear to be at the extremity of the major axis, we must make in the expression for the parallax in longitude, namely, in

$$-\text{ sec. } \lambda \text{ sin. } (3^s\ 11^\circ\ 23' - \odot),$$

$$3^s\ 11^\circ\ 23' - \odot = 3^s;\ \text{whence } \odot = 11^\circ\ 23':$$

the time, corresponding to this longitude, is April 1: at the interval of half a year, the star is at the other extremity of the axis major.

With regard to Arcturus, the expressions for his parallaxes in latitude and longitude are, respectively,

$$-1''.\text{sin. }\lambda.\text{cos. }(6^s\ 21^0\ 30' - \odot),$$

$$\text{and } -1''.\text{sec. }\lambda.\text{sin. }(6\ \ 21\ \ 30 - \odot);$$

consequently, he is at the extremities of his minor axis, either when $\odot = 6^s\ 21^0\ 30'$, or $= 21^0\ 30'$: that is, on October 15th, and April 11th; and he is at the extremities of the major axis of the ellipse of parallax, either when $\odot = 3^s\ 21^0\ 30'$, or $= 9^s\ 21^\circ\ 30'$, that is, on July 14, and January 12. The minor axis of the ellipse is $2''.\text{sin. } 30^0\ 52'\ 17''$, or $2 \times .513 = 1''.026$.

It was in the observations of the pole star that Flamstead thought he discovered the existence of parallax. Now (see p. 278,) the aberration in N. P. D. $= 20''.06 \text{ sin.} (\odot + 11^s\ 18^0\ 17')$;

$$\therefore \text{ (see p. 313,) the parallax} = \frac{20''.06}{20.25}\text{sin.}(\odot + 14\ 18\ 17)$$

$$= 0''.99 . \text{sin. } (\odot + 2\ 18\ 17).$$

The parallax, therefore, is nothing when $\odot = 3^s\ 11^0\ 43'$, and very small, when $\odot$ is nearly of the above value; that is, about the middle of summer. In winter the same thing will take place; that is, the parallax in declination, (supposing the star to

have an annual parallax) will be extremely small. Now, Flamstead, from his observations, found the declination of the pole star to be less in summer than in winter by about $40''$; or, which is the same thing, he found the diameter of the small circle, described by Polaris round the pole, to be larger in summer than in winter by about $1'\ 20''$. But this phenomenon could not, as we have shewn, arise from parallax: still it was a phenomenon: in other words, the observations of Flamstead were just*: there was such a difference as he noted, but it arose not from parallax but aberration: which it is easy to shew: thus, by the expression in the preceding page,

$$\text{when } \odot = 3^s\ 11^\circ\ 43',$$

$$\text{the aberration in N. P. D.} = 20''.06 \,.\, \sin.\ 15^s = 20''.06,$$

$$\text{and, at the opposite point of the Earth's orbit,} = -20''.06.$$

Hence, the north polar distance was greater, or the declination less, in the former period than in the latter by $20''.06 + 20''.06$, or $40''.12$, which agrees exactly with Flamstead's Observations, but overturns his inferences.

The preceding part of this Chapter relates to the *fixed* stars. Should these, or any of them, have an annual parallax, their *apparent* places in right ascension and north polar distance will, by reason of such parallax, differ from their *mean*. The corrections, for reducing the one to the other are furnished by the preceding formulæ: which formulæ (see p. 315,) are, with regard to the analytical law of their construction, the same as the formulæ of aberration. We could easily, then, correct the

* 'The observations of Mr. Flamstead of the different distances of the pole star at different times of the year, which were through mistake looked upon by some as a proof of the annual parallax of it, seem to have been made with much greater care than those of Dr. Hook. For though they do not all exactly correspond with each other, yet from the whole Mr. Flamstead concluded that the star was 35″, 40″, or 45″ nearer the pole in December than in May or July; and according to my hypothesis it ought to appear 40″ nearer in December than in June.' Bradley, *Phil. Trans.* No. 106, p. 661.

right ascension and north polar distance of a star on account of parallax, if a certain annual parallax were assigned to it; or, from observing the maxima of parallax in north polar distance, we could (as in the case of aberration, see p. 292,) assign the radius of that circle of parallax which a star situated in the pole of the ecliptic would apparently describe. There is no difficulty, indeed, in correcting an observed distance on an assumed quantity of parallax, nor, should any differences in the places of stars, not accounted for on established theories, be observed, any difficulty in determining, whether such differences can be imputed to parallax. The real difficulty is to make observations that can be relied on to the fractions of a second of space: since the question is, whether there can be shewn in observations parts of a second of space not accounted for on known theories, and not attributable to the errors of observation.

The subject has, at various times, occupied the attention of Astronomers. Before the discovery of the aberration of light, the main object, in the search after parallax, was the establishment of the Copernican System * as far as that could be effected by the proof of the Earth's motion. This was Hook's object, Flamstead's, and Bradley's. The first asserted the existence of parallax by relying on his own faulty observations: the second by faulty inferences. From good observations Bradley shewed the errors of Hook's observations, and of Flamstead's reasonings; he made it evident that the latter Astronomer, in his search after parallax, had stumbled on the effects of aberration, which he mistook for those of the former inequality. Bradley himself thought † that the stars had no sensible parallax, and that he

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* 'To furnish the learned with an Experimentum Crucis to determine between the Tychonic and Copernican Systems.' Hooke's Treatise entitled, An attempt to prove the Motion of the Earth by Observations.

† 'I am of opinion, that if it were 1″, I should have perceived it, in the great number of observations that I made especially of γ Draconis: which agreeing with the hypothesis (without allowing any thing for parallax) nearly as when the Sun was in conjunction with, as in opposition to this star, it seems very probable that the parallax of it is not so great as one single second; and consequently, that it is above 400000 times farther from us than the Sun.' *Phil. Trans.* for Dec. 1728. p. 660.

must have discovered such an inequality in γ Draconis and η Ursæ majoris, had the parallax in either of these stars amounted to 1″. It is right, however, to observe that this remark of Bradley is not much to be relied on, since it was made at a time when the stars were subject to an inequality, of the existence of which he was then ignorant.

The question of parallax has, of late years, been revived by Dr. Brinkley* who thinks that he has found parallax in α Aquilæ, α Lyræ, α Cygni. In consequence of this opinion, fixed telescopes have been directed, at the Greenwich Observatory, towards certain stars, with this special object in view: namely, that each telescope should take into its field of view, at least, two stars differing from each other in right ascension. One of these telescopes is directed towards α Cygni, of which the north polar distance is about 45° 22′ and right ascension $20^h\ 35^m$. But the north polar polar distance of β Aurigæ is about 45° 5′. Therefore, since the difference of their declinations does not exceed 18′, the telescope can be so placed that each star, when it passes the meridian, shall be in the field of view: one passing to the north of a middle wire, the other to the south. Now the right ascension of β Aurigæ is about $5^h\ 46^m$, and, therefore, it will pass the meridian about $14^h\ 40^m$ before α Cygni. The effects of parallax will in certain seasons be to decrease the north polar distances of both stars: in other seasons to increase them: and in others to increase the north polar distance of one whilst it decreases that of the other: and conversely. These combined effects are shewn by means of a micrometer which measures every day (every day on which an observation can be made) the *difference* of the declinations of the two stars. We ought rather to have said, that the combined effects of parallax (should there be any) may be extricated from the differences of declination which, by means of the micrometer attached to the telescope, are instrumentally shewn. For, besides the substantial difference of the *mean* declinations, the apparent distance of the two stars arises from the different effects of aberration, nutation, &c. on the two stars. These latter effects being accounted for, or the

* Irish Transactions, vol. XII. *Trans. Royal Society*, 1818.

observations being corrected, if there should appear to be any other difference than that of the mean declination, the causes of such difference would become matter of enquiry. In order to ascertain whether parallax is one, or a sole cause, we must compare the *unaccounted for* differences with the mathematical calculations of the combined effects of parallax. This is easily effected: Thus, computing, on the principles and according to the methods of pp. 274, &c. the *numbers* due to the parallaxes in north polar distance of α Cygni and β Aurigæ, and the maxima, we have

the number for α Cygni $= 5^s\ 0^0\ 53'$, maximum $= 0''.913$
for β Aurigæ $= 0\ 17\ 17$, maximum $= 0''.367$;

consequently, the combined effect of parallax in north polar distance on these two stars is

$$0''.913 \times \sin.(\odot + 5^s\ 0^0\ 53') - 0''.367 . \sin.(\odot + 0^s\ 17^0\ 17').$$

For instance, if we wish to find the combined effect on July 1, October 1, and Oct. 11, we have

	α Cygni, July 1.		β Aurigæ, July 1.	
$\odot =$	$3^s\ 9^0\ 23'$	....	$3^s\ 9^0\ 23'$	
	5 0 53	....	0 17 17	
	8 10 16	log. sin. = 9.9737	3 26 40	log. sin. = 9.9549
		log. 913 = 9.9549		log. 367 = 9.5652
		log. (– .848) 9.9286		(log. .327) 9.5157

Hence, the combined effect $= -0''.848 - 0''.327 = -1''.75$.

Again,

	α Cygni, Oct. 1.		β Aurigæ, Oct. 1.	
$\odot =$	$6^s\ 8^0\ 8'$	....	$6^s\ 8^0\ 8'$	
	5 0 53	....	0 17 17	
	11 9 1	log. sin. = 9.5540	6 25 25	log. sin. = 9.6326
		log. 913 = 9.9549		log. 3678 = 9.5652
		(log. —,318) 9.5089		log. (– .157) 9.1978

∴ combined effect $= -0''.318 + 0''.157 = -0''.161$.

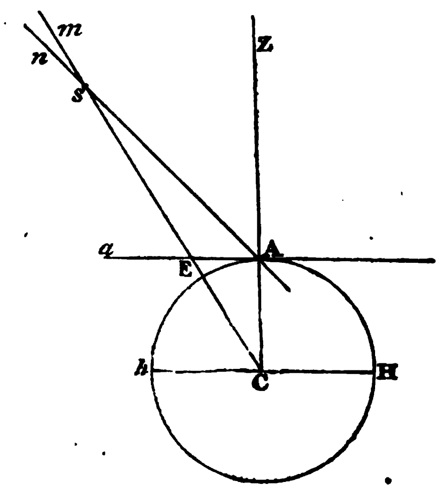
Again,

	α Cygni, Oct. 11.		β Aurigæ, Oct. 11.	
$\odot =$	$6^s\ 18^\circ\ 0'$	....	$6^s\ 18^\circ\ 0'$	
	$5\ \ 0\ \ 53$	....	$0\ \ 17\ \ 17$	
	$11\ \ 18\ \ 53$	log. sin. $= 9.2851$	$7\ \ 5\ \ 17$	log. sin. $= 9.7616$
		log. $913 = 9.9549$		log. $.3678 = 9.5652$
		(log. $- .173$) 9.2400		($- .212$) 9.3268

$\therefore$ combined effect $= -0''.173 + 0''.212 = 0''.039$.

But we will now dismiss this class of parallaxes, the discussion of which is not suited to an Elementary Treatise, and turn our attention to the parallaxes of the planets, which can not only be proved to exist, but which are, in Practical Astronomy, used for determining the distances of those planets that are affected by them.

We have already (see p. 209.) entered on this subject. If s be a planet, C the centre of the Earth, Z the zenith of the spectator placed at A, then s is seen from A in the direction Asm,



and from C in the direction Csm. If the latter be held, or be defined, to be the *true* or *Astronomical* direction, then Asn is the *apparent*. As far as parallax is concerned, m is the planet's true

place, n its apparent, and the angular distance of these places, or the angle nsm ($=AsC$) is the *diurnal parallax*.

By Plane Trigonometry,

$$\sin. CsA = \frac{AC}{Cs} \times \sin. CAs = \frac{AC}{Cs} . \sin. ZAs;$$

but ZAs is the star's apparent zenith distance, consequently, if CA, Cs be held to be constant,

$\sin. CsA$, or sin. parallax $\propto$ sin. apparent zenith distance.

Hence, the parallax is greatest when the zenith distance $=90^\circ$, or when the planet appears in the horizon. Let P be the greatest parallax, p the parallax at any other zenith distance, then

$$\sin. P : \sin. p :: 1 : \text{sin. apparent zenith distance};$$

$$\therefore \sin. p = \sin. P . \text{sin. zenith distance}$$
$$= \sin. P . \sin. (D + p),$$

D being the angle ZCs.

We may thus approximate to the value of p in terms of P and D,

$$\sin. p = \sin. P \sin. D \cos. p + \sin. P \cos. D \sin. p;$$
$$\therefore \tan. p = \sin. P \sin. D + \sin. P \cos. D . \tan. p.$$

Instead of $\tan. p$ in the last term, substitute its value as expressed by the equation just obtained; then

$$\tan. p = \sin. P . \sin. D + \sin.^2 P . \sin. D \cos. D + \sin.^2 P . \cos.^2 D \tan. p.$$

Repeat the operation, and

$$\tan. p = \sin. P . \sin. D + \sin.^2 P . \sin. D . \cos. D$$
$$+ \sin.^3 P . \cos.^2 D \sin. D + \sin.^4 P \cos.^3 D . \sin. D$$
$$+ \sin.^4 P \cos.^4 D \tan. p:$$

the law of the terms is evident; but, for almost every case that can occur, the summation of the three first terms will be sufficient, P not exceeding 1°.

If, instead of $\tan. p$, we wish to have an expression for p, we may easily obtain such by means of the expression

$$p = \tan. p - \tfrac{1}{3} \tan.^3 p,$$

which is the approximate expression for the arc in terms of the tangent, when the arc is small. The second term of this expression ($-\frac{1}{3}\tan.^3 p$) will produce from the equation

$$\tan. p = \sin. D.\sin. P + \sin. D.\cos. D.\sin.^2 P$$
$$+ \sin. D.\cos.^2 D.\sin.^3 P$$

a term such as $-\dfrac{\sin.^3 D.\sin.^3 P}{3}$; if, therefore, we do not continue the series beyond terms involving $\sin.^3 P$, we have

$$p = \sin. D.\sin. P + \tfrac{1}{2}\sin. 2D \sin.^2 P +$$
$$\sin.^3 P\left(\sin. D \cos.^2 D - \frac{\sin.^3 D}{3}\right);$$

but $\sin. D \cos.^2 D - \dfrac{\sin.^3 D}{3} = \sin. D - \sin.^3 D - \dfrac{\sin.^3 D}{3}$

$$= \sin. D - \frac{4 \sin.^3 D}{3};$$

(see *Trigonometry*, p. 47.) $= \dfrac{\sin. 3D}{3}$.

Hence,

$$p = \sin. D.\sin. P + \tfrac{1}{2}\sin. 2D.\sin.^2 P + \tfrac{1}{3}\sin. 3D.\sin.^3 P,$$

which, as it may be fairly conjectured, is only part of a series obeying the same law.

In an assigned instance, the resulting arithmetical value of p would be in terms of the radius. It is more convenient to have such value expressed in seconds of angular space. Now, p being small,

$$p : \text{arc} (= p) :: \sin. 1'' : 1'';$$

$$\therefore \text{arc} (= p) = \frac{p}{\sin. 1''};$$

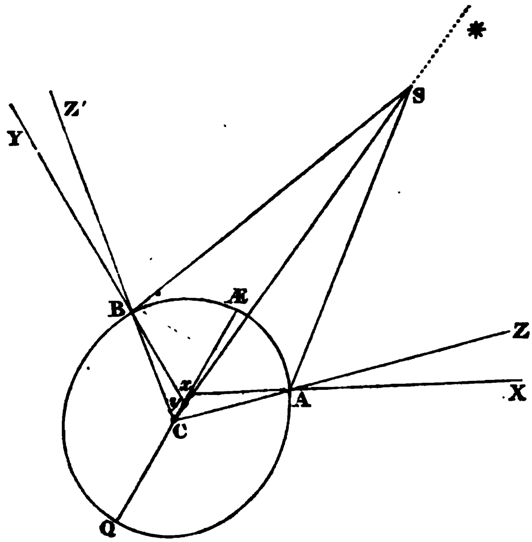
$\therefore$ expressed in seconds,

$$p = \frac{\sin. D}{\sin. 1''}.\sin. P + \frac{\sin. 2D}{2.\sin. 1''}.\sin.^2 P + \frac{\sin. 3D}{3.\sin. 1''}.\sin.^3 P.$$

There are, besides what we have given, other series and expressions for computing the parallax, from D the zenith distance, and P the *horizontal* parallax. D can always be determined (very nearly at least) by observation, but hitherto no method has been given of determining P. In short, although we know what parallax is, can symbolically express it, and can compute

the parallax at a given zenith distance from the horizontal, yet we are at present without a practical method of determining it. We will now turn our attention to that point.

Let A and B be two places on the Earth's surface situated in the same meridian; and suppose, by the methods described



in pp. 12, 129, their latitudes to be determined. When the planet S is on the meridian, let its zenith distances ZAS (z), $Z'BS$ (z'), be, respectively, observed at A and B; then, since ACB the difference or sum of the latitudes (in the diagram the sum) is known, we have

$$\angle ASB = 360^\circ - (180^0 - z + 180^\circ - z' + ACB)$$
$$= z + z' - ACB;$$

hence the angle ASB, (sometimes called the parallax, being the angle which a chord AB subtends at S,) is known: call this angle A, and the angles CSB, CSA, p', p, respectively.

A is not the angle (see the former Figure) which we are seeking: it is, either the angle CSB (p') or the angle CSA (p). Now

$$\sin. p' = \sin. z' . \frac{CB}{CS}, \text{ and } \sin. p = \sin. z . \frac{CA}{CS} = \sin. z \frac{CB}{CS}:$$

$$\frac{CB}{CS} = \frac{\sin p}{\sin z}$$

$$\text{hence, } \sin. p', \text{ or, } \sin. (A-p) = \sin. p \cdot \frac{\sin. z'}{\sin. z}, \text{ or,}$$

$$\sin. A \cdot \cos. p - \cos. A \cdot \sin. p = \sin. p \frac{\sin. z'}{\sin. z}:$$

whence, dividing by $\sin. A \cdot \sin. p$, and transposing,

$$\cot. p = \cot. A + \frac{\sin. z'}{\sin. z \cdot \sin. A}.$$

This formula may be thus adapted to logarithmic computation:

$$\cot. p = \cot. A \left(1 + \frac{\sin. z'}{\sin. z \cdot \cos. A}\right);$$

make $\dfrac{\sin. z'}{\sin. z \cdot \cos. A} = (\tan. \theta)^2$; $\therefore \cot. p = \cot. A \cdot (\sec. \theta)^2$;

and consequently,

$$\log. \cot. p = \log. \cot. A + 2 \log. \sec. \theta - 20;$$

θ being determined from

$$\log. \tan. = \tfrac{1}{2} (30 + \log. \sin. z' - \log. \sin. z - \log. \cos. A).$$

From this formula, p may be computed; but since, in point of fact, the parallax of all heavenly bodies that are observed is very small, a much simpler formula, and accurate enough for computation, may be exhibited:

Thus, A, p, p', being very small, are nearly equal their sines; instead of

$$\sin. (A-p) = \sin. p \cdot \frac{\sin. z'}{\sin. z}, \text{ we may write}$$

$$A - p = p \cdot \frac{\sin. z'}{\sin. z}; \text{ whence}$$

$$p = \frac{A \sin. z}{\sin. z + \sin. z'};$$

$$\text{or } = \frac{A \cdot \sin. z}{2 \cdot \sin. \frac{z+z'}{2} \cdot \cos. \frac{z-z'}{2}}.$$

If we wish to express the *horizontal* parallax, since

$$\sin. p = \sin. P . \sin. z, \text{ or } p = P . \sin. z,$$

$$P = \frac{A}{\sin. z + \sin. z'};$$

and, if we restore the value of A, making $\angle ACB = L \pm L'$

$$P = \frac{z + z' - (L \pm L')}{\sin. z + \sin. z'}.$$

As an example to this formula, we may take the observations of Lacaille, at the Cape of Good Hope, and of Wargentin, at Stockholm:

1751, Oct. 6.

At the Cape, zen. dist. (z) of ♂ 25° 2′ 0″...... sin. z = .4231
At Stockholm, zen. dist. (z') .. 68 41 6...... sin. z' = .9287

$z + z'$.... 93 16 6 sin. z + sin. z' = 1.3518,

lat. (L) of the Cape (south)..........33° 55′ 5″
lat (L') of Stockholm..............59 20 30

$L + L'$..93 15 35

$$\therefore z + z' - (L + L') = 31'';$$

$$\therefore P, \text{ the horizontal parallax, } = \frac{31''}{1.3518} = 22''.9.$$

This Example is, in appearance, solved somewhat differently by Lacaille. Instead of computing the latitudes, he immediately computes the angle A: thus, if a star λ ♒ were on the meridian with *Mars* (*S*), *Mars* would appear *below* λ ♒ to an observer at B, or Stockholm; below, in this case by 1′ 26″: it would also appear, to an observer A at the Cape, below λ ♒, and by 1′ 57″; the difference of 1′ 57″ and 1′ 26″ is 31″ the angle A.

λ ♒, whose declination in 1751 was about 8° 50′, in fact, was not on the meridian with *Mars;* therefore, Lacaille says, "*Mars* was below the *parallel* of λ ♒": now, the point at which this parallel crossed the meridian, he could easily ascertain by observing the declination of λ; it was simply the place of λ on the meridian.

The two places of observation are the Cape of Good Hope and Stockholm: and, the longitudes of these two places are, respectively, $18^\circ\ 23'\ 7''$ E., $18^\circ\ 3'\ 51''$E; consequently, they are not under the same meridian; therefore, a condition of the method (see p. 325,) is not preserved: and indeed it is not essentially necessary to preserve it. For, the difference of longitude $19'\ 16''$, in time, answers to $1^m\ 17^s$: accordingly, *Mars* would be on the meridian of the Cape $1^m\ 17^s$, before he had been on that of Stockholm. If, in that interval, his *declination* had not altered, no correction would be necessary: but, if in 24 hours his declination should have altered one minute, then the change of declination due to $1^m\ 17^s$ would be $\dfrac{60''}{24 \times 60 \times 60} \times 77$, or $\dfrac{77''}{24 \times 60}$, or $.0534''$; that is, if *Mars* had been on the meridian at the Cape when observed at Stockholm, the zenith distance instead of being $25^\circ\ 2'\ 0''$ would have been $25^\circ\ 2'\ 0'' \pm .0534''$. Hence it appears that it is of no use, in an example like the preceding, to notice the very small correction arising from a difference of longitudes: it also appears that the method itself is applicable, even if the difference of longitudes should be greater than in the example.

By the result of the computation (p. 327,) the parallax of *Mars* was found to be about twenty-three seconds. Of planets more distant than *Mars*, the parallax must, it is plain, be less. Hence, for such planets, the above method, although in theory very exact, can practically be of little use. It cannot be relied on: for, when the parallax does not exceed ten or twelve seconds, the probable errors of observation will bear so large a proportion to it, as materially to affect the certainty of the result. Hence, the method cannot be successfully applied to the Sun, whose parallax is less than nine seconds: neither to *Jupiter*, *Saturn*, nor the *Georgian* Planet.

The Moon, however, the parallaxes of which are considerable, the greatest being $61'\ 32''$, the least $53'\ 52''$, and the mean, (or rather the parallax at the mean distance,) $57'\ 11''.4$, is a proper instance for the method. Yet, with the Moon, the method

requires some modification. We must take into consideration, the spheroidical figure of the Earth.

Suppose the meridian $A\text{Æ}B$ not to be circular; then, the produced radii CA, CB, are not necessarily perpendicular to it, and consequently, Z, Z' are not the zeniths of the observers at A and B. But, if XAx, YBy, be perpendicular to the meridian, or vertical, or in the direction of a plumb-line, then X, Y are the true zeniths, and the angles SAX, SBY, are the observed zenith distances: now

$$\sin. ASC, \text{ or, } \sin. p = \frac{CA}{CS} \times \sin. CAS =$$

$$\frac{CA}{CS} \times \sin. (SAX - ZAX;)$$

$\therefore$ if z still represents the angle SAZ, it will equal the difference of the zenith distance and the angle contained between the radius and vertical. Hence,

$$\sin. p = \frac{CA}{CS} . \sin. z, \text{ similarly, } \sin. p' = \frac{CB}{CS} . \sin. z';$$

and, hence, if we take, instead of sin. p, sin. p', p and p',

$$p + p', \text{ or } A = \frac{CA \sin. z + CB . \sin. z'}{CS};$$

and since P, the horizontal parallax, $= \frac{\text{rad.} \oplus}{CS}$, (p. 327,)

$$P = \frac{\text{rad.} \oplus \times A}{CA . \sin. z + CB . \sin. z'}.$$

Let us take, as an example to this method, the observations of Lacaille and Wargentin, see *Mem. Acad. des Sciences.* Paris 1761:

1751, Nov. 5.

				Correct.
At the Cape, zen. dist. ☽ 's north limb	56°	39′	40″....	13′ 54″
parallel of ζ ♉ more north than ☽ .	1	46	32.8	
at Stockholm zen. dist. ☽ 's limb	38	4	52....	14 14
parallel of ζ ♉ more north than ☽	0	18	37.2.	

Hence, (see p. 327,) the difference of the quantities in the second and fourth line being $1^\circ\ 27'\ 55''.6$,

$$A = 1^\circ\ 27'\ 55''.6.$$

Now to find z, z', we must from the zenith distances subtract the corrections $13'\ 54''$, $14'\ 14''$, which are the angles between the vertical and the radius. Accordingly,

$$z = 56^\circ\ 25'\ 46'' \ldots\ldots\ldots \sin. z = .8332$$
$$z' = 37\ \ 50\ \ 38 \ldots\ldots\ldots \sin. z' = .6135$$
$$\sin. z + \sin. z' = 1.4467$$

Hence if we suppose CA, CB equal, we shall have (p. 329,) the horizontal parallax $= \dfrac{1^\circ\ 27'\ 55''}{1.4467} = 1^\circ\ 0'\ 46''$: the only difference, between this and the preceding method, consisting in the reduction of the zenith distances.

The reduction, or the value of the angle of the vertical, is taken from one of Lalande's Tables, computed for an *Ellipticity* $\dfrac{1}{230}$, and is, in fact, too large.

The expression or formula, from which the table just alluded to is computed, may be easily deduced. It is merely requisite to investigate the angle contained between the normal and radius vector, in an ellipse of small eccentricity.

In a sphere, the horizontal parallax $P = \dfrac{CA}{CS}$, and, consequently, the distance CS remaining the same, the horizontal parallax, whatever be the place of observation, would be the same. In a spheroid,

$$P = \frac{A . \times \text{rad.} \oplus}{CA \sin. z + CB \sin. z'},$$

consequently, the horizontal parallax, observed at different places, would be different. And with the Moon this is found to be the case: so that, (and there is something curious in the circumstance), this planet which, by her eclipses, shews, in a general

way, the Earth to be *round*, by her parallaxes, proves the Earth not to be *spherical* (see p. 38).

The preceding method, by which the parallaxes of *Mars* and the Moon have been determined, is not sufficiently accurate in practice, to determine the Sun's. That, however, is a most important Astronomical element, and requires to be exactly determined: which it has been by Dr. Maskelyne, and by means of the transit of Venus; a method of determination, not immediate and direct, but which infers the quantity required, on the supposition that the planetary motions are known to a very considerable degree of exactness *.

It is the distance of an heavenly body, as it is clear from pages 329, 330, that causes its parallax to be small: and the Sun's distance is so great, that its parallax, equal to $8''.75$, ($8''.81$, according to Laplace) cannot accurately be determined by the preceding method (p. 330.). The same method therefore, will not apply to bodies more distant from us than the Sun; neither to *Jupiter*, to *Saturn*, nor to the *Georgian* planet.

The smaller the parallax of a body, the greater is its distance: and, if we take, which we may do by reason of its smallness, the parallax instead of its sine, the mathematical relation between the parallax and distance (d), is

$$d = \frac{\text{rad.} \oplus}{P}.$$

This last expression is not, as it stands, fit for computation. It was deduced from sin. $P = \frac{\text{rad.} \oplus}{d}$, in which the radius is

* It is with this, as with many other parts in Astronomy, described in the following passage by the Abbè Lacaille: "Dans l'Astronomie on ne parvient à donner une certaine précision a quelque théorie qu'en revenant incessament sur ces pas et en remaniant tous les Calculs, a mesure que l'on decouvre quelque nouvel element, qui y devoit entrer, ou que l'on perfectionne quelqu'un de ceux qui se compliquent avec les autres." *Mem. de l'Acad.* 1757, p. 108.

supposed 1. But to a tabular radius r, (see *Trig.* p. 17,)
$\frac{\sin. P}{r} = \frac{\text{rad.} \oplus}{d}$; hence,

$$d = \frac{r}{\sin. P} \times \text{rad.} \oplus, \text{ or } = \frac{r}{P} \times \text{rad.} \oplus.$$

Now, since P is to be expressed in degrees, minutes, seconds, &c. we must express the radius r also, in degrees, minutes, &c.: and since, to a radius 1, the circumference $= 2\,(3.14159)$, we have

$$2(3.14159) : 360^\circ :: 1 : r = \frac{180^\circ}{3.14159} = 57^\circ.2957795.$$

Hence, the last of the two expressions for d becomes

$$d = \frac{57^\circ.2957795}{P} \times \text{rad.} \oplus:$$

and from this or the former, $d = \frac{r}{\sin. P} \times \text{rad.} \oplus$, may the distances of heavenly bodies be computed.

If we express the radius r, in degrees, minutes, &c. of French measure (*Trig.* p. 23), we shall have

$$d = \frac{63^\circ.6619}{P} \times \text{rad.} \oplus.$$

Hence, in the case of the Sun, if $P = 8''.81$, or, in French measure, $= 27''.2$,

$$d = \frac{57^0.2957795}{8''.81} \times \text{rad.} \oplus, \text{ or } = \frac{63^0.6619}{27''.2} \times \text{rad.} \oplus = 23405 \,\text{rad.} \oplus$$

In the case of *Mars*, $P = 24''.624$, or in French measure, $= 76''$,

$$d = \frac{57^\circ.2957795}{24''.624} \text{rad.} \oplus, \text{ or} = \frac{63^\circ.6619}{0^0.0076} \text{rad.} \oplus = 8376 \,.\, \text{rad.} \oplus$$

the distance of *Mars* from the Earth at the time of observation.

In the case of the Moon, $P = 57'\ 11''.4$, or, in French measure, $= 1^0.059$,

$$d = \frac{57^\circ.2957795}{57'\ 11''.4} \text{rad.} \oplus, \text{ or} = \frac{63^\circ.6619}{1^\circ.059} \text{rad.} \oplus = 60.1 \,.\, \text{rad.} \oplus.$$

Hence the mean distance of the Moon is about 60 radii of the Earth.

Here, the greatest and least distances are, respectively,

$$\frac{63^{\circ}.6619}{0.99567}\ \text{rad.}\ \oplus,\ \text{and}\ \frac{63^{\circ}.6619}{1.13714}\ \text{rad.}\ \oplus,\ \text{or}$$

$$63.94145 \times \text{rad.}\ \oplus,\ \text{and}\ 55.98725\ \text{rad.}\ \oplus^{*}.$$

The general use of parallax is, then, to determine the distances of heavenly bodies: but the special object for which it has been here introduced, is the reduction or correction, which must be made, by means of it, to the observed place of a body; to *prepare*, for instance, an observed altitude of the Moon, for the deducing its declination. Now since, by the principle of the reduction, we imagine a spectator to be in the centre of the Earth, it is plain, from the inspection of the Figure, p. 322, that the planet seen from the surface, must be lower, that is, nearer to the horizon than seen from the centre. But, this last is assumed to be the true place, or, it is made the place in Astronomical computations: and, accordingly, a body seen from the surface must be said to be below its true place, or to be *depressed by parallax*.

This depression takes place in a plane passing through, the centre of the Earth, the spectator, and the observed heavenly body; it takes place, therefore, like refraction, in the plane of a vertical circle. Now, the meridian is a vertical circle; the declination of an heavenly body then, as determined by its meridian altitude (see p. 151,) will be affected by the whole quantity of parallax; but its right ascension, as determined by the time of transit over the meridian, will not be at all affected.

We will now subjoin two instances, in the first of which the Sun's declination is deduced from his observed zenith distance: in the second the Moon's declination from her observed altitude: both observations are corrected for refraction and parallax: in

* It is plain from the above instances, that it is shorter to compute by the French than by the English expression: for, in the former, we may immediately divide the numerator (63°.6619) by the denominator; which we cannot do in the latter.

the former instance, then, the refraction is added to the zenith distance, the parallax subtracted: in the latter the refraction is taken away from the altitude, the parallax added.

Example I.

Altitude of Sun's upper limb......	62°	30′	30″.5	
error of collimation.............	0	0	34.5	
	62	29	56	
apparent zenith distance..........	27	30	4	sin. .4617
refraction.....................	0	0	29	
	27	30	33	
($8''\frac{3}{4}$ × .4617) *Parallax*.........	0	0	4	
	27	30	29	
semi-diameter of the Sun.........	0	15	46	
	27	46	15	
latitude of the place of observation ..	48	50	14	
declination of the Sun...........	21	3	59	

Example II.

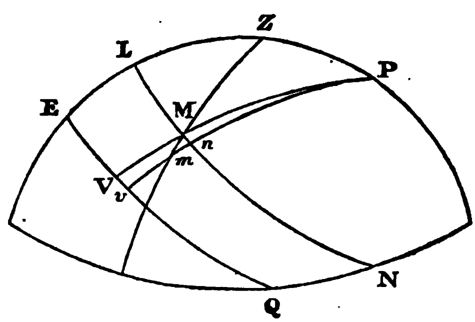
Altitude of Moon's upper limb.............	51°	11′	24″
refraction..........................	0	0	45
	51	10	39
(55′ 24″ × .6246) *Parallax*..............	0	34	36.2
	51	45	15.2
semi-diameter.........................	0	15	8.8
altitude of Moon's center.................	51	30	6.4
co-latitude of Greenwich.................	38	31	21.5
declination of the Moon.................	12	58	44.9

In this case, the horizontal parallax for Greenwich is taken =*55′ 24″*; and the multiplier .6246 is the natural sine of

38° 39′ 18″, which is the zenith distance 38° 49′ 21″ diminished by 10′ 3″, the value of the vertical angle (see p. 330.)

55′ 24″ represents the horizontal parallax for Greenwich, being the parallax on a spheroid at the latitude 51° 28′ 40″, deduced from, what is called, the *Equatoreal* parallax; which is the difference of the Moon's place in the Heavens seen from the equator and the Earth's centre: the Moon being in the horizon of the spectator. But this equatoreal parallax is deduced from the equatoreal parallax at the *mean* distance of the Moon*, which according to Mayer, is 57′ 11″.4. There is, therefore, the equatoreal parallax at the mean distance; the horizontal equatoreal at any distance; the horizontal for any latitude, and the common parallax for any altitude: and, in observations of the Moon and in calculations from them, all these circumstances must be attended to.

The quantity of parallax has been computed (see p. 327,) by means of observations made in the meridian. It may also be computed, as refraction was (p. 238,) by observations out of the plane of the meridian; for, in these latter, parallax, which causes a variation in the right ascension, may be computed from such variation. For instance, let *M* be a planet in its true place, *m*



in its apparent place, *Mm* lying in a vertical circle *ZMm* (see p. 238.). Now, *m* being the place instead of *M*, the time from the passage over the meridian, will be represented by the angle

* The Moon's greatest parallax is 61′ 32″; her least 53′ 52″.

ZPm, instead of the angle ZPM: the change, therefore, in the time, or in the apparent right ascension of the planet, caused by parallax, is represented by the angle VPv; and this change may be thus estimated: if M were a fixed star, Mm would be nothing, and there would be no parallax affecting the time, or the right ascension: two fixed stars then, near to each other, that crossed the vertical wire of a telescope in the plane of the meridian, after an interval of t seconds, would also cross the vertical wire of the telescope in a plane, not that of the meridian, after the same interval t. But if, instead of one of the fixed stars, we take a planet having parallax, then if the above-mentioned interval were t seconds on the meridian (where parallax does not affect the right ascension,) it could not be t seconds out of the meridian, but, as the figure shews, something more; for instance, $t+\epsilon$ seconds. Now ϵ is reckoned, or known, by means of a chronometer; and thence, a horizontal parallax (P) may be computed from this formula

$$P = \frac{15 \times \epsilon \times \text{cos. dec.}}{\text{cos. lat.} \times \text{sin. hour angle}},$$

which may be thus proved:

$$\begin{aligned} Vv &= Mn\,.\,\text{sec.}\ VM = Mm\,.\,\text{sin.}\ ZMP\,.\,\text{sec.}\ VM \\ &= P\,.\,\text{sin.}\ ZM\,.\,\text{sin.}\ ZMP\,.\,\text{sec.}\ VM \\ &= P\,.\,\text{sin.}\ ZP\,.\,\text{sin.}\ ZPM\,.\,\text{sec.}\ VM, \end{aligned}$$

(for sin. ZM . sin. ZMP = sin. ZP . sin. ZPM *Trig.* p. 155.)

Hence,

$$P = \frac{Vv}{\text{sin.}\ ZP\,.\,\text{sin.}\ ZPM\,.\,\text{sec.}\ VM},$$

or, since $360^\circ : 24^h :: Vv : \epsilon$; and since sin. ZP = cos. latitude,

sin. ZPM = sin. hour angle (h), sec. $VM = \frac{1}{\text{cos.}\ VM} = \frac{1}{\text{cos. dec.}}$

$$P = \frac{15\,.\,\epsilon\,.\,\text{cos. dec.}}{\text{cos. lat.} \times \text{sin. hour angle}}.$$

This expression applies to the case when the planet and star are observed, firstly, on the meridian, and afterwards when they have passed it: if they are observed before they are on the me-

ridian, then a similar expression would obtain for a line $V'v'$ analogous to Vv; and we should have

$$V'v' = \frac{P \text{ cos. lat. sin. } h'}{\text{cos. dec.}}.$$

Hence, if the difference ϵ belongs to two observations of the star and planet, the one made to the east, the other to the west of the meridian, we have

$$Vv + V'v', \text{ or } \epsilon \times 15 = \frac{P \text{ cos. lat. sin. } h}{\text{cos. dec.}} + \frac{P \text{ cos. lat. sin. } h'}{\text{cos. dec.}},$$

and accordingly,

$$P = \frac{\epsilon \times 15 \text{ cos. dec.}}{\text{cos. lat.} \times (\text{sin. } h + \text{sin. } h')},$$

$$= \frac{\epsilon \times 15 \times \text{cos. dec.}}{2 \text{ cos. lat.} \left(\text{sin. } \frac{h+h'}{2}\right) \text{ cos. } \left(\frac{h-h'}{2}\right)}, \text{ (Trig. p. 31.)}$$

In the preceding investigation it has been supposed, that ϵ arises solely from parallax: but since, during the observations, the planet will have moved either from, or towards the star, the noted difference of time, or excess above t seconds, will be compounded of the effect of parallax, and of the time due to the planet's motion, during the interval of the observations.

Example.

Aug. 15, 1719. Paris. By the observations of M. Maraldi at $9^h\ 18^m$, *Mars* passed the vertical wire $10^m\ 17^s$ after a small star in *Aquarius;* and, seven hours being elapsed, $10^m\ 1^s$ after.

But in this interval (seven hours) *Mars* had approached the star by fourteen seconds; that is, had there been no parallax, the former difference of passage, which was $10^m\ 17^s$, would have been reduced to $10^m\ 17^s - 14^s$, or, $10^m\ 3^s$: but, by the second observation, the difference of passage is only $10^m\ 1^s$, consequently, the effect of parallax is $(10^m\ 3^s) - (10^m\ 1^s)$, or 2^s: and this is the value to be substituted for ϵ in the preceding expression: and since, by observations at the time, it appeared that

$$
\begin{array}{rlr}
\text{Declination} = 15^\circ\ \ 0'\ \ 0'' & & \text{log. cos.} \ldots 9.9849438 \\
h = 56\ \ 39\ \ 0 & & (\text{log. } 15 \ldots 1.1760913) \\
h' = 49\ \ 15\ \ 0 & & \\
\dfrac{h+h'}{2} = 52\ \ 57\ \ 0 & & \text{log. sin} \ldots 9.9020628 \\
\dfrac{h-h'}{2} = \ \ 3\ \ 42\ \ 0 & & \text{log. cos} \ldots 9.9990938 \\
\text{latitude of Paris} = 48\ \ 50\ \ 12 & & \text{log. cos} \ldots 9.8183630
\end{array}
$$

We have, from the logarithmic formula of p. 337,

$$
\begin{aligned}
\text{log. } P = {} & \text{log. } 15 + 20 + \text{log. cos. } 15^\circ \\
& - (\text{log. cos. } 48^0\ 50'\ 12'' + \text{log. sin. } 52^0\ 57' + \text{log. cos. } 3^\circ\ 42') \\
& = 1.4415155\,;
\end{aligned}
$$

$\therefore$ P, the horizontal parallax of *Mars*, is $27''.638$ (See *Mem. de l'Acad.* 1722; and Lalande's *Astron.* tom. II. p. 356).

Some additions to the preceding investigations will be subsequently given in the Chapters on the 'Occultations of fixed Stars,' and '*the Transit of Venus.*'

CHAP. XIII.

ON PRECESSION.

Formula for computing the Precession in North Polar Distance and Right Ascension. Uses of the Formulæ in correcting Observations.

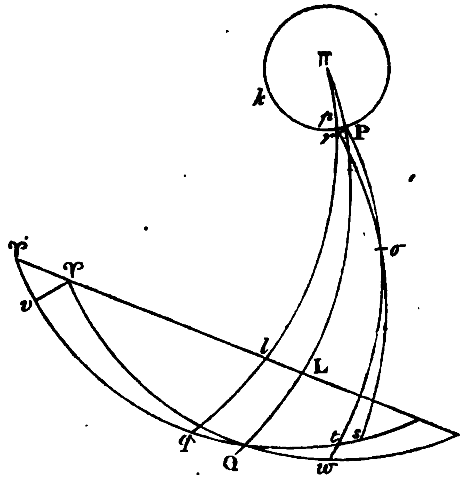
We have already shewn in pp. 189, &c. by a mere comparison of the catalogues of stars formed for different epochs, that the north polar distances and right ascensions of stars are continually varying; the latter, generally increasing with the time: the former, however, decreasing, if the stars be in either the first or fourth quadrant of right ascension, but increasing, if the stars be in the second or third.

It was also shewn in the pages just referred to, that the above phenomena, of the changes in the north polar distances and right ascensions, could be accounted for, by attributing to the pole of the equator a slow circular motion, and contrary to the order of the signs, round the pole of the ecliptic.

This circular motion of the pole produces a corresponding change in the equator. It makes to vary, the intersection of the equator and ecliptic, and produces, in that point of intersection, a retrograde motion denominated the *Precession of the Equinoxes*.

The philosophers, who observed the phenomena of precession and of the changes in the positions of stars, conjectured that such phenomena could be accounted for, by attributing a motion to the pole of the equator round that of the ecliptic. Physical Astronomy has shewn this conjecture to be true. We may, therefore, recur to the diagram of p. 192, in which the path of the pole of the Earth is represented by a small circle described round the pole of the ecliptic, and, by means of it, compute the formulæ of the changes of the north polar distance and right ascensions of stars.

If σ then be a star situated in the second quadrant of right ascension, the polar distance, by the translation of the pole from



P to p, is changed from $P\sigma$ to $p\sigma$; and the variation of north polar distance is

$$p\sigma - P\sigma = pr, \text{ nearly,}$$
$$\text{but } pr = Pp \,.\, \cos.\, Ppr$$
$$= Pp \,.\, \sin.\, qps$$
$$= \angle P\pi p \,.\, \sin.\, \pi P \,.\, \sin.\, qps;$$

now, the angle $P\pi p$, is the angle described in a given time (during the translation of P to p) by P round π: its measure is equal to the arc that represents the retrogradation of the point ♈, or, in other words, the *precession* ♈ ♈′. Suppose Pp to represent the motion of P during a year, then (see p. 187.) ♈ ♈′ $= 50''.1$: again, qps is equal to (according to the position of the star in the present diagram)

$$*\text{'s Æ} - 3^s:$$

and πP measures the obliquity (I) of the ecliptic, hence,

$$p\sigma - P\sigma = 50''.1 \sin.\, I \,.\, \sin.\, (*\text{'s Æ} - 3^s)$$
$$= -\, 50''.1 \,.\, \sin.\, I \,.\, \cos.\, *\text{'s Æ}.$$

According to the construction of the present diagram, the star is in the second quadrant of right ascension: consequently, cos. Æ is negative, and $p\sigma - P\sigma$ is positive, as it appears to be in the Figure. $p\sigma - P\sigma$, the variation in north polar distance, or the *precession in north polar distance* will also be positive in the third quadrant, but negative in the first and fourth quadrants. These results will immediately appear to be true results by constructing three diagrams like the preceding. But we may avoid this prolixity of investigation by employing one of the general equations of page 182, and by taking its fluxion or differential.

Thus, (Equation 2.) is

$$\cos.\,\delta = \sin.\,L.\cos.\,\lambda.\sin.\,I + \sin.\,\lambda\cos.\,I;$$

$$\therefore -d\delta.\sin.\,\delta = dL\cos.\,L.\cos.\,\lambda\sin.\,I\ (\lambda,\ I,\ \text{invariable})$$

$$(\text{by Eq}^{n}.\ 7.)\ = dL.\cos.\,Æ.\sin.\,\delta.\sin.\,I;$$

consequently,

$$d\delta = -\,dL.\cos.\,Æ.\sin.\,I,$$

which is a general expression for the *precession* ($d\delta$) in north polar distance, whatever be the star's place. Hence, when the right ascension is $< 90°$, or $> 270°$, $d\delta$ is negative; when $> 90°$ and < 270, positive. When Æ $= 90°$, or $= 270°$, that is, when the star is in the solstitial colure, cos. Æ$=0$, and there is no *precession* in north polar distance. When Æ $= 0$, or $= 180°$, cos. Æ $= \pm 1$, consequently, the *precession* in north polar distance of stars situated in the equinoctial colure is the greatest. γ Pegasi is nearly so situated, and its precession in north polar distance, nearly equals to

$$-50''.1\ \sin.\ 23°\ 27'\ 50'' = -19''.9:$$

all which conclusions agree with those of Chapter VIII.

By a like way we may arrive at the precession in right ascension. Thus, the equation (4) of p. 182, is

$$\sin.\,\lambda = \cos.\,\delta.\cos.\,I - \sin.\,\delta.\sin.\,I.\sin.\,Æ;$$

$$\therefore 0 = -\,d\delta\,(\sin.\,\delta\cos.\,I + \cos.\,\delta\sin.\,I\sin.\,Æ)$$

$$-\,dÆ.\sin.\,\delta\sin.\,I\cos.\,Æ$$

In this equation, instead of $d\delta$, write its value $dL.\sin. I.\cos. Æ$: after which substitution, each side of the equation is divisible by $\sin. I.\cos. Æ$; accordingly,

$$dÆ \sin. \delta = dL (\sin. \delta.\cos. I + \cos. \delta.\sin. I.\sin. Æ),$$

$$\text{and } dÆ = 50''.1 (\cos. I + \cot. \delta \sin. I.\sin. Æ),$$

which is the expression for the *precession* in right ascension during a year*.

The precession in north polar distance depends, as the expression for it shews, on the right ascension and not on the declination. The precession in right ascension depends both on the star's right ascension and its declination.

The first term in the preceding formula is

$$50''.1 . \cos. I,$$

which involves neither the right ascension nor the declination. It is independent, therefore, of the star's place, or, in other words (and as it is commonly stated) it is expressive of that part of the precession in right ascension which is *common* to all stars.

Let a star be situated in the equator, then $\delta = 90^{\circ}$, and $\cot. \delta = 0$, consequently,

$$dÆ = 50''.1 . \cos. I = 50''.1 \times .9173 = 45''.95.$$

The precession, therefore, of an equatoreal star in right ascension is expressed by that part of the precession which is said to be common to all the stars, and which is also the same as the retrogradation of the *first point of Aries* in right ascension.

---

* We may easily obtain the same expression from the diagram of p. 340. Thus, the right ascension $♈Qw$ becomes, by the effect of precession, $♈'qs$; and

$$♈'qs = ♈'v + vt + ts$$
$$= ♈'v + ♈Qw + ts.$$

The variation in right ascension, therefore, is

$$= ♈'v + ts,$$

$$♈'v = ♈♈'.\cos. I = 50''.1 . \cos. I,\ ts = Pr.\frac{\sin. s\sigma}{\sin. P\sigma} = Pr.\frac{\cos. P\sigma}{\sin. P\sigma}$$
$$= Pr \cot. P\sigma = Pp.\sin. Ppr.\cot. P\sigma =$$
$$♈♈'.\sin. \pi P.\sin. Ppr.\cot. P\sigma = 50''.1 .\sin. I \sin. ✱\text{'s } Æ \cot. \text{N. P. D.}$$
$$\therefore \text{precession in } Æ = 50''.1 (\cos. I + \sin. I.\sin. Æ.\cot. \text{N. P. D.})$$

We are enabled, by means of the preceding formulæ, to extend the uses of the catalogues of fixed stars. For instance, from a catalogue constructed for the epoch of 1800, we can take the mean right ascensions and mean polar distances of stars for the 1st of January 1800, and by adding to, or subtracting from these mean distances, the annual precessions in right ascension and north polar distance, we obtain the *mean* right ascensions and the *mean* north polar distances for the 1st of January 1801, and 1799, respectively. By adding and subtracting twice and thrice the annual precessions, we obtain, in like manner, and *nearly*, the *mean* right ascensions and *mean* north polar distance for the beginnings of the years 1802, 1803, 1798, 1797. But the greater the interval of years between the epoch of the catalogue and that for which we deduce, by this method, the right ascensions and north polar distances, the less exact are the results. The reason is plain from the inspection of the formulæ. Those formulæ involve the right ascension and north polar distance. If we compute the annual precession for 1800, we use the right ascension and north polar distance for 1800: but, if we compute the precession for 1804, we ought to use the right ascension and north polar distance for 1804, both which quantities are changed from what they were in 1800: for instance, if Æ be the star's right ascension in 1800,

$$\text{the precession in N. P. D.} = -50''.1 \,.\, \sin. I \,.\, \cos. Æ\,;$$

but in 1804, the right ascension will have become $Æ + \Delta Æ$: therefore if I the obliquity be supposed to be the same,

$$\text{the pre}^{\text{n}}\text{. in N. P. D. for } 1804 = -50''.1 \,.\, \sin. I \,.\, \cos. (Æ + \Delta Æ).$$

If the subject needed any farther illustration we might take the instance of γ Draconis. In 1760 the right ascension of this star was $267^\circ\ 45'\ 50''$, and its precession in north polar distance, thence computed, was equal to $0''.78$. In 1815, the star's right ascension had increased to $268^\circ\ 4'\ 40''.2$, and the precession, accordingly, decreased to $0''.7$. When the star, in consequence of the precession of the equinoxes, shall have reached the solstitial colure, it is clear that the precession in north polar distance will be nothing.

For the reasons that have been just stated, if we wish to make a catalogue of stars serviceable for 10, 20, &c. or more years, we must add to the registered mean right ascensions and north polar distances not ten times or twenty times, of &c. the computed annual precessions, (which are the *differentials* of the right ascensions and north polar distances), but the real increments of such right ascensions and north polar distances, or some quantities that approximate to the values of those increments. Such approximations we may derive from the preceding expressions. Thus, since

$$d\delta = -\ dL\,.\,\text{sin.}\ I\,.\,\text{cos.}\ Æ,$$

$$d^2\delta = \quad dL\,.\,dÆ\,.\,\text{sin.}\ I\,.\,\text{sin.}\ Æ$$

$$= dL^2(\text{sin.}\ I\,.\,\text{cos.}\ I\,.\,\text{sin.}\ Æ + \text{sin.}^2 I\ \text{sin.}^2 Æ\ \text{cot.}\ \delta),$$

$$\text{but,}\quad \Delta\delta = d\delta + \tfrac{1}{2}d^2\delta,\ \text{nearly};$$

$$\therefore\ \Delta\delta = -\ dL\,.\,\text{sin.}\ I\,.\,\text{cos.}\ Æ$$

$$+\tfrac{1}{2}dL^2\,.\,(\text{sin.}\ I\,.\,\text{cos.}\ I\,.\,\text{sin.}\ Æ + \text{sin.}^2 I\,.\,\text{sin.}^2 Æ\ \text{cot.}\ \delta).$$

If t be the time, $dL = 50''.1 \times t$,

$$\Delta\delta = -\ 50''.1 \times t\,.\,\text{sin.}\ I\,.\,\text{cos.}\ Æ$$

$$+\tfrac{1}{2}\,.\,(50''.1)^2.\,t^2.\,\text{sin.}\ I\,.\,\text{cos.}\ I\ \text{sin.}\ Æ\,(1+\text{tan.}\ I\,.\,\text{sin.}\ Æ\ \text{cot.}\ \delta).$$

If we apply this to the pole star for the year 1800, and take its place from pages 167, &c.

$$\Delta\delta = -\ 19''.5t + 0''.028878t^2.$$

In computing the above formula the obliquity I has been supposed to be constant: if, as it really is the case, I be supposed to vary we must add to the above value of $\Delta\delta$ the term

$$-\ 50''.1\,.\,dI\,.\,\text{cos.}\ I\ \text{cos.}\ Æ,$$

dI being equal to $0''.457$.

On like principles we may compute the value of $d^2 Æ$, and from it complete, or, rather, more accurately determine, the value of the precession in right ascension: thus, since

$$dÆ = dL\,.\,\text{cos.}\ I + dL\,.\,\text{sin.}\ I\,.\,\text{sin.}\ Æ\,.\,\text{cot.}\ \delta,$$

$$d^2Æ = dL\,.\,dÆ\,.\,\text{sin.}\ I\,.\,\text{cos.}\ Æ\,.\,\text{cot.}\ \delta$$

$$-\ dL\,.\,d\delta\,.\,\text{sin.}\ I\,.\,\text{sin.}\ Æ\,.\,\text{co-sec.}^2\ \delta,$$

neglecting the terms that would involve dI.

The differences of the precessions in north polar distance and right ascension thus determined will enable us, from the north polar distances and right ascensions *tabulated* for a certain epoch, to deduce, with considerable exactness, the real changes undergone by those quantities during the intervals of several years. But, if the intervals should be very large, it would be a more sure operation to compute from the right ascension, north polar distance and obliquity, the latitudes and longitudes of the stars. Now the precession (dL) is known very exactly; $\therefore L \pm dL$ is known, from which, by some of the formulæ of p. 182, we may compute

$$Æ \pm dÆ, \text{ and } \delta \pm d\delta.$$

The general expression for the precession in right ascension is

$$50''.1 . (\cos. I + \sin. I . \sin. Æ . \cot. \delta).$$

In the third and fourth quadrant of right ascension, that is, if the star's right ascension should be $> 12^h$, the sin. Æ becomes negative, and consequently, the second term of the above formula is negative. If it should exceed the first term, the precession in right ascension would be negative: and this happens with one (β Ursæ minoris) of the forty-five principal stars inserted in the Nautical Almanack. Its annual precession in right ascension is nearly $-0^s.267$ *.

This circumstance (that of a *negative* precession in right ascension) will not take place with any of the thirty-six principal stars formerly inserted in Dr. Maskelyne's Catalogue: for, amongst the last twenty stars, the right ascensions of which are

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* According to the subjoined computation,

log. cot. δ, or log. cot. 15°	6′	24″	....	10.5690
log. sin. Æ, or log. sin. 42	35	0	....	9.8303
log. sin. I, or log. sin. 23	27	50	....	9.6000
				29.9993; ∴ No. = .9985,

$\cos. I = .9172$; $\therefore$ precession $= 50''.1\ (.9172 - .9985)$
$= -\ 50''.1 \times .0813 = -4''.07$, nearly, and in time
$= -\ 0^s.267$, nearly.

greater than 12^h, the star that has the least north polar distance, is α Lyræ, the co-tangent of which distance ($51^\circ\ 24'$) is < 1: consequently, since sin. Æ cannot, in its greatest negative value, exceed -1, and since cos. $23^\circ\ 27'\ 50''$ is $>$ sin. $23^\circ\ 27'\ 50''$,

$$\cos. 23^\circ\ 27'\ 50'' + \sin. 23^\circ\ 27'\ 50'' . \sin. Æ . \cot. 51^\circ\ 24',$$

must be positive.

In Wollaston's Catalogue of circumpolar stars there are abundant instances of stars, the annual precessions of which in right ascension are negative.

Since the precessions in right ascension of some stars are positive, of others negative, there must be some stars so situated as not, during short periods, at least, to be affected in their right ascensions by the precession. The places of such stars must depend on the equation,

$$\cos. I + \sin. I . \sin. Æ \cot. \delta = 0,$$

which equation, in other terms, is

$$\sin. Æ + \tan. \delta \cot. I = 0.$$

But the equation (5) of p. 182, is

$$\cot. P = \cos. \delta \tan. Æ + \sin. \delta . \sec. Æ . \cot. I,$$

$$\text{or } \frac{\cot. P . \cos. Æ}{\cos. \delta} = \sin. Æ + \tan. \delta . \cot. I;$$

therefore cot. P must $= 0$, or P the angle of position must $= 90^\circ$.

The sixth equation of p. 182, is

$$\cot. P = \cos. \lambda \sec. L \cot. I - \sin. \lambda . \tan. L;$$

therefore if $P = 90^\circ$,

$$\cot. \lambda = \tan. I . \sin. L.$$

Hence, by assuming certain values of the longitude, we may determine the corresponding latitudes: for instance,

$$\begin{array}{rr} \text{let } L = 10^\circ, \text{ then log. sin. } 10^\circ \ldots\ldots = & 19.23967 \\ \text{log. tan. } 23^\circ\ 28' \ldots\ldots = & 9.63761 \\ \hline & 28.87728 \end{array}$$

$$\therefore \cot. \lambda = 8.87728, \text{ and } \lambda = 85^\circ\ 41'.$$

In like manner we may form other corresponding values, and arrange them thus

long^s.	0,	$10^\circ\ 0'$,	$20^\circ\ 0'$,	$30^\circ\ 0'$,	$50^\circ\ 0'$,	$70^\circ\ 0'$,	$90^\circ\ 0'$,
lat^s.	90,	85 41,	81 34,	79 44,	71 36,	67 47,	66 32.

We will now give one or two Examples of the formulæ of precession.

Example I.

Required the annual precession in Æ of γ Pegasi (Algenib). supposing its right ascension to be $0^h\ 2^m\ 56''.79$, and its north polar distance to $= 75^\circ\ 55'\ 44''$,

$50''.1 \,.\, \cos. I$, computed.

$$\begin{array}{ll}
-\log. r \ldots\ldots\ldots\ldots & -10 \\
\log. 50''.1 \ldots\ldots\ldots\ldots & = 1.69983 \\
\log. \cos. I \ldots\ldots\ldots\ldots & = \underline{9.96251} \\
 & 11.66234 = \log. 45.95.
\end{array}$$

$50''.1 \,.\, \sin. I \,.\, \sin.$ Æ $\cot. \delta$, computed.

$$\begin{array}{ll}
-\log. r^3 \ldots\ldots\ldots\ldots\ldots & -30 \\
\log. 50''.1 \ldots\ldots\ldots\ldots\ldots & 1.69983 \\
\log. \sin. 23^\circ\ 28' \ldots\ldots\ldots\ldots & 9.60012 \\
\log. \sin. 2^m\ 56^s \ldots\ldots\ldots\ldots & 8.10716 \\
\log. \cot. 75^\circ\ 55'\ 44'' \ldots\ldots\ldots & \underline{9.39906} \\
 & \bar{2}.80617 = \log. 0.640
\end{array}$$

Hence the annual precession in right ascension is equal to

$$45''.95 + 0''.0640 = 46''.014,$$

and, in time, $= 3^s.067$, nearly.

EXAMPLE II.

Required the annual precession in north polar distance of the same star.

$-$ log. r^2	-20
log. $50''.1$	1.6998
log. sin. $23^\circ\ 28'$.	9.6001
log. cos. $2^m\ 56^s$	9.9999
	$1.2998 = \log.\ 19''.94.$

EXAMPLE III.

The right ascension of α Serpentis being, in 1800, $= 15^h\ 34^m\ 25^s.2$, and its north polar distance $= 82^\circ\ 56'\ 9''.2$, it is required to find its precessions in right ascension and north polar distance.

$50''.1$. sin, I . sin. Æ . cot. δ, computed.

$-$ log. r^3	-30
log. $50''.1$	1.6998
log. sin. $15^h\ 34^m\ 25^s$.	9.9057
log. sin. $23^\circ\ 28'\ 0''$	9.6001
log. cot. 82 56 9.	9.0933
	$.2989 = \log.\ 1''.99.$

But, since Æ $> 12^h$, this part of the precession must be taken negatively, and written $-\ 1''.99$.

Hence, since the *common part* of the precession (see p. 342,) is $45''.98$, we have the

annual precession of α Serpentis in Æ $= 45''.98 - 1''.99 = 43''.99$,

$50''.1$. sin. I. cos. Æ, computed.

$-$ log. r^2 . . . . $=$	-20
log. $50''.1$ . . . . $=$	1.6998
log. sin. $23^\circ\ 28'$. . . . . $=$	9.6001
log. cos. $15^h\ 34^m$. . . . . . . $=$	9.7733
	$1.0732 = \log.\ 11''.83$

and since the Æ is > 12^h, the precession in north polar distance is negative and = − 11″.83.

By such easy computations, may the *annual* precessions be found, and as easily may the precessions for parts of a year be found. In fact, if t be the number of days elapsed since the beginning of the year, the precession must be equal to the annual precession multiplied into the fraction $\frac{t}{365}$; for instance,

Example IV.

Let it be required to find the precession in north polar distance of α Arietis on May 22, 1812.

$-\log. r^2 \ldots\ldots$	-20
$\log. 50''.1 \ldots\ldots$	1.6998
$\log. \sin\ 23^\circ\ 28'\ 0''$	9.6001
$\log. \cos.\ 29\ \ 8\ \ 56$	9.9412
	1.2411 = log. 17″.42.
Again, log. 142	2.1522 (142 days from Jan. 1, to May 22.)
	3.3933
log. 365	2.5622
	.8311 = log. 6″.778.

Hence, the annual precession is − 17″.42, and the precession up to May 22 = − 6″.78, nearly, on the supposition of an equable generation of precession.

The uses of the formulæ of precession are like those of aberration; they enable us to correct observations: to reduce north polar or zenith distances, observed at different times to the same time. For, since the pole of the Earth is, within certain and narrow limits, continually pointing to different parts of the Heavens, the distances of the pole from the stars must be continually changing. The distance, therefore, of the zenith of a place from any particular star is continually varying; for the distance of the zenith from the pole must remain the same, whilst the Earth preserves its axis of rotation. If, therefore, we had

to determine the difference of the latitudes of Greenwich and Blenheim, from two observed zenith distances of the same star, we should be unable to determine the difference, except, amongst other conditions, we knew that of the times at which the observations were respectively made.

For instance, if the star γ Draconis were, on the 1st of January 1800, $19' 23''$ south of the zenith of Blenheim, it would the next year be $19' 23''.7$ south: the succeeding year $19' 24''.4$ south. The difference, therefore, of the latitudes of Greenwich and Blenheim, determined by adding the *mean* zenith distance of γ Draconis at Blenheim on March 1, 1800, to the mean zenith distance of the same star at Greenwich on April 30, 1801, would be altogether an erroneous determination. In order to procure a right one, we must *reduce*, by other corrections, as well as by that of precession, the zenith distance of γ Draconis observed at Greenwich on April 30, 1801, to that which was its zenith distance on March 1, 1800; or the zenith distance of the same star observed at Blenheim on March 1, 1800, to what would be its distance on April 30, 1801: or, both the zenith distances must be reduced to those zenith distances which would be, or were, the true zenith distances at some common epoch; either, for instance, the 1st of January 1798, or the 1st of October 1803.

As far then as the preceding matter of the Treatise informs us concerning the influence of the inequalities, that make the apparent place of a star different from its mean place, we must, in order to use observations like the preceding, and for the purposes specified, know the states of the barometer and thermometer at each place of observation, that we may thence determine the respective quantities of refraction. We must also know the days of the months, in order to determine the *difference* of the effects of aberration, which inequality, with a given star, is independent of latitude and of every condition save that of the Sun's longitude: and, in the third place, we must know the year and the number of days elapsed from its beginning, in order to know how much, past a given epoch, the zenith of the place has altered with respect to the star, by reason of the pole's motion. In the ensuing Chapters, we shall see the necessity of correcting the star's place for other reasons than those already stated.

From the formulæ that have been given, Tables have been constructed. In the Greenwich Observations for 1812, two Tables of M. Zach's are inserted, by which the precession in north polar distance of any star for any day in the year may be found. The first, from the star's right ascension, which is the *argument*, gives the annual precession in north polar distance: the second Table gives a decimal number, corresponding to the day of the month (on which it is required to find the precession) by which the annual precession is to be multiplied. Thus, the number of seconds in the first Table belonging to the *argument* $29^0\ 8'$ (which is the right ascension of α Arietis) is $17''.47$. The decimal number corresponding to May 22, is .386: therefore, the precession of α Arietis from January 1, to May 22, is $17''.47 \times .386$ equal to $6''.7$ (see p. 349.)

In order to shew the usefulness of the formulæ of precession in right ascension, we will take, as an instance, the method of regulating Astronomical Clocks.

In order to know (see pp. 103, &c.) whether a clock be too fast or too slow, we observe the hour, minute, and second noted by it, when a known star is, or is computed to be, on the meridional wire of the Transit Telescope. If the clock were neither too fast nor too slow, it would, at that instant, denote the star's apparent right ascension. In order to ascertain this circumstance, we must *compute* the star's apparent right ascension. The first step in such computation, is to take the star's *mean* right ascension from a catalogue of stars constructed for a certain epoch; the next step is to add the increase of right ascension that has accrued, in the interval between the above epoch and the time of observation. This increase, in other words, is (leaving out of consideration any proper or peculiar motion which the star may have) the star's *precession in right ascension*. So that, if we would make the comparison of the clock's time with time computed from Astronomical elements, we must, in the second step of our computation, be able to assign the star's precession in right ascension.

For instance, suppose α Arietis to be on the meridional wire of the transit telescope on May 20th, 1822, when the sidereal

clock indicates 1^h 58^m 0^s, and that we are obliged to use a catalogue of stars computed for the epoch of Jan. 1, 1819: then, from such catalogue, we have,

firstly, mean right ascension, January 1, 1819	1^h	56^m	$59^s.36$
secondly, from the same catalogue, and by the formula of p. 344, the annual precession = $3^s.34$; therefore for three years, precession........	0	0	10.02
precession to May 20th, (= $3^s.34 \times .381$)......	0	0	1.27
mean right ascen. of *a* Arietis on May 20, 1822...	1	57	10.65

If we stopped at these corrections, the clock would appear to be too fast by $48^s.93$: but, as we have shewn, in the Chapter on Aberration, the star, by the effect of that inequality, will appear *sooner* on the meridional wire than it otherwise would do, and by $1^s.205$. From that cause, therefore, the apparent right ascension will be greater than the mean: in computing, therefore, the former from the latter, we must subtract $1^s.205$ from the latter; and, accordingly, since

Mean right ascension, on May 20, 1822, is =	1^h	57^m	$10^s.65$
Aberration......................... =	0	0	1.205
Apparent right ascension.................	1	57	9.445

We have not, at present, theory and formulæ, to continue farther the process of corrections, and to compute, to a greater degree of exactness, the star's apparent right ascension. If the last result were true and final, it would make the clock too fast by $50^s.555$.

But it will be soon our business, to explain the existence of two other inequalities, and to assign their quantities and laws. It will, then, appear that, in the instance before us, the apparent right ascension of *a* Arietis must be increased by *lunar nutation* (by $0^s.584$) and diminished by Solar nutation. What are the causes of these two inequalities, and the laws to which they are subjected, we will proceed to explain in the following Chapter.

CHAP. XIV.

ON SOLAR AND LUNAR NUTATION.

The Origin of the Nutations in the Inequable Generation of the Precession.—Formulæ of the Lunar Nutation in Right Ascension and North Polar Distance; made similar to the Formulæ of Aberration and Parallax. Formulæ of the Solar similar to those of the Lunar Nutation.—History of the Discovery of Nutation.

THE two inequalities that give the title to the present Chapter, are intimately connected with that of the preceding. For the purpose of pointing out that connexion, we must look to the physical causes of these inequalities; and, in the *inequable* action of the cause of precession, we shall be able to trace the origin of the Lunar and Solar Nutations.

The actions of the Sun and Moon on the excess of the Earth, which is an oblate spheroid, above the inscribed sphere, produce the retrogradation of the equinoctial points, or, as it is technically called, the *Precession of the Equinoxes.* The material circumstance in the production of this phenomenon, is that excess of matter, just spoken of. The other circumstances, scarcely less material, and, indeed, essential to the phenomenon, are the inclination of the Sun's orbit to the equator, and the inclination of the Moon's orbit to that of the Sun's, and, consequently, to the Earth's equator. If the Sun and Moon were constantly in the plane of the equator, there would, notwithstanding the Earth's spheroidical form, be no precession. When either luminary is in the equator, its action, in producing precession, is nothing. Twice a year, therefore, namely, at the two equinoxes, the Sun's force in causing precession is nothing, and twice a year, at the solstices, it is the greatest. It must, therefore, be of some mean value, in the intermediate times. The retrogradation, therefore, of the equinoctial points, inasmuch as it arises from the

Sun, cannot be equable, since the cause producing it is, on no two successive days of the year, exactly the same. There arises, therefore, an *inequality of precession*. In consequence of such inequality, the precession in right ascension of α Arietis (taking one of the instances of the last Chapter, see p. 352,) on May 20th, will not bear that proportion to the annual precession ($3^s.34$) which the number of days, between January 1, and May 20, bears to three hundred and sixty-five days; and, generally, the precession for fifty days, whether it be in right ascension or in north polar distance, will not be necessarily equal to $\frac{50}{365} \times p$, p representing the precession. The exact portion of the annual precession (in right ascension or in north polar distance) to which it is equal, or the correction necessary to be made to the *mean* portion, will depend on the season of the year to which the fifty days belong.

The precession, therefore, after being used as a correction, itself requires to be corrected. This, however, is easily effected by altering the number by which (see p. 349,) it is necessary to multiply the annual precession, in order to obtain its proportional part. Thus, of the star α Serpentis, the annual precession in right ascension of which is $2^s.935$, the *mean* proportional precession on April 30, would be $\frac{120}{365} \times 2^s.935 = .328 \times 2^s.935$, and .328 would be the multiplier: but this is too large, the actual precession generated from January 1st to April 30th, being less than the proportional part of the *mean*. It may be made duly less then, by merely lessening the multiplier .328: in the present instance, it would be reduced to .30, which number, and like numbers, in like instances, are furnished by proper Tables (see Wollaston's *Fasciculus*, Appendix, p. 42). This, however, it is to be noted, is not the sole method for correcting the precession.

The inequable retrogradation of the equinoctial points, or the inequality of precession, is not the sole effect produced by the unequal action of the Sun on the Earth's excess of matter above an inscribed sphere. The obliquity of the ecliptic, which, were the precession uniform, would not be affected by the cause pro-

ducing precession, is subject to a *semi-annual* equation: since, as in the inequality of precession, the force causing a change in the obliquity arrives, twice in a year, at its maximum.

These two effects, one of an inequality of precession, the other of an oscillation of the plane of the equator, constitute, what technically is called, the *Solar Nutation*.

There is also, as it may be conjectured from the arguments just alledged, a *Lunar Nutation*. The *precession of the equinoxes* is produced by the joint action of the Sun and Moon. As the Sun not being in the equator, causes that part of the precession, which is due to his action, to be inequably generated, so the Moon, continually altering her declination, is continually causing precession with an unequal force. But the period of the inequalities of its action, from their evanescent state to a state of maximum, is different from the period of the inequality of the Sun's action. It is no semi-annual period. The lunar period depends, however, on principles the same as those that regulate the solar. When the Moon's orbit, which is continually changing its position, returns, and not, at the end of an interval, to the same position which it had at the beginning, the interval so circumstanced is the period required. Now this is regulated by the motion of the Moon's nodes. The Moon's orbit is inclined to the ecliptic, and its nodes *retrograde* in the ecliptic. The period of this retrogradation is about eighteen years and seven months. At the beginning, suppose the Moon's node to have been in the node of the equator and ecliptic, then, at the end of eighteen years and seven months, the same node will have described 360° contrary to the order of the signs, and returned to the *first point of Aries*, and, during this retrogradation of the node, the lunar orbit will have occupied every position which it can occupy relative to the equator. The inequality of the Moon's action, then, in causing precession, will have passed through all its vicissitudes.

But, as in the former case, this is not the sole effect of the inequality of the Moon's force. The plane of the equator will be made to oscillate; so that, according to the longitude of the node of the Moon's orbit, it will be necessary to correct the mean obliquity on account of the lunar nutation.

We have seen in pp. 192, 193, that the phenomena of precession can be accounted for, by supposing the pole of the equator to describe uniformly a small circle round the pole of the ecliptic in a period of 25869 years. But these new phenomena of precession render some modification necessary in the preceding hypothesis. By reason of the solar nutation, the pole of the equator will oscillate, during half a year, about its mean place in the above-mentioned small circle, and the retrogradation of the pole will not be uniform. There will be a like oscillation and a like inequability of precession from the lunar nutation, but during a longer period. From both causes then, the north polar distances and the right ascensions of stars will be changed. Their precessions in north polar distance and right ascension computed according to the methods of pp. 344, &c. will not be the true precessions. In order to make the former the true precessions, we must correct them both for solar and for lunar nutation.

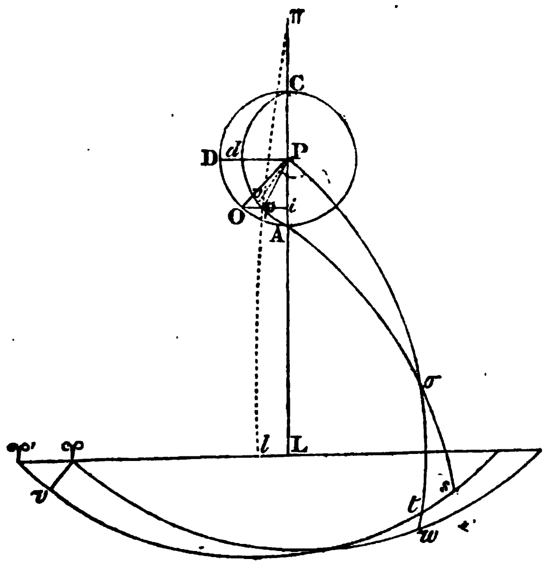
We have, in the preceding pages, described the causes of the lunar and solar nutations. But the lunar nutation, which is, by far, the most considerable, was not sought for and found out from a previous persuasion, or belief of the existence of its cause. Bradley, soon after the discovery of the *aberration of light*, noticed it as a phenomenon, and then assigned its cause, and the laws of its variation. But the solar nutation has never appeared to Astronomers as a *phenomenon*. It could scarcely be expected to be noticed as such, since its maximum is less than half a second. Its existence and quantity are derived from Physical Astronomy; and, on such authority, it is introduced as a *correction* of Astronomical observations.

We will now proceed to the deduction of the formulæ of the lunar nutation. Similar formulæ will express the laws of the solar nutation: the formulæ, considered as variable expressions, will differ only in their coefficients. One set of formulæ belong to the inequality of the Moon's action on the *Earth's excess* above an inscribed sphere; the other to the inequality of the Sun's action on the same excess.

In deriving these formulæ, we must begin with borrowing certain results established by Physical Astronomy. It has been

proved, in confirmation of Bradley's conjectures, that the phenomena of nutation are explicable on the hypothesis of the pole of the Earth, describing, round its mean place (that place which, see p. 337, it would hold in the small circle described round the pole of the ecliptic, were there no *inequality* of precession) an ellipse, in a period equal to the revolution of the Moon's nodes. The major axis of this ellipse is situated in the solstitial colure and equal to 19″.296; it bears that proportion to the minor axis (such are the results of theory) which the cosine of the obliquity bears to the cosine of twice the obliquity: consequently, the minor axis will be 14″.364 *.

Let CdA represent such an ellipse, P being the mean place of the pole, π the pole of the ecliptic. $CDAO$ is a circle



described with the centre P and radius CP. ♈ L is the ecliptic, ♈ w the equator, πPL the solstitial colure. In order to determine the true place of the pole, take the angle APO equal to

* These are M. Zach's numbers. Bradley's are 18″, 16″. Maskelyne's 19″.1, and Laplace's 19″.16. See *Mecanique Celeste*, Liv. V. p. 351.

the retrogradation of the Moon's ascending node from ♈ : draw Oi perpendicular to PA, and the point in the ellipse, through which Oi passes, is the true place of the pole. This construction being admitted, the *nutations* in right ascensions and north polar distance may, Pp being very small, be thus easily computed.

Nutation in North Polar Distance.

$$\begin{aligned} \text{The nutation in N. P. D.} &= P\sigma - p\sigma = Pp.\cos.\, pP\sigma, \text{ nearly,} \\ &= Pp.\cos.\,(APp + AP\sigma) \\ &= Pp.\cos.\,(APp + \mathrm{AR} - 90^0) \\ &= Pp.\sin.\,(APp + \mathrm{AR}). \end{aligned}$$

Nutation in Right Ascension.

The right ascension of a star is, by the effect of nutation, changed from $\text{♈}\, w$ into $\text{♈}'ts$. Now

$$\text{♈}'ts = \text{♈}'v + \text{♈}\, w + ts, \text{ nearly,}$$

$$\therefore \text{♈}\, w - \text{♈}'ts \doteq -\text{♈}'v - ts$$

$$= -\text{♈}\,\text{♈}'.\cos.\,\text{♈}\,\text{♈}'v - Pp.\sin.\, pP\sigma.\frac{\sin.\,\sigma s}{\sin.\, P\sigma},$$

in which expression $\text{♈}'v\ (= \Upsilon\Upsilon' \cos.\ \Upsilon\Upsilon' v)$ is, as in the case of precession, common to all stars.

In order to reduce farther the above expression, we have $pP\sigma = APp + AP\sigma =$ (in the present figure) $APp + \mathrm{AR} - 90^0$,

$$\text{and } \text{♈}\,\text{♈}' = Ll = Pp.\frac{\sin.\, APp}{\sin.\, P\pi};$$

$$\begin{aligned} \therefore -\Upsilon'v - ts &= -Pp.\sin.\, APp.\text{co-tan.}\, I \\ &\quad - Pp.\sin.\,(APp + \mathrm{AR} - 90^0).\cot.\ \text{N. P. D.} \\ &= -Pp.\sin.\, APp.\cot.\, I + Pp.\cos.\,(APp + \mathrm{AR}).\cot.\,\delta, \end{aligned}$$

δ representing the north polar distance.

But these forms are not convenient for computation. In order to render them convenient, we must, from the properties of the ellipse, deduce the values of Pp, and of the tangent of APp, and then substitute such values in the above expressions: thus,

$$\frac{Pp}{PO} = \frac{\text{sec. } APp}{\text{sec. } APO} = \frac{\text{cos. } APO}{\text{cos. } APp} = \frac{\text{cos. } (12^s - \Omega)}{\text{cos. } APp}$$

$= \dfrac{\text{cos. } \Omega}{\text{cos. } APp}$, Ω designating the longitude of the Moon's ascending node.

Again,

$$\frac{\text{tan. } APp}{\text{tan. } APO} = \frac{pi}{Oi} = \frac{Pd}{PD} = \frac{Pd}{PO};$$

$$\therefore \text{tan. } APp = \frac{Pd}{PO} . \text{tan. } APO = \frac{Pd}{PO} . \text{tan. } (12^s - \Omega)$$

$$= - \frac{Pd}{PO} . \text{tan. } \Omega .$$

Now substitute, and there will result

The Nutation in North Polar Distance

$$= \frac{PO . \text{cos. } \Omega}{\text{cos. } APp} (\text{sin. } APp . \text{cos. } Æ + \text{cos. } APp . \text{sin. } Æ)$$
$$= PO \ (\text{tan. } APp . \text{cos. } Æ \ \text{cos. } \Omega + \text{cos. } \Omega \ . \text{sin. } Æ)$$
$$= - Pd \ \text{cos. } Æ \ \text{sin. } \Omega + PO . \text{cos. } \Omega \ . \text{sin. } Æ$$
$$= -7''.182 \ \text{cos. } Æ \ \text{sin. } \Omega + 9''.648 . \text{cos. } \Omega \ \text{sin. } Æ,$$

which is the difference, as far as nutation is concerned, between the *mean* and *apparent* north polar distance. The *apparent* north polar distance, therefore, must be had by adding the preceding quantity, with its sign changed, to the mean.

Nutation in right ascension $= Pd . \text{sin. } \Omega \ \text{cot. } I$
$+ PO . \text{cos. } \Omega \ . \text{cos. } Æ \ \text{cot. } \delta + Pd . \text{sin. } \Omega \ \text{sin. } Æ \ \text{cot. } \delta$,
which, as far as nutation is concerned, is the difference of the mean and apparent right ascensions: and, consequently, the above expression must be subtracted from the mean, in order to obtain the apparent right ascension; or, which is the same, must be added after a negative sign has been prefixed; in which case, we have, substituting for PO, Pd their numerical values,

The Nutation in Right Ascension

$$= - 7''.182 . \text{sin. } \Omega \ \text{cot. } I$$
$$-9''.648 . \text{cos. } \Omega \ \text{cos. } Æ . \text{cot. } \delta - 7''.182 . \text{sin. } \Omega \ \text{sin. } Æ \ \text{cot. } \delta.$$

Of the expressions for the nutations in north polar distance and right ascension, each admits of a maximum value: in order to find that value of ☊ which gives the nutation in north polar distance a maximum, we have

$$0 = 7''.182 \,.\, \cos.\ \text{Æ} \cos.\ ☊ + 9''.648 \sin.\ ☊ \sin.\ \text{Æ};$$

$$\therefore\ \tan.\ ☊ = -\frac{7''.182}{9.648} . \cot.\ \text{Æ} = -\frac{b}{a} \cot.\ \text{Æ},$$

which is the value of tan. ☊, when the nutation is a maximum.

Let X be the corresponding value of ☊, M the maximum, then (see p. 359, l. 16, &c.)

$$\begin{aligned} M &= b \,.\, \cos.\ \text{Æ} \sin.\ X - a \,.\, \sin.\ \text{Æ} \cos.\ X \\ &= -a \,.\, \sin.\ \text{Æ} \left(-\frac{b}{a} \text{ co-tan. } \text{Æ} \sin.\ X + \cos.\ X\right) \\ &= -a \sin.\ \text{Æ} (\sin.\ X \,.\, \tan.\ X + \cos.\ X) \\ &= -a \,.\, \frac{\sin.\ \text{Æ}}{\cos.\ X}. \end{aligned}$$

We may now express the nutation at any time (what takes place with any given longitude of the Moon's node) in terms of the maximum, and of the corresponding value of the longitude of the node: thus,

The Nutation in North Polar Distance

$$\begin{aligned} &= b \cos.\ \text{Æ} \,.\, \sin.\ ☊ - a \,.\, \sin.\ \text{Æ} \cos.\ ☊ \\ &= -a \,.\, \sin.\ \text{Æ} \left(-\frac{b}{a} \cot.\ \text{Æ} \sin.\ ☊ + \cos.\ ☊\right) \\ &= M \,.\, \cos.\ X (\tan.\ X \sin.\ ☊ + \cos.\ ☊) \\ &= M (\sin.\ X \sin.\ ☊ + \cos.\ X \cos.\ ☊) \\ &= M \,.\, \cos.\ (☊ - X) \\ &= M \,.\, \cos.\ (☊ + 15^s - X - 15^s) \\ &= M \,.\, \sin.\ (☊ + 15^s - X), \end{aligned}$$

or (should M, when its arithmetical value is deduced from the expression of l. 13, be negative)

$$= M \,.\, \sin.\ (☊ + 21^s - X)^*.$$

---

* Since X cannot exceed 12^s, we are sure, by using 15^s and 21^s, of having $15^s - X$, and $21^s - X$, expressed by a positive arc. If the resulting arc exceeds 12^s we may cast out 12^s: for $\sin.\ (12^s + A) = \sin.\ A$.

Hence, as in the case of aberration (see pp. 274, &c.) we can always find the nutation by adding, to the longitude of the Moon's ascending node, an arc equal to $15^s - X$, or $= 21^s - X$, the value of which arc will depend on the star's right ascension.

In the same way we may reduce the expression for the nutation in right ascension. Thus, the nutation in right ascension,

$$= -b \,.\, \sin.\, \Omega \,\cot.\, I$$
$$- a \,.\cos.\, \Omega \,.\cos.\, \text{Æ} \cot.\, \delta - b \,.\sin.\, \Omega \,\sin.\, \text{Æ} \cot.\, \delta.$$

In order then to obtain that value (Y) of Ω which shall make the nutation a maximum, we have

$$b \cos.\, \Omega \,.(\cot.\, I + \sin.\, \text{Æ} \cot.\, \delta) = a \,.\sin.\, \Omega \,.\cos.\, \text{Æ} \cot.\, \delta\,;$$

therefore, writing Y instead of Ω,

$$\tan.\, Y = \frac{b}{a} \,.\, \frac{\cot.\, I + \sin.\, \text{Æ} \,.\cot.\, \delta}{\cos.\, \text{Æ} \,.\cot.\, \delta}.$$

Hence the maximum (m) of nutation in right ascension,

$$= -b \,.\sin.\, Y .(\cot.\, I + \sin.\, \text{Æ} \cot.\, \delta) - a \,.\cos.\, Y \cos.\, \text{Æ} \cot.\, \delta$$
$$= -\frac{a \,.\cos.\, \text{Æ} \cot.\, \delta}{\cos.\, Y}.$$

By means of this expression we may, as in the case of the nutation in north polar distance, (see p. 360.) express the nutation in right ascension in terms of m, and of the corresponding value of Ω: thus, the nutation in right ascension, (n)

$$= -b \,.\sin.\, \Omega \,(\cot.\, I + \sin.\, \text{Æ} \cot.\, \delta) - a \,.\cos.\, \Omega \,\cos.\, \text{Æ} \cot.\, \delta$$
$$= -a \,.\cos.\, \text{Æ} \cot.\, \delta .\tan.\, Y \sin.\, \Omega - a \,.\cos.\, \text{Æ} \cot.\, \delta \cos.\, \Omega$$
$$= -a \,.\cos.\, \text{Æ} \cot.\, \delta \left(\frac{\sin.\, Y \sin.\, \Omega + \cos.\, Y \cos.\, \Omega}{\cos.\, Y}\right)$$
$$= m \,.\cos.\, (\Omega - Y)$$
$$= m \sin.\, (\Omega + 15^s - Y),$$

or (should the value of m, when arithmetically deduced, be negative)

$$= m \,.\sin.\, (\Omega + 1^s - Y).$$

Hence, as in the former case, (see p. 360.) and in the case

of aberrations, the nutation may be expressed by the sine of a positive arc.

We may, then, thus symbolically express the formulæ of the Nutation.

Formulæ of the Nutation in North Polar Distance.

$$\tan. X = -\frac{7''.182}{9.648} \cot. Æ = -0''.744 \cot. Æ \ldots\ldots (1)$$

in logarithms, log. tan. $X = 9.87182 + \log. \cot. Æ - 10$;

next,

$$M = -9.648 . \frac{\sin. Æ}{\cos. X} = -9''.648 . \sin. Æ \sec. X \ldots\ldots (2)$$

in logarithms, log. $M = .984437 + \log. \sin. Æ + \log. \sec. X - 20$,

and thirdly,

$$\text{(nutation) } N = M \sin. (\Omega + 15^s - X) \ldots\ldots\ldots\ldots (3)$$

in logarithms, log. $N = \log. M + \log. \sin. (\Omega + 15^s - X) - 10$.

Formulæ of the Nutation in Right Ascension.

$$\tan. Y = 0''.744 \left(\frac{\cot. I + \sin. Æ . \cot. \delta}{\cos. Æ \cot. \delta}\right) \ldots\ldots\ldots (4)$$

$$m = -\frac{9''.648 . \cos. Æ . \cot. \delta}{\cos. Y} \ldots\ldots\ldots\ldots (5)$$

in logarithms,

log. $m = .984437 + \log. \cos. Æ + \log. \cot. \delta - \log. \cos. Y - 10$,

and thirdly, $n = m . \sin. (\Omega + 15^s - Y) \ldots\ldots\ldots\ldots (6)$

in logarithms, log. $n = \log. m + \log. \sin. (\Omega + 15^s - Y) - 10$.

It now remains to assign, in particular instances, the peculiar values of the arcs $15^s - X$, $15^s - Y$, or $21^s - X$, $21^s - Y$, which are to be added to Ω.

EXAMPLE I. γ *Pegasi*, (1800.)

X computed.	*Y* computed.
$-10 \ldots\ldots -10$	$-40 = -40$
$Æ = 44' \; 11''.85 \ldots \cot. = 11.890852$	$\log. .744 = \;\; 9.871820$
(see p. 362.).......... 9.871820	$\cot. I \ldots = 10.362458$
$\log. \tan. (89^0 \; 0' \; 38'') \ldots \; 11.762672$	$\sec. Æ \ldots = 10.000036$
but $\tan. X$ is negative; $\therefore X = 90^0 \; 59'$, nearly	$\tan. \delta \ldots = 10.601938$
and $+ 15^s - X = 11^s \; 29^0 \; 1'$	$(= \log. 6.858) \;\; .836252$

Again,

$$-20 = -20$$

$$\begin{array}{lr} \log. .744 \ldots\ldots\ldots\ldots\ldots\ldots = & 9.871808 \\ \tan. Æ \ldots\ldots\ldots\ldots\ldots\ldots = & 8.109147 \\ \hline (\log. .0095) \ldots\ldots\ldots\ldots\ldots\ldots\ldots & \bar{3}.981955 \end{array}$$

But,

$$6.858 + .0095 = 6.868,$$

$$\text{and } 6.868 = \tan. 81^\circ \; 43', \text{ nearly},$$

$$\therefore Y = 2^s \; 21^0 \; 43'.$$

M computed.	*m* computed.
see p. 362.	
$-20 \ldots -20$	$-30 \ldots -30$
$\log. 9.648 \ldots\ldots \;\; .984437$	$\ldots\ldots\ldots\ldots \;\; .984431$
$\log. \sin. Æ \ldots \;\; 8.109111$	$\cos. Æ \ldots 9.999963$
$\log. \sec. X \ldots \;\; 11.765443$	$\cot. \delta \ldots 9.398061$
$(\log. M) \ldots\ldots \;\; .858985$	$\sec. Y \ldots 10.819448$
	$(\log. m) \ldots 1.201903$

Hence,

$$N = M . \sin. (☊ + 11^s \; 29^0 \; 1'),$$

$$\text{and } \log. N = .8589 + \log. \sin. (☊ + 11^s \; 29^0 \; 1').$$

Again,

$$\begin{aligned} n &= -m . \sin. (☊ + 15^s - 2^s \; 21^\circ \; 43') \\ &= \;\; m . \sin. (☊ + 21 - 2 \; 21 \; 43) \\ &= \;\; m . \sin. (☊ + 18 \quad 0 \; 8 \; 17) \\ &= \;\; m . \sin. (☊ + 6 \quad 0 \; 8 \; 17), \end{aligned}$$

$$\text{and } \log. m = 1.202 + \log. \sin. (☊ + 6^s \; 8^0 \; 17')$$

EXAMPLE II. *a Arietis* (1815.)

X computed.

$$-10 = -10$$
$$Æ = 29^0\ 11'\ 28'' \ldots \text{cot.} = 10.252838$$
$$9.871820$$
$$(\text{tan. } 53^\circ\ 6'\ 44'') \ldots\ldots\ 10.124658$$

but tan. X is negative; $\therefore X = 126^\circ\ 53'\ 16''$

$$= 4^s\ 6^\circ\ 53'\ 16''$$

Y computed.

$$-40 = -40$$
$$\text{log. } .744 = 9.871820$$
$$\text{cot. } I \ldots = 10.362458$$
$$\text{sec. } Æ \ldots = 10.058987$$
$$\text{tan. } \delta \ldots = 10.381991$$
$$(\text{log. } 4.7342)\ \ .675256$$

$$\therefore\ 15^s - X = 10^s\ 23^0\ 7', \text{ nearly.}$$

Again,

$$-10 = -10$$
$$\text{log. } .744 \ldots\ldots\ldots = 9.871820$$
$$\text{tan. } Æ \ldots\ldots\ldots = 9.747161$$
$$(\text{log. } .41588) \ldots\ldots\ldots\ 9.618981$$

Now, $4.7342 + .4158 = 5.15008$,

and $5.15008 = \text{tan. } 79^\circ\ 0'\ 41''$;

$$\therefore\ Y = 2^s\ 19^0\ 1', \text{ nearly,}$$

M computed.

see p. 362.

$$-20 = -20$$
$$.984437$$
$$\text{sin. } Æ = 9.688174$$
$$\text{sec. } X = 10.221662$$
$$(\text{log. } M) \ldots\ .894273$$

m computed.

$$-30 = -30$$
$$.984437$$
$$\text{cos. } Æ\ \ 9.941012$$
$$\text{cot. } \delta\ \ 9.618008$$
$$\text{sec. } Y\ \ 10.719845$$
$$(\text{log. } m)\ \ 1.263302$$

Hence,

$$N = M.\text{sin.}(\Omega + 10^s\ 23^0\ 7'),$$

and $\text{log. } N = .8943 + \text{log. sin.}(\Omega + 10^s\ 23^0\ 7')$,

Again,

$$n = -m.\text{sin.}(\Omega + 15^s - 2^s\ 19^\circ\ 1')$$
$$= m.\text{sin.}(\Omega + 21^s - 2^s\ 19^\circ\ 1')$$
$$= m.\text{sin.}(\Omega + 18^s\ 10^\circ\ 59')$$
$$= m.\text{sin.}(\Omega + 6^s\ 10^\circ\ 59'),$$

and $\text{log. } n = 1.2633 + \text{log. sin.}(\Omega + 6^s\ 10^\circ\ 59')$.

EXAMPLE III. *Polaris*, (1800).

X computed.	Y computed.
$-10 = -10$	$-40 = -40$
$Æ = 13^0\ 5'\ 15''$ .. cot. $= 10.633762$	log. .744 $= 9.871820$
9.871820	cot. I. . $= 10.362458$
tan. $(72^0\ 39'\ 35'')$ 10.505582	sec. $Æ$ .. $= 10.011422$
but tan. X is negative; $\therefore X = 107^0\ 20'\ 25''$	tan. δ .. $= 8.486050$
$= 3^s\ 17^0\ 20'\ 25''$	(log. .05392) $\bar{2}.731750$

$$\therefore\ 15^s - X = 11^s\ 12^0\ 39'\ 35''.$$

Again,

$$-10 = -10$$

log. 744 .... $=$	9.871820
tan. $Æ$ .... $=$	9.366237
(log. .1730)	9.238057

Now $\qquad .1730 + .05392 = .2269,$

and $.2269 = \tan.\ 12^0\ 47'$;

$\therefore\ Y = 12^0\ 47'$.

M computed.		m computed.	
$-20 = -20$		$-30 = -30$	
	.98443		.98443
sin. $Æ$ $=$	9.35481	cos. $Æ$ . .	9.98857
sec. X $=$	10.52588	cot. δ . .	11.51261
(log. M)	$.86512$	sec. $Y =$	10.01091
		(log. m)	2.49652

Hence,

$$N = M.\sin.(\Omega + 11^s\ 12^0\ 39'\ 35''),$$

and log. $N = .8651 + \log.\sin.(\Omega + 11^s\ 12^0\ 39'\ 35'')$.

Again,

$$\begin{aligned} n &= -\ m.\sin.(\Omega + 15^s - 12^0\ 47') \\ &= \quad m.\sin.(\Omega + 21 - 12\ \ 47) \\ &= \quad m.\sin.(\Omega + 20^s\ 17^0\ 13') \\ &= \quad m.\sin.(\Omega + 8\ \ 17\ \ 13), \end{aligned}$$

and log. $n = 2.4965 + \log.\sin.(\Omega + 8^s\ 17^0\ 13')$.

EXAMPLE IV. *a Aquarii*, (1800), see pp. 281, &c.

X computed.

$$-10 = -10$$

$$\text{Æ} = 328^\circ\ 52'\ 26'' \ldots \cot. = 10.21906$$

$$9.87182$$

$$\tan. (50^\circ\ 57'\ 6'') \ldots\ldots\ldots 10.09088$$

now, since cot. Æ is negative

tan. *X* (see p. 362.) is positive

Y computed.

$$-40 = -40$$

$$\log.\ .744 = \ 9.87182$$

$$\cot.\ I \ldots = 10.36246$$

$$\sec.\ \text{Æ} \ldots = 10.06751$$

$$\tan.\ \delta \ldots = 11.65989$$

$$(\log.\ 91.556)\ \ 1.96168$$

$$\therefore\ X = 1^s\ 20^\circ\ 57', \text{ nearly.}$$

Again,

$$-20 = -20$$

$$\log.\ .744 \ldots\ldots\ldots\ldots\ldots\ldots\ 9.87182$$

$$\tan.\ \text{Æ} \ldots\ldots\ldots\ldots\ldots\ldots\ 9.78037$$

$$(= \log.\ .4489) \ldots\ldots\ldots\ldots\ \bar{1}.65219$$

Now
$$91.556 + .4489 = 92.005,$$
and $92.005 = \tan.\ 89^\circ\ 22'$;
but tan. *Y* is negative;
$$\therefore\ Y = 6^s - (2^s\ 29^\circ\ 22')$$
$$= 3\ \ 0\ \ 38.$$

M computed.

$$-20 = -20$$

$$.98443$$

$$\sin.\ \text{Æ} \ldots\ 9.71342$$

$$\sec.\ X \ldots\ 10.20067$$

$$(\log.\ M) \ldots\ .89852$$

m computed.

$$-30 = -30$$

$$.98443$$

$$\cos.\ \text{Æ}\ \ 9.93248$$

$$\cot.\ \delta\ \ 8.34010$$

$$\sec.\ Y\ \ 11.95665$$

$$(\log.\ m)\ \ 1.21366$$

Hence,

$$N = M.\sin.(\Omega + 15^s - 1^s\ 20^\circ\ 57'),$$
$$= M.\sin.(\Omega + 1^s\ 9^\circ\ 3')$$

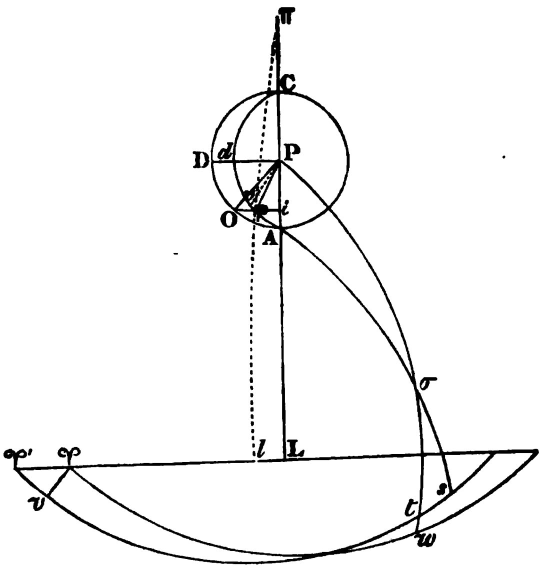
and $\log.\ N = .8985 + \log.\sin.(\Omega + 1^s\ 9^\circ\ 3')$

Again,

$$n = m.\sin.(\Omega + 21^s - 3\ \ 0\ \ 38)$$
$$= m.\sin.(\Omega + 17^s\ 29^\circ\ 22')$$
$$= m.\sin.(\Omega + \ 5\ \ 29\ \ 22),$$

and $\log.\ n = 1.2136 + \log.\sin.(\Omega + 5^s\ 29^\circ\ 22')$.

The *Solar Nutation* arises from like causes as the Lunar, and admits of similar formulæ. As an ellipse, made the locus of the true place of the pole, served to exhibit the effects of the lunar nutation, so an ellipse, of different, and much smaller dimensions may be made to represent the path which the true pole of the equator would, by reason of the Sun's inequality of force in causing precession, describe about the mean place of the pole. Thus, in the present Figure, the ellipse AdC will serve to re-



present the locus of the pole, when $AP = .435$, $Pd = .399$, and APO, instead of being $= ☊$, is equal to $2 ⊙$, or twice the Sun's longitude, according to the order of the signs; the equations, therefore, for the solar nutation in north polar distance, and right ascension, analogous to those of p. 362, will be

The Solar Nutation in North Polar Distance

$$= - .399 \cos. Æ . \sin. 2 ⊙ + .435 \sin. Æ . \cos. 2 ⊙ ,$$

The Solar Nutation in Right Ascension

$$= - .399 \sin. 2 ⊙ \cot. I$$
$$- .435 \cos. 2 ⊙ \cos. Æ . \cot. \delta - .399 . \sin. 2 ⊙ \sin. Æ \cot. \delta,$$

which equations admit of transformations precisely similar to

those which the equations of the lunar nutation (see pp. 360, &c.) were made to undergo. Hence, we have these formulæ similar to those of p. 362. for north polar distance,

$$\tan. X' = -\frac{.399}{.435}\cot. Æ, \text{ and}$$

$$\log. \tan. X' = 9.96248 + \log. \cot. Æ - 10,$$

$$M' = -.435 . \sin. Æ . \sec. X'$$

$$\text{and } \log. M' = .63849 + \log. \sin. Æ + \log. \sec. X' - 20,$$

$$N' = M' . \sin. (2\odot + 15^s - X'),$$

$$\text{or (see p. 361.)} = M' \sin. (2\odot + 21^s - X'),$$

$$\log. N' = \log. M' + \log. \sin. (2\odot + 15^s - X') - 10,$$

for right ascension,

$$\tan. Y' = -\frac{.399}{.435}(\cot. I \sec. Æ . \tan. \delta + \tan. Æ);$$

$$\therefore \tan. Y' = \tan. Y \times \frac{917}{744},$$

$$m' = -.435 \cos. Æ \cot. \delta \sec. Y',$$

$$\text{and } \log. m' = 9.63849 + \log. \cos. Æ + \log. \cot. \delta + \log. \sec. Y' - 30,$$

$$n' = m' . \sin. (2\odot + 15^s - Y'),$$

$$\log. n' = \log. m' + \log. \sin. (2\odot + 15^s - Y') - 10.$$

Hence it appears, that we may very easily deduce the *solar nutations*, if we have computed the lunar, since in the processes of computation, there are several parts nearly the same. Thus, if we take the last instance, that of α Aquarii, (see p. 366.) we have

	$-10 = -10$		
tan. Æ.	10.21906	log. tan. $Y =$	11.96381
	9.96248		.09166
(log. tan. X'). . . .	10.18154	log. tan. $Y' =$	12.05547

$$\therefore X' = 56^\circ\ 38'\ 28'',$$

$$15^s - X' = 13^s\ 3^\circ\ 22';$$

but $12.05547 = \log. \tan. 89^\circ\ 28'$, nearly,

and since tan Y' is negative,

$$Y' = 3^s . 0^\circ\ 32',$$
$$15^s - Y' = 11\ \ 29\ \ 28.$$

M′ computed.		*m′* computed.	
$-20 = -20$		$-30 = -30$	
log. ratio of axes. .	.63849	. . . .	.63849
sin. Æ	9.71342	cos. Æ.	9.93248
sec. X'.	10.25973	cot. δ	8.34010
(log. M')	.61164	sec. Y'.	12.03113
		(log. m').	.94220

Hence,

$$N' = M' \sin. (2\odot + 13^s\ 3^\circ\ 22')$$
$$= M' \sin. (2\odot + 1\ \ 3\ \ 22),$$
$$\text{and log. } N' = .6116 + \text{log. sin.} (2\odot + 1^s\ 3^\circ\ 22') - 10,$$
$$n' = - m' . \sin. (2\odot + 15^s - 3^s\ 0^\circ\ 32')$$
$$= m' \sin. (2\odot + 5^s\ 29^\circ\ 28'),$$
$$\text{and log. } n' = .942 + \text{log. sin.} (2\odot + 5^s\ 29^\circ\ 28') - 10\text{*}.$$

---

* The expressions for the solar nutation are thus made similar to the expressions for the lunar: but they require a separate investigation. There is not the same ratio between the axes of the ellipse that belongs to the solar nutation, as between the axes belonging to the ellipse of the lunar nutation. M. Zach, however, (see p. 120. *Tabulæ Speciales Aberrationis et Nutationis*) and M. Delambre commenting on him (see *Connoissance des Tems* of 1810. p. 463.) derive the solar from the lunar corrections by merely multiplying the latter by a constant quantity: which is no just operation.

The lunar and solar nutations are now then expressed by formulæ similar to those by which the aberrations of stars (see pp. 283, &c.) have been expressed; and, we might form Tables like that of which a specimen has been given in p. 283. Thus,

	Lunar Nutation.		Solar Nutation.	
	North Polar Dist.	Right Ascen.	North Polar Dist.	Right Ascen.
Number. γ Pegasi. log. max^{m}.	$11^{s}\ 29^{\circ}\ 1'$ .8589	$6^{s}\ 8^{\circ}\ 17'$ 1.202	$11^{s}\ 29^{0}\ 12'$ 9.6026	$6^{s}\ 6^{0}\ 43'$ 9.968
a Arietis.	$10^{s}\ 23^{\circ}\ 7'$ .8943	$6^{s}\ 10^{0}\ 59'$ 1.2633	$10^{s}\ 28^{\circ}\ 39'$ 9.61050	$6^{s}\ 8^{\circ}\ 55'$ .00733
Polaris.	$11^{s}\ 12^{0}\ 40'$ .8651	$8^{s}\ 17^{\circ}\ 13'$ 2.4965	$11^{s}\ 15^{\circ}\ 13'$ 9.603	$8^{s}\ 14^{\circ}\ 21'$ 9.1561
a Aquarii.	$1^{s}\ 9^{0}\ 3'$ .8985	$5^{s}\ 29^{\circ}\ 22'$ 1.2136	$1^{s}\ 3^{0}\ 22'$ 9.6116	$5^{s}\ 29^{0}\ 28'$ 9.942

By means of a Table like the preceding, and of a Table like the one of p. 283, we may easily compute the corrections which it is necessary to apply to the *mean* north polar distance and *mean* right ascension of a star, in order to deduce the *apparent* north polar distance and *apparent* right ascension. There will be indeed another correction, besides those just mentioned, to be taken account of, namely, the correction for precession. Suppose, for instance, it were required to express the *apparent* north polar distance of a Aquarii for some time in the year 1800: its mean north polar distance (see p. 281.) is $91^{\circ}\ 16'\ 58''$: its mean right ascension $328^{0}\ 52'\ 26''$; $\therefore$ its *precession*, proportional to a time t elapsed from the beginning of the year, is (see p. 341.)

$$-50''.1 \times t \sin.(23^{\circ}\ 27'\ 50'') \times \cos.(328^{\circ}\ 52'\ 26'') = \ \ 17''.07t.$$

Hence,

the apparent north polar distance $= 91^{\circ}\ 16'\ 58'' - 17''.07\ t$

$+7''.856 \cdot \sin. (\odot + 3^{s}\ 2^{\circ}\ 49'\ 52'')$......(aberration, p. 281.)

$+7''.915 \cdot \sin. (\Omega + 1\ \ 9\ \ 3'\ \ 0)$....(lunar nutation, p. 370.)

$+0''.408 \cdot \sin. (2\odot + 1\ \ 3\ \ 22\ \ 0)$....(solar nutation, p. 370.)

and, in a specific instance, when the values of the longitudes of the Sun and of the Moon's ascending node would be assigned, the resulting value of the apparent north polar distance would agree with the observed and instrumental distance cleared of the effects of refraction.

On the footing of mere theory, we ought to add to the preceding terms (see ll. 2, &c.) that express the several corrections, a similar term (see p. 313, &c.) on account of parallax. But its coefficient is, at present, either insensible or unknown. We do not, therefore, correct for parallax: but we must correct on account of the *star's proper motion:* the quantity of which correction, resting on no theory, is determined solely by observation.

We have assigned the formula for determining the apparent north polar distance of α Aquarii for some time (t) in the year 1800. But, as it has been explained, the same formula (excepting its first and fourth term) will serve to express the north polar distance of α Aquarii for any time in any other year; provided such year be not too remote from the epoch for which the numbers and maxima (see p. 366.) have been computed. Thus, the *numbers* and *maxima* have been computed for α Arietis, supposing the epoch to be 1815: but, the same *numbers* and *maxima* will serve to compute, with no practical error, the aberrations and nutations of that star for any time during 1822. The like will happen with other stars; for instance, suppose it were necessary to express and compute the apparent right ascension of α Arietis on May 20th, 1822: then we have, from the Catalogue of 1819.

Star's mean right ascension, Jan. 1, 1819......	1^{h}	56^{m}	$59^{s}.36$
three years precession ($= 3 \times 3^{s}.34$)..........	0	0	10.02
proportional precession to May 22 ($= 3.34 \times .3801$)	0	0	1.27

and from the Nautical Almanack and Tables,

	$\odot =$ 1^s	28°	50′	39″
	☊ = 10	20	20	0
also see p. 283, No. for aberration	= 7	28	39	0
p. 370, for lunar nutation	= 6	10	59	0
p. 370, for solar.	= 6	8	55	0

consequently, see pp. 283, &c.

the argument for aberration is........	9^s	27°	29′.5, nearly,
..........for lunar nutation.......	17	1	19
..........for solar nutation.......	10	6	36

whilst, by the same pages, the maxima (expressed in time) are, respectively,

$$\tfrac{1}{15}(20^s.564),\quad \tfrac{1}{15}(18^s.335),\quad \tfrac{1}{15}(1^s.017).$$

Hence on May 20th, 1822, the apparent right ascension of *a* Arietis is

$1^h\ 57^m\ 10^s.65$........................ =	$1^h\ 57^m\ 10^s.65$
$+\frac{1}{15}(20.564)$ sin. (9^s 27° 29′.5).......... =	− 1.21
$+\frac{1}{15}(18.335)$ sin. (17 1 19)............ =	+ 0.58
$+\frac{1}{15}(\ 1.017)$ sin. (10 6 36)............ =	− 0.05
	1 57 9.97

The apparent right ascension, therefore, of *a* Arietis will be very nearly $1^h\ 57^m\ 10^s$.

Of the four corrections that have been applied, in the preceding instance (and of like corrections with other stars) three are dependent on the time elapsed from the beginning of the year; namely, the proportional part of the precession, the aberration, and the solar nutation. If these corrections, therefore, be computed for every day of a certain year and their results taken, such results will serve for every day on succeeding years, and, without material error, will be right results during a century. Such is, nearly, the practice of Astronomers. They compute to every tenth day of the year, and insert in Tables, the results of precession, and aberration. Thus, in the preceding instance, we have

the precession, in right ascension . . . . =	$1^s.27$
the inequality of preces.^{n} in Æ, or *solar nutat.* in Æ =	-0.05
the aberration in right ascension . . . . =	-1.21
	$+0.01$

and this result is called, (see Table X, in the first Volume of the Greenwich Observations) the *Correction of the Star's Right Ascension in time.*

The fourth correction, that of the lunar nutation, depends on the longitude of the Moon's ascending node, and consequently will not, like the other corrections, be the same on the corresponding days of successive years. It is computed from a separate Table, of which the *argument* is the longitude of the Moon's node.

The deduction of formulæ for *correcting*, on account of the lunar and solar nutations, the apparent north polar distances and right ascensions of stars, ought to be considered as the essential business of the present Chapter; which, therefore, might here be closed. But, before this is done, we wish to make a short digression towards certain collateral objects: some of little moment, or merely curious: others distinguished solely by their celebrity in Astronomical history.

In aberration, we pointed out its origin and cause, and then, with such means as we were using, deduced its formulæ. Nothing was then borrowed from a foreign or an unexplained theory. But it has been otherwise in the subject of nutation. Some general idea, indeed, was given of its cause, but no formulæ deduced from such explanation. The means of deducing them were borrowed from Physical Astronomy and taken on trust. And, in order to obtain the most *convenient* means of computing such formulæ, we supposed (which indeed is one of the results of Physical Astronomy on this subject) the locus of the pole to be an ellipse. But, it is to be observed, this is only one way of viewing the subject: it is neither the essential nor the only way. All the computations might have been conducted, and their results arrived at, without an ellipse to represent either the solar or lunar nutation. The inequality of the lunar force in causing precession

produces an *equation* of precession, and an *equation* of the obliquity. The inequality of the solar force does the same thing.

Let

the lunar equation of precession, or $d\Upsilon = -18''.03584 \sin. \Omega$
the equation of obliquity, or $dI \ldots\ldots = - 9''.6 . \cos. \Omega$,

and from these two equations, by the means either of spherical triangles, (as Cagnoli has done pp. 439, &c. of his *Trigonometry*) or by taking the differentials of some of the equations of page 182, (as Suanberg has done, pp. 108, &c. of his *Exposition des Operations*, &c.) the corresponding *variations* of the right ascension and north polar distance, or, technically, the *nutations* in right ascension and north polar distance, may be deduced.

In like manner, if we represent the inequality or equation of precession arising from the Sun by

$$d\Upsilon' = -1''.002 . \sin. 2\odot,$$

and the equation of obliquity, by $dI = 0''.435 . \cos. 2\odot$,

we may deduce, by the method just described, the corresponding variations in the right ascensions and north polar distances of stars, which, technically, will be the *solar* nutations in right ascensions and north polar distance, or which, as it is sometimes said, arise from the *solar inequality of precession*.

Instead of this, which is, perhaps, the most direct method, we have followed Bradley's. This latter is usually adopted in Astronomical Treatises, and, certainly, possesses the merit of being clear and intelligible. But it is apt to cause the student to form erroneous conceptions: to make him view as a fact, or phenomenon, what is merely a mathematical fiction. If we could trace out in the Heavens the path of the pole of the equator it would not be an ellipse. It *would* be such a curve were there no inequality of the Sun's force, and were not the mean place of the pole itself in motion along a circular arc. But this latter motion takes place, and, besides, the path of the true pole, by reason of the solar nutation, would, were other causes abstracted, itself be elliptical.

The path, therefore, described by the true pole, by virtue of three motions, or in consequence of precession, and (for such

they are) its two *inequalities*, is some epicycloidal curve *, of no very easy description.

Instead of deducing the nutations in north polar distance and right ascension from the *nutation of the obliquity*, and *the nutation of longitude*, (see pp. 374, 357.) we have deduced them from the assumption of the locus of the pole being an ellipse. From formulæ so deduced we may derive, as consequences, the *nutations* of *obliquity* and *longitude*.

Thus,

N (in N. P. D.) $= 9''.64 . \sin. Æ . \cos. ☊ - 7''.18 \cos. Æ . \sin. ☊$.

Now the change produced, by nutation, in the north polar distance of a star situated on the solstitial colure, will be equal to the change of obliquity from the same cause. Let the right ascension, therefore, $= 90^0$, in which case $\sin. Æ = 1$, $\cos. Æ = 0$,

and the nutation of obliquity $= 9''.64 . \cos. ☊$.

Hence, the nutation of the obliquity is the greatest when ☊ either $= 0$, or $= 180^0$, that is, either when the Moon's node is in ♈, or in ♎. Again, (see p. 359.)

$$n \text{ (in } Æ) = -9''.648 . \cos. Æ \cos. ☊ . \cot. \delta$$
$$- 7''.182 . \sin. Æ . \sin. ☊ . \cot. \delta - 16''.544 . \sin. ☊ .$$

Let $\delta = 90^0$, then

$$n \text{ (in } Æ) = - 16''.544 . \sin. ☊ ,$$

* In point of practical accuracy, nothing would be gained by investigating such a curve, and thereby deducing the changes in the north polar distances and right ascensions of stars. The present method of allowing for those changes consists in adding together three terms: one due to the precession, a second due to the lunar inequality of precession, a third due to the solar inequality: each term, if we speak of theoretical exactness, inaccurate: but so slightly inaccurate that their sum will differ, by no difference of moment, from the single term or formula which, computed on exacter principles, shall express either the change in north polar distance, or in right ascension.

which is the lunar inequality of precession in right ascension, hence, the nutation in longitude $= n \times$ secant obliquity

$$= 16''.544.\sin.\ \Omega \times \sec.\ 23^\circ\ 28'$$
$$= 18''.034.\sin.\ \Omega,$$

which nutation is technically called the *Equation of the Equinoxes in Longitude*, (see Maskelyne's Tables, Tab. XLII.)

Similar equations for the changes in the obliquity and precession may be deduced from the formulæ of solar nutation. Thus, since

the solar nutation (in N. P. D.) =

$$.435\ \sin.\ Æ\ \cos.\ 2\odot\ -\ .399\ \cos.\ Æ.\sin.\ 2\odot,$$

we have, taking $Æ = 90^\circ$,

$$\text{the solar nutation of obliquity} = .435\ \cos.\ 2\odot,$$

which is sometimes called, the *Solar Equation of obliquity.*

Again, (see p. 367.)

$$\text{the solar nutation (in } Æ) = -\ .435\ \cos.\ Æ\ \cos.\ 2\odot\ \cot.\ \delta$$
$$-\ .399\ \sin.\ Æ\ \sin.\ 2\odot\ \cot.\ \delta - 0''.918.\sin.\ 2\odot.$$

Hence, making $\delta = 90^\circ$,

$$\text{the solar nutation of the equinoxes in } Æ = -0''.918\ \sin.\ 2\odot$$
$$\text{and in longitude} = -.918.\sin.\ 2\odot\ \sec.\ \text{obliquity}$$
$$= -\ 1''.\sin.\ 2\odot, \text{ nearly.}$$

The former equation, the solar equation of obliquity, is, in Maskelyne's Tables, combined with the *secular* diminution of the obliquity caused by the action of the planets; the effect of which action is a change not, as in the case of nutation, of the equator but of the ecliptic. Thus, if the secular diminution of the obliquity be $45''.7$: the annual diminution will be $0''.457$, and the diminution for half a year, or about June 22, will be $0''.229$: if we represent the Sun's longitude at that time by 3^s, we shall have the whole diminution from the beginning of the year,

$$= -0''.229 + .435\ \cos.\ 6^s$$
$$= -0''.229 - .435 = -\ .714.$$

Again, on March 21, the proportional part of the *annual* diminution is nearly $-.189$, and since $\odot = 0^s$, the whole diminution is

$$-.189 + .435 \,.\, \cos.\, 2\odot = .246,$$

on January 10, it will be

$$-.457 \times \frac{10}{365} + .435 \cos.\, 2 \times (9^s\ 20^\circ) = -.34, \text{ nearly,}$$

(see Table XXXI, and its explanation in the 1st Volume of Maskelyne's Observations.)

The *apparent* obliquity of the ecliptic is inserted in the first pages of the Nautical Almanack: it is equal to the mean obliquity at a given epoch + the proportional quantity of the *secular* diminution + the solar nutation of obliquity + the lunar nutation. Thus, to find the *apparent* obliquity on Jan. 1, 1820.

The mean obliquity in 1813..................	$= 23^\circ\ 27'\ 50''$
proportional part of secular diminution $(= 7 \times .457) =$	-3.2
$\odot = 9^s\ 10^\circ\ 3'\ 48''$, sol. nutat. $= .435 \times \cos.\ (18^s\ 20^\circ\ 7') =$	-0.4
$\Omega = 6^s\ 21'$, lunar nutation $= 9''.64 \,.\, \cos.\ (6^s\ 22') =$	$+9.58$
hence, the *apparent* obliquity in Jan. 1........	$= 23\ \ 27\ \ 55.98$

and in the same way we must compute the apparent obliquity on April 1, July 1, October 1, and December 31.

There are several occasions on which it is necessary to know the *apparent*, or the *true* obliquity of the ecliptic; for instance, when (as in pp. 151, &c.) the Sun's right ascension is computed from his observed zenith distance and the obliquity; and, generally, in all cases, where it is necessary, at any assigned time, to compute the corresponding position of an heavenly body.

But there are other occasions when the *mean* value of the obliquity is employed: for instance, in the catalogues of the longitudes and latitudes of stars; which longitudes and latitudes (see pp. 160, &c.) are computed from that mean value which the obliquity had at the catalogue's epoch.

Some stars are more affected in their positions, by nutation, than others. In order to determine the places of those stars that are either the most or the least affected, we have

$$(N) \text{ nutation in N.P.D.} = 9''.648 \sin. Æ . \cos. \Omega$$
$$- 7''.182 \cos. Æ . \sin. \Omega .$$

If $Æ = 90^\circ$, or 270°, or, if the star be situated in the solstitial colure,

$$N = \pm 9''.648 . \cos. \Omega .$$

If $Æ = 0$, or $= 180^\circ$, or, if the star be situated in the equinoctial colure,

$$N = \mp 7''.182 . \sin. \Omega .$$

Again,

$$n(\text{in } Æ) = - 9''.648 \cos. Æ . \cos. \Omega \cot. \delta$$
$$- 7''.182 \sin. Æ \sin. \Omega . \cot. \delta - 16''.544 . \sin. \Omega .$$

If the star be in the solstitial colure,

$$n = (\mp 7''.182 \cot. \delta - 16''.544) \sin. \Omega .$$

If the star be in the equinoctial colure,

$$n = \mp 9''.648 \cos. \Omega . \cot. \delta - 16''.544 . \sin. \Omega .$$

The formulæ that have been in pp. 362, 369. deduced are sufficient, in all cases, for the computations of the quantities of the solar and lunar nutation. They have been propounded also as the most *convenient* formulæ of computation. There are, indeed, other formulæ of computation, some of which (although this is, in a slight degree, to neglect the main and essential business of the Treatise) we will now consider.

By *Trigonometry*, p. 21,

$$\sin. Æ \cos. \Omega = \tfrac{1}{2} \sin. (Æ + \Omega) + \tfrac{1}{2} \sin. (Æ - \Omega)$$
and $\cos. Æ \sin. \Omega = \tfrac{1}{2} \sin. (Æ + \Omega) - \tfrac{1}{2} . \sin. (Æ - \Omega)$.

Hence, the above formula for the nutation in north polar distance, becomes

$$N = 1''.233 . \sin. (Æ + \Omega) + 8''.415 \sin. (Æ - \Omega),$$

in this expression, substitute, instead of Æ, $180^0 + Æ$, and the resulting expression will be one for the nutation of a star having a right ascension opposite to the former, that is,

$$N' = -1''.233 . \sin. (Æ + ☊) - 8''.415 \sin. (Æ - ☊),$$

an expression equal in quantity to the former, but in a different direction (see *Phil. Trans.* No. 435, pp. 12, 13, also p. 294. of this Treatise).

By a similar transformation, the expression for the nutation in right ascension becomes

$$n = -8''.415 . \cot. \delta . \cos. (Æ - ☊) - 1''.233 . \cot. \delta . \cos. (Æ + ☊) - 16''.544 \sin. ☊,$$

and under these two latter forms, which are Lambert's, the nutations in north polar distance and right ascension are usually expressed (see Delambre, Chap. xxx. tom. 3. *Connoissance des Tems.* 1810. p. 463. Cagnoli's *Trigonometry*, p. 440. Vince's *Astronomy*, Vol. II. p. 132.)

With regard to the Astronomical uses of the theory of nutation and of its formulæ, the same may be said, both for explanation and illustration, as has been already said on the subjects of precession and aberration. The aberration will be nearly the same, on the same days of different years: so will be the solar nutation. The lunar nutation will, almost certainly, be different. If, therefore, to take our old instance, we would determine, the difference of the latitudes of the Observatories of Greenwich and Blenheim, from two recorded instrumental observations of γ Draconis, we must know, besides the zenith distances determined by the instruments, the states of the thermometer and barometer at the instants of the two observations in order thence to determine the refractions; secondly, the interval of years between the two observations in order to determine the difference of the precessions in north polar distance; thirdly, the days of the year in order to determine the respective proportional changes of precession; the *inequalities* of precession (or the solar nutations in north polar distance) and the aberrations; and, lastly, we must know the particular years and days, in order to know the respective positions of the Moon's ascending node and thence to determine the lunar nutations.

But it is not on the occasions of determining the latitudes, which are rare occasions, that the knowledge of the nutation, and of the other corrections is chiefly necessary. Such knowledge is required in the daily business of an Observatory. The first operation is to observe the star and to register the observation: the second is to *clear* the observation of its inequalities, or to *reduce* it. For instance, in order to determine the error of the line of collimation of the brass quadrant of Greenwich, there were observed with it forty-six zenith distances of γ Draconis, at different times of the year, from Feb. 21, 1811, to Dec. 29, 1811. Each of these distances was an *apparent* distance, and each different, the one from the other. *Reduced* to the same epoch, which was the beginning of the year, each would express the *mean* zenith distance, and (were the observations exactly made, and the theories by which they are reduced, correct) by the same quantity. This is not the case, if it were, one observation would do as well as forty-six. But the accuracy which cannot be hoped for from one observation is attained by taking the mean of many. In order to effect this, each zenith distance must, as it has been said, be *reduced*, or corrected for *aberration*, *solar nutation*, *lunar nutation*, and *precession*. The following is a specimen of registering the observations and their reductions:

Observed Zenith Distances of γ Draconis reduced to the beginning of the Year.

	Observed Z. D.	Aberration.	Solar Nutation.	Lunar Nutation.	Precession.	Sum of Equations.	Mean Z. D. Jan. 1, 1811.
Feb. 21.	2′ 7″	+17″.42	+0″.30	−9″.54	+0″.09	+8″.27	2′ 15″.27
22,	2 6.8	17.56	0.31	−9.54	0.1	8.43	15.23
26,	2 4.8	18.08	0.36	−9.54	0.1	9.01	13.81
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
Aug. 2,	2 36	−12.70	0.0	−9.41	0.40	−21.79	2′ 14.21
6,	2 37	−13.66	0.02	−9.41	0.41	−22.66	14.34

The corrections in the several columns are, in practice, taken from appropriate Tables: but they may, amongst other methods of computation, be computed by those which have been deduced in this Treatise: for instance, we may compute the numbers under the third column from $19''.55 \sin. (\odot + 3^s\ 1^\circ\ 42'\ 15'')$,

$$5\text{th, from } 0''.966 \sin. (☊ + 3.\ 1\ 30'),$$

$$6\text{th, from } 50''.1 \times \frac{52}{365} . \sin. (23^\circ\ 28') . \cos. 268^\circ,$$

when, instead of $\odot$ and ☊, the respective values of the longitudes of the Sun and of the Moon's node are substituted on Feb. 21, 22, &c. and Aug. 2, 6, &c.

The preceding expressions for computing the aberration, &c. are adapted (see p. 380.) to reduce the *mean* to the *apparent* polar distance. Now, γ Draconis is north of the zenith of the Observatory of Greenwich. The resulting numerical values, therefore, of the aberration, &c. if additive of the north polar distance, are subtractive of the zenith distance; and conversely: but, the above Table reduces the *apparent* zenith distance to the *mean;* consequently, the numbers in that Table will have the same sign as the numbers computed by the preceding formulæ.

The numbers expressing the aberration, &c. being either taken from Tables, or computed, are added together with their proper signs. The results are inserted in the seventh column. The numbers in the seventh column added to those in the second (which express the observed zenith distances) give the mean zenith distance on the beginning of the year 1811. For instance, the *sum* of the equations in the first horizontal line is $+8''.27$; the observed zenith distance, in the second column, is $2'\ 7''$; consequently, the mean zenith distance is $2'\ 15''.27$; which number, as well as all similarly obtained numbers, is inserted in the eighth column. The *numbers* in the eighth column added together and divided by their number (46 in the Table of which we have given a specimen) give the *mean* value of the mean zenith distances*.

* In the above specimen and its explanation, we may perceive the use of the theories of the inequalities in deducing certain results. The result,

The corrections, the theories of which have been investigated, are precession, aberration, the lunar and solar nutations. By these the instrumental and apparent zenith distances of γ Draconis observed in March, April, &c. of any particular year may be reduced to the mean zenith distance at the beginning of the same year; and, consequently, the mean zenith distances of the same star on the beginnings of different years may be compared together. A like reduction and comparison may be made of the right ascension of a star. With regard to the mean zenith distances and mean right ascensions of stars at the beginnings of different years, if these differ, they ought to differ, supposing all the corrections to have been accurately assigned, solely from the effect of precession. If the differences should not be accounted for from that effect, a new source of inequality would be indicated. If the effects of precession will account for the differences in the mean zenith distances of some stars, but not of others, there would, in that case, arise an indication of some peculiarity affecting these latter stars. But instead of describing, in general terms, the

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result, in the adduced instance, is the mean zenith distance of γ Draconis on Jan. 1, 1811. We may be required to go a step farther, and to shew the use of the results obtained in the above Table. Those results were obtained for the purpose of thence deducing the error of the line of collimation of the brass mural quadrant of Greenwich. But, see p. 67, such results alone are not sufficient. They must be compared with other results obtained by the zenith sector (see p. 68.) Thus, in the Volume of Observations, a few pages after those we have quoted, there is a Table of the zenith distances of γ Draconis observed with the zenith sector, partly with its face towards the east, and partly towards the west. All these observed zenith distances are *reduced*, precisely as the preceding ones have been, to the beginning of 1811. The mean of such reduced zenith distances is (see p. 68.) the *true* mean zenith distance of γ Draconis. The difference of this last mean and the mean obtained by the brass mural quadrant is the error of the line of collimation. We may infer, and not wrongly, from the preceding instance (which is one of many similar ones) that the business of an Observatory does not admit of being very leisurely and not laboriously conducted. The method of finding the *error of the line of collimation* is, usually described in four or five lines. The actual finding of it requires, as we have seen, the observations of fifty days, many arithmetical computations and the use of extensive Tables.

method of detecting inequalities, it will be better to exemplify it. And, as a first instance, we will describe the method which Bradley followed in extricating, from certain observed differences in the declinations of stars firstly, the inequality of aberration, and, secondly, that of nutation. This being done we will shew, on like principles, that there is still some change in the places of stars to be explained, or at least to make account of, even when they are reduced to the same epoch by the corrections that are due to the *inequalities* of precession, aberration, lunar and solar nutation.

These investigations will be carried on in the next Chapter, in which, we will also briefly advert to the methods which were first resorted to, for representing the lunar and solar nutations.

CHAP. XV.

On the Means by which Bradley separated Nutation from the Inequalities of Precession and Aberration.—On the successive Corrections applied to the Apparent Place of a Star.—On the Secular Diminution of the Obliquity.

In treating of the several inequalities of precession, aberration, and nutation, it is necessary, in order to avoid being perplexed by the mere words of a theory, to recur to the simple facts of observation. Now, the observations of Bradley were on the Declinations of Stars, or, what amounts to the same thing at a given place, on their zenith distances*; and, the *phenomena* of his observations, were *changes* in the observed zenith distances of the same stars; happening sometimes, at different parts of the same year, and at other times, at corresponding seasons of different years.

The star γ *Draconis*, passing the meridian very near the zenith of Bradley's Observatory, and being, consequently, little affected by refraction, was the chief star of his observations. This star (see pp. 288, &c.) in March passed more to the south of the zenith by about $39''$ than it did in September: that is, whatever was its mean place, the difference of its two zenith distances, or of its declinations, was, in half a year, observed to be about $39''$. Other stars, also, changed their declinations. The changes of declination of a small star in *Camelopardalus* (the 35th of Hevelius), with an opposite right ascension to that of γ *Draconis*, were observed at the same times as those of the latter star: and, it was Bradley's argument, that, if these phe-

* Bradley's observations were made with a *Zenith Sector*, adapted to measure the small portions, or minutes of declination, and of zenith distances, near the zenith.

nomena (changes of declination) arose from a real nutation of the Earth's axis, the pole must have moved as much towards γ *Draconis*, as from the star in *Camelopardalus;* but (see p. 295.) this not being the case, the hypothesis of a nutation of the Earth's axis would not account for the observed phenomenon: more strictly speaking, it would not completely account for it, for, in fact, some part of the observed changes of declination was due to the effect of nutation.

Bradley, as we have seen, (p. 295.) solved the above phenomena by the theory of aberration. Now, if such theory, with the known one of precession, would account for all observed changes of zenith distances, or, of north polar distances, then, there could be no changes but what arose from precession and aberration. Hence, since (p. 276.) the aberration is the same, at the same season of the year, the distance of γ *Draconis*, in September 1728, ought to have differed from his distance, in September 1727, only by the annual precession in north polar distance: the distance, in September 1729, from the distance, in September 1727, by twice the annual precession in north polar distance; and so on. Such, however, was not the observed fact. In 1728, after the effect of precession had been allowed for, γ *Draconis* was nearer the north, by about $0''.8$ than in 1727. In 1729, nearer than in 1727, by $1''.5$. In 1730, by $4''.5$. In 1731, by nearly $8''$. Here then was a new phenomenon, a change of north polar distance, indicating an inequality not yet discovered.

Bradley observed other stars besides γ *Draconis;* amongst others, the small star above-mentioned (p. 295.) of *Camelopardalus:* and, it is not a little worthy of notice, this same star, which, in the case of the former inequality, (that of aberration) directed him to reject the hypothesis of a nutation of the Earth's axis, here determined him to adopt it. For, within the same periods, the changes in north polar distance of γ *Draconis* and of the star in *Camelopardalus*, were equal and in contrary directions: that is, whilst the former, through the years 1728, 1729, 1730, 1731, was approaching the zenith, and consequently, the pole, the latter was, by equal steps, receding from the zenith, and consequently from the pole. These phenomena then of the

changes in north polar distances, which (not like those of aberration that take place in different parts of the same year, and recur in the corresponding parts of different years) were observed, through a term of years, could adequately be explained by supposing, during that term, a nutation in the Earth's axis, *towards* γ *Draconis*, and, *from* the small star in *Camelopardalus*.

After 1731, Bradley observed contrary effects to happen; that is, γ *Draconis* receded from the zenith and north pole, and the star in *Camelopardalus*, by equal steps, approached those points; and this access and recess continued till 1741, (a period of more than nine years); after which, the former star again began to approach the zenith, and the latter to recede from it. These phenomena, then, that took place between 1731 and 1741, could be adequately explained by supposing, during that term, a nutation in the Earth's axis, *from* γ *Draconis* and *towards* the small star in *Camelopardalus*.

The mere hypothesis of a nutation, or vibratory motion in the Earth's axis, would have found little reception amongst men of science, if no arguments had been adduced to render such nutation probable: that is, if some physical cause, likely to produce it, had not been suggested. Previously, however, to the suggestion of the real and immediate physical cause, Bradley enquired, whether this seeming nutation of the Earth's axis was connected with any concomitant circumstance, or phenomenon: such circumstance he found to be the position of the nodes of the Moon's orbit.

The star γ *Draconis* was (after the effects of precession had been allowed for) most remote from the pole, when the Moon's node was in *Aries*, and least, when in *Libra*: and, after a complete revolution of the Moon's nodes, the distances of all the observed stars, at the end, differed from the distances at the beginning, by the effect of precession only. Hence, the phenomenon of a nutation, and the longitude of the Moon's node were connected. But, the inclination of the Moon's orbit varies with the longitude of the node: the former is greatest, when the latter is equal to nothing; and least, when the latter is six signs. Hence, the nutation and inclination were connected together.

But, the Moon's action, on the bulging equatoreal parts of the Earth, is greater the more distant the Moon is from the equator; and her mean action greater, the greater the inclination of her orbit. Hence, the phenomenon of the nutation was connected, with the variable action of the Moon in causing precession; and this last connexion made nutation the effect, and the variation of the Moon's action the cause. And this was the physical cause which seemed to Bradley to afford an adequate solution of the phenomenon he observed: and subsequent researches have confirmed the sagacity of his conjectures.

The real distance of any star (γ *Draconis* for instance) from the north pole of the equator, is changed continually and constantly, by the effect of precession only. The variations in that distance from aberration and nutation are periodical, and recur, the former in the space of a year, the latter in the time of a revolution of the Moon's nodes. Hence, although, in any phenomenon of a change in the north polar distance of a star, the effects of several causes may be blended together and compounded; yet the method is plain, by which we may disengage and separate them. For instance, since the revolution of the Moon's nodes is completed in about eighteen years, and since the aberration and the solar inequality are the same, at the same time of the year, the north polar distance of γ *Draconis* in 1745, ought to differ from its north polar distance in 1727, almost solely by the effect of precession: that is, since the latter north polar distance was $38° 28' 10''.2$, and the precession $0''.8$, the north polar distance in 1745 ought to have been $38° 27' 56''$. And such difference was, by Bradley's observations, (see *Phil. Trans.* No. 485, p. 27.) found very nearly to exist.

Again, between September 6, 1728, and September 6, 1730, the aberration and solar inequality being the same, the respective north polar distances of γ *Draconis* at those periods ought to differ, by twice the annual precession in north polar distance, and by the effect of nutation: and hence the effect of nutation in an interval of two years, between two known positions of the Moon's ascending node, would be known.

Again, between September 6, 1728, and March 6, 1729, the

solar inequality being the same, the respective north polar distances of γ *Draconis* ought to differ from each other by the precession in north polar distance due to half a year, by the nutation for the same time, and (see p. 122.) nearly by the sum of the greatest aberrations in north polar distance; and the whole difference would consist almost entirely of aberration, since the precession and nutation together would not amount to a second.

Again, the Moon's ascending node being, March 28, 1727, in *Aries*, and July 17, 1736, in *Libra*; the respective north polar distance of γ *Draconis* would differ by the precession due to nine years three months, by the solar inequality of precession, by aberration, and by the sum of the two maximum effects of nutation. But, between March 28, 1727, and March 28, 1736, (since then the solar inequality and the aberration would be the same) the north polar distances would differ by the effect of precession, (a known quantity) and, nearly, by the sum of the two maximum effects in nutation. Hence, it would be easy to disengage, and numerically exhibit, (what is a material element), the maximum effect of nutation.

By examining various and numerous observations and by discriminating those that happened at particular conjunctures, Bradley found abundant confirmation of the truth of his two theories, aberration and nutation. During a period of more than twenty years, he accounted for the phenomena of observation, that is, the changes in the declinations of various stars, by making those changes or variations consist of three parts; the first due to precession; the second to aberration; and, the third to nutation: the quantities and laws of the two latter, being assigned on the principles and by the formulæ of his theories.

We cannot sufficiently admire the patience, the sagacity, and the genius of this Astronomer, who, from a previously unobserved variation not amounting to more than forty seconds, extricated, and reduced to form and regularity, two curious and beautiful theories.

The following Table exhibits the coincidence of his theories with observations. (See *Phil. Trans.* No. 485, p. 27.)

γ Draconis.	South of 38° 25'.	Precession.	Aberration.	Nutation.	Mean Distance.
1727 Sept. 3	70″.5	−0″.4	+19″.2	−8″.9	80″.4
1728 Mar. 18	108.7	−0.8	−19	−8.6	80.3
Sept. 6	70.2	−1.2	+19.3	−8.1	80.2
1729 Mar. 6	108.3	−1.6	−19.3	−7.4	80.0
Sept. 8	69.4	−2.1	+19.3	−6.9	80.2
1730 Sept. 8	68.0	−2.9	+19.3	−3.4	80.5
1731 Sept. 8	66.0	−3.8	+19.3	−1.0	80.5
1732 Sept. 6	64.3	−4.6	+19.3	+2.0	81.0
1733 Aug. 29	60.8	−5.4	+19.0	+4.8	79.2
1734 Aug. 11	62.3	−6.2	+16.9	+6.9	79.9
1735 Sept. 10	60.0	−7.1	+19.3	+7.9	80.1
1736 Sept. 9	59.3	−8.0	+19.3	+9.0	79.6
1737 Sept. 6	60.8	−8.8	+19.3	+8.5	79.8
1738 Sept. 13	62	−9.6	+19.3	+7.0	78.7
1739 Sept. 2	66.6	−10.5	+19.2	+4.7	80.0
1740 Sept. 5	70.8	−11.3	+19.3	+1.9	80.7
1741 Sept. 2	75.4	−12.1	+19.2	−1.1	81.4
1742 Sept. 5	76.7	−12.9	+19.3	−4.0	79.1
1743 Sept. 2	81.6	−13.7	+19.1	−6.4	80.6
1745 Sept. 5	86.3	−15.4	+19.2	−8.9	81.2
1746 Sept. 17	86.5	−16.2	+19.2	−8.7	80.8
1747 Sept. 2	86.1	−17.0	+19.2	−7.6	80.7

A brief explanation will suffice for this Table. The *apparent* place of a star is deduced from the *mean*, by applying to the latter the several corrections: or, the mean is deduced from the apparent, by applying the same corrections with contrary signs.

If therefore γ *Draconis* were, at the beginning of any period, a certain number of seconds, south of the zenith, or south of

any particular division in the zenith sector; it would, at the end of the period, be *really* farther from the zenith by precession; *really* farther or nearer, by nutation; and *apparently* nearer or farther by aberration. By the *mean distance* of the star from the division $38^{0}\ 25'$ of the zenith sector (see last column in preceding Table), Bradley means the distance on March 27, 1727, such as would have been the distance, had there been neither nutation, nor aberration. But, in that year, the nutation, (the node of the Moon's orbit being in *Aries*) was the greatest. Hence, in September 1727, (see the first horizontal row of the preceding Table) the *observed* or *apparent* distance of γ *Draconis* would differ from the mean, by the effect of precession ($\frac{1}{2} \times .8$) in half a year, by the maximum effect of aberration, and by nearly the greatest effect of nutation. The *apparent* distance then of the star being $70''.5$, the *mean* (according to Bradley) would be

$$70''.5 - 0''.4 + 19''.2 - 8''.9 = 80''.4.$$

Again, reversing the process. If $80''$ were the mean distance, then, on March 6, 1729, the star would appear by aberration farther distant about $19''.3$: would really be more distant by the effect of two years' precession in north polar distance ($2 \times .8$;) and would really be more distant than it would be if the Moon's orbit were at its mean inclination (the ☊ being either in ♋ or in ♑) by the effect of nutation ($7''.4$). The apparent* distance therefore would be

$$80'' + 1''.6 + 19''.3 + 7''.4 = 108''.3.$$

The mean distances deduced according to the preceding explanation, by means of corrections, from Bradley's two theories of aberration and nutation, and from the known effect of precession, ought, if the theories be true, to be invariably the same: and their very near equality (see last column in Table, p. 389.) establishes, almost beyond a doubt, the truth of those theories.

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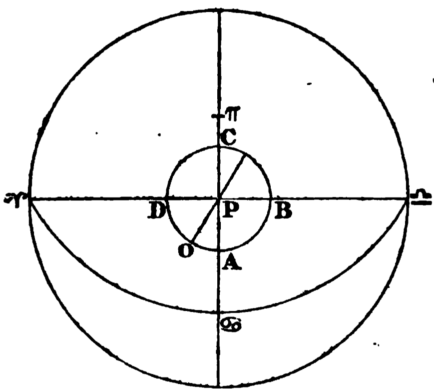
* There is some violation of the propriety of language in calling that *apparent*, which depends on real causes, viz. the changes of the place of the pole from precession and nutation. In strictness, apparent should have been confined to *aberration*, refraction.

The division in the zenith sector, from which, as a fixed point, Bradley measured the distances of γ *Draconis*, (the north polar distance of which he calls 38° 25′) is not the division corresponding to the zenith of the Observatory at Wansted. If it had been, the apparent north polar distances of γ *Draconis* on Sept. 3, 1727, and on March 6, 1729, would have been, respectively, 38° 26′ 10″.5, and 38° 26′ 48″.3.

Having now explained the method by which Bradley detected, in the small differences of the declinations of certain fixed stars, the existence of several inequalities, we will briefly state, after what manner, he first thought that the effects of nutation could be represented.

Bradley supposed the path described by the true pole round its mean place, by reason of the inequality of the Moon's force



in causing precession, to be a circle. Thus, in the subjoined Figure, let π be the pole of the ecliptic, P the *mean* place of the pole of the equator, and let $DOAB$ be a circle described round P as a centre and with a radius PA ($= 9''.6$). Moreover, A is to be the *true* place of the pole, when the ascending node of the Moon's orbit is ♈ (the first point of Aries). The other positions of the true pole are to be determined by supposing it to move equably along the circle AOD, &c. in a direction contrary to the order of the signs, and to describe the circle, in a period equal to that of the retrogradation of the Moon's nodes.

The Moon's node then being at any distance from ♈, take the angle APO equal to that distance, and O is the true place of the pole.

Such point being assumed to be the true place, the changes in the north polar distances and in right ascensions of stars are to be computed, exactly as they were when p_2 a point in the ellipse, was assumed to be the pole's place (see p. 358.)

This Memoir of Bradley's is inserted in No. 485, of the Philosophical Transactions. Towards the end of it, its Author suggests that the effects of nutation would be more truly represented, by supposing the locus of the pole to be an ellipse, instead of a circle, the transverse and conjugate axes, AC and DB, being nearly $18''$ and $16''$ respectively. Not, however, perfectly satisfied of the justness of this last suggestion, Bradley wished it to be tried by theory; and, such trial, as we have seen in the last Chapter, has, since Bradley's time, been made.

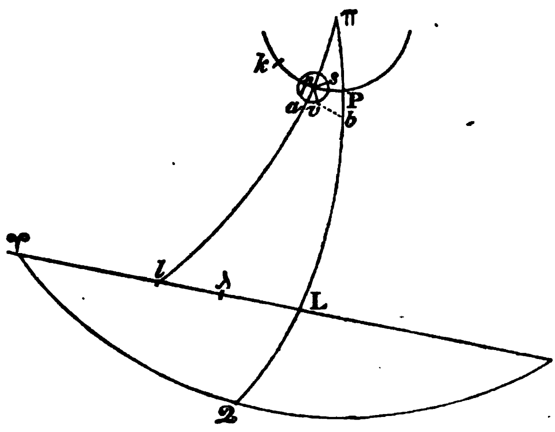
In the inequalities of precession, aberration, and the lunar nutation, observation has preceded theory. These inequalities were first detected as phenomena, and then their physical causes assigned. It has not been so with the solar nutation, which was never, (such is its minuteness), distinctly perceived as a phenomenon. It was first conjectured to exist from analogy. The *inequality* of the Moon's force in generating precession being found to cause a lunar nutation, the inequality of the Sun's force it was presumed, would also cause a *solar* nutation resembling the lunar. Its law and quantity have, accordingly, been computed, and the numerical results applied as corrections of observations.

In the solar, as in the lunar nutation, the true pole describes, round the mean place of the pole, an ellipse, the semi-axis major of which ellipse, in the case of the solar nutation, is less than half a second. The corrections, therefore, of the north polar distances and right ascensions of stars, in consequence of this deviation of the pole, are very small. And this last circumstance makes it of little consequence, whether, in computing the above

errors in the places of stars, we consider the pole to be *erratic* in an ellipse or in a circle* which Dr. Maskelyne† and other writers consider as its locus.

We will again mention, for the sake of preventing any false conceptions on this subject, that the two ellipses, as the respective curves of deviation of the true pole, in consequence of the inequalities of the lunar and solar force in causing precession, are merely mathematical schemes and contrivances for the convenient computation of the changes produced in the places of stars. The changes to be computed are very small: which is the reason

* The solar inequality has been thus represented: π, P, are, respectively, the poles of the ecliptic and the equator. By virtue of the precession, P will describe, and contrary to the order of the signs, the arcs of a



small circle Ppk. After a lapse of time, suppose p to be the *mean* place of the pole: the true place will be nearer to π, or farther from π, or to the right or left of p, according to the position of the Sun. In order to determine its place, describe round p as a centre, and with a radius $=0''.435$, avs, and take, according to the order of the signs, the angle apv equal to twice the Sun's longitude: then, v is the *true* place of the pole, the pole being at b (Pb = radius of the small circle) when the Sun was at ♈.

† Explanation and use of the Tables inserted in the 1st Volume of the Greenwich Observations.

why we may separately compute the effects of precession and of the two nutations, combine them and obtain a result scarcely different from the true result; the *true* result being that which would be obtained by placing the pole in that curve which would be described, by the combination of its three movements; one circular round the pole of the ecliptic, and representing the mean effect of the *luni-solar* precession: the second elliptical by reason of the inequality of the lunar precession: the third also elliptical and caused by the inequality of the solar precession.

In the next Chapter we will consider whether the theories of the preceding inequalities completely explain the differences of the declinations and of the right ascensions of stars, either computed or observed, at different epochs.

CHAP. XVI.

On the proper Motions of Stars: the Means of discovering such; their ambiguous Nature; arising from the Means used for determining the Precession.—Instances of the Methods used for finding the proper Motions of a Star in Right Ascension and Declination.

In order to compute the right ascensions and declinations of stars, there are necessary, firstly, a catalogue of their mean right ascensions and mean declinations at a certain epoch; and, secondly, Tables, computed by the aid of theory and observation, for supplying the amount of those differences which will be found to exist between the above-mentioned right ascensions and declinations, and their values at a different epoch, whether such values be the mean or the true values.

If the comparison between the right ascensions and declinations be a comparison between their mean values at different epochs, then certain inequalities will be rescinded and have none effect in producing differences of the mean values: for instance, aberration and the two nutations (or the two sources of inequalities of precession) will be in such predicament.

The differences of the mean values will depend solely (if there be no other inequalities than those treated of in the preceding part of the Treatise) on Precession. Thus, if in July 9, 1821, the north polar distance of α Lyræ be observed, such north polar distance, is an *apparent* distance, or *true* distance. When corrected for aberration, for the solar and lunar nutation, and for that part of the annual precession which is proportional to the interval between January 1, and July 9, it will express the *mean north polar distance* of α Lyræ for January 1, 1821. And it ought, were the preceding enumeration of the sources of inequality complete, to differ from the mean value of 1815 (supposing that to be the epoch of the existing or standard catalogue)

by the sum of six annual precessions. And the like is to be said of the polar distances and right ascensions of other stars.

Now the fact is that, after the completion of the above process, the differences of the polar distances and right ascensions of stars are not found to be exactly accounted for by the quantity and law of precession: in some stars the differences are greater than what they ought to be by the effect of precession, in others less. And this fact being ascertained, our attention is drawn towards the mode by which the quantity of precession is ascertained.

The precession, or the retrogradation of the intersection of the equator and ecliptic on the ecliptic, is not a phenomenon of immediate observation. It requires, in all cases, some slight computation; which computation may be made either from the changes it produces on the right ascensions of stars, or from the changes in north polar distances, or from the differences of the longitudes of stars computed, for different epochs*, and from the respective values of the right ascensions and polar distances of stars belonging to those epochs (see Chapter VIII). Now, whichever be the method used, the mean quantity of the precession is that which results from a great number of stars, three or four hundred, for instance; and even if there were any undetected inequality, equally affecting, however, all the stars, yet, since the effect of such inequality would be blended with that of precession, the quantity of the precession (or what is so deemed) obtained from all the stars, ought to agree with the mean precession deduced from numerous observations of any one star. But, if we suppose any peculiar movements to belong to any one, or to more stars, such peculiar movements would affect the quantity of precession determined by the preceding method, and vitiate it. Reversely, if the mean quantity of the precession deduced from the comparison of three hundred stars should differ from the quantity resulting from the comparison of fifty longitudes of the star Arcturus, for instance, it would infallibly follow

* See pp. 184, &c. of this Work. M. Zach computed the longitudes of thirty-five principal stars, in order to determine the precession.

as a consequence, either that Arcturus was subjected to some motion to which all other stars were not, or that some or all of those stars were subject to motions from which Arcturus was exempt.

Such motions, not generally affecting all stars, are called by Astronomers, *Proper Motions,* and are to be assigned to stars not from any theory but solely by observation.

It is clear, however, that if there exists no other method of detecting these proper motions than what has been just described, that they can never be entirely disengaged from the effect of precession and exhibited separately; since they themselves enter into the composition of precession. All that can be done is to determine how much the annual changes of the mean right ascension and mean north polar distance of each star differ from its *mean* precession in right ascension and *mean* precession in north polar distance; understanding, thereby, those values which are computed, by the formulæ of pp. 340, 341, and from a quantity (50″.1) held to be the mean quantity of the precession.

These annual changes, which are compounded of the precession and certain proper motions, are technically denominated the *Annual Variations* in right ascension and north polar distance, and are inserted as such in the catalogues of stars. The annual precessions in north polar distance and right ascension are then subjected to a certain law (see pp. 190, &c. 340, 341.) but the annual variations are altogether irregular, never differing, however, from the former, except by minute quantities.

We have seen, then, that the precession, as its results from Astronomical methods, is not the actual retrogradation of the intersection of the equator and ecliptic on the ecliptic. We will now consider another point. Is the retrogradation (supposing it capable of being determined) produced solely by the influence of the Sun and Moon on the excess of the Earth above a sphere? Is it, in fact, a *luni-solar* precession? This is a question which we must go out of the precincts of Plane Astronomy to find an answer to.

We have already seen, (Chap. XV.) that the obliquity of the ecliptic, besides its periodical variations (see pp. 375, 376.) is subject to an inequality of a very long period * and called a *Secular* Inequality. The effect of this inequality is the diminution of the mean inclination at the rate of $45''.7$ in a century. It is caused by the action of the planets on the Sun: the effect of which action is to draw the Sun out of the plane of the curve in which he is moving; so that, unlike the periodical changes of obliquity, which arise from the oscillations of the equator, or the nutations of the Earth's axis, the *secular* diminution of the obliquity arises from the displacement of the ecliptic itself. But this is not the sole effect of the action of the planets; for, besides the changes of obliquity, the intersection of the equator and ecliptic is made to move, not by a retrograde, but by a direct motion, or according to the order of the signs along the equator. And, estimated in that direction, its annual amount (a quantity too small to be determined by observation) is $0''.20174$: in the direction of the ecliptic its quantity is $0''.18505$ ($=.20174 . \cos. 23° 28'$).

Now this inequality is under the predicament described in p. 396: it equally affects the longitudes of all stars: and, consequently, in determining the precession from the differences of the longitudes of stars at different epochs, (see p. 186, &c.) we determine not the *luni-solar* precession, but the luni-solar precession diminished by this quantity $0''.18505$. If, therefore, $50''.1$ be the precession determined by observation, the precession due solely to the action of the Sun and Moon is

$$50''.1 + 0''.18505 = 50''.28505.$$

In like manner the precession in right ascension, determined by observation, is less than the *luni-solar precession* in right ascension, by the quantity $0''.20174$, and, consequently, the actual change of right ascension is (see p. 344.)

$$50''.28505 (\cos. I + \sin. I \, \sin. Æ . \cot. \delta) - 0''.20174.$$

* *Physical Astronomy*, Chap. XXII.

This is the expression for the precession in right ascension, on the supposition that the obliquity of the ecliptic remains the same: but, if that should be variable, it would be necessary to add the term

$$- dI . \cos. Æ . \cot. \delta$$

to the preceding: so that, if dL represent the *luni-solar* precession, the change in right ascension equals to

$$dL (\cos. I + \sin. I . \sin. Æ \cot. \delta)$$
$$- 0''.20174 - dI . \cos. Æ . \cot. \delta\,*,$$

* It may be right to explain the grounds and the method of deducing this and similar expressions, especially, since the subject, as it is found in some Authors, is not free from ambiguity.

The equations from which the variations in right ascension, north polar distance, longitude and latitude may be readily deduced, are the first four equations of p. 182: to wit,

(1.) $\tan. Æ = \tan. L \cos. I - \tan. \lambda \sec. L . \sin. I.$

(2.) $\sin. (90^\circ - \delta) = \sin. \lambda \cos. I + \sin. L . \cos. \lambda . \sin. I.$

(3.) $\tan. L = \tan. Æ \cos. I + \tan. (90^\circ - \delta) . \sec. Æ . \sin. I.$

(4.) $\sin. \lambda = \sin. (90^\circ - \delta) \cos. I - \sin. Æ . \cos. (90^\circ - \delta) . \sin. I.$

Which equations, by a slight transformation, are made similar, for a purpose which will be soon explained.

If in the equation (1) we make L and $Æ$ to vary, λ and I remaining constant, we have, as in p. 341,

$$dÆ = dL (\cos. I + \cot. \delta . \sin. I . \sin. Æ);$$

but, if we suppose I to vary, then there will be introduced a term such as $- dI . \cos. Æ . \cot. \delta$: so that

(a) $dÆ = dL (\cos. I + \cot. \delta . \sin. I \sin. Æ) - dI . \cos. Æ . \cot. \delta.$

In like manner, from the equation (2), we have, λ and I being constant, (see p. 341.)

$$d\delta = - dL . \cos. Æ . \sin. I;$$

but if we make I to vary, we have

(b) $d\delta = - dL . \cos. Æ \sin. I - dI . \sin. Æ.$

Now

which is Laplace's expression, (see *Mecanique Celeste*, tom. II. p. 350.)

Now the operations of reduction by which these differential expressions have been deduced from the equations (1) and (2), will, when applied to the equations (3) and (4), (which see p. 399. are similar), produce similar differential equations; and accordingly,

(c) $dL = d\text{Æ}(\cos. I - \tan. \lambda . \sin. I . \sin. L) - dI . \cos. L \tan. \lambda,$

(d) $d\lambda = - d\text{Æ} . \cos. L . \sin. I - dI . \sin. I.$

We must now consider the conditions or circumstances under which these variations take place, and the several values which dI, the change of obliquity, will, according to those circumstances, possess.

In the first place we may observe, that, the expressions (a), (b) were obtained by supposing the latitude (λ) to be constant. The star, therefore, being fixed, the ecliptic must be supposed not to change its position. But I the obliquity is variable: such variation, therefore, must be supposed to arise from a change in the position of the equator; and the preceding expression (a), by means of its last term, will express the change of right ascension due to that variation, of whatever kind the variation be, whether it be periodical or secular: provided its quantity be very small. The expression, therefore, will represent those variations of the right ascension and declination which arise from the changes in the obliquity produced by the solar and lunar nutations; for, these changes are produced by an oscillation of the equator: but they will not serve (for the reasons alledged in l. 13, &c.) to represent the variations produced by that change of the obliquity which a change in the position of the plane of the ecliptic gives rise to. The expressions will also serve to represent any variations of right ascension and north polar distance, produced by a secular change of the obliquity, provided such secular change arise from a change in the plane of the equator.

Now it is such a change which Laplace and other writers must have had in their minds, when they represented the variation in right ascension, by the expression (a) of p. 399. For, such expression is used in determining, from an assigned mean right ascension of a star at a certain epoch, its mean right ascension at another epoch. In such kind of calculation (used, for instance, in determining the proper motion of a star) all periodical inequalities, such as the *nutations of the obliquity*, are rescinded or accounted

The right ascension of a star will be affected, as we have seen, by that *Progression* of the first point of Aries which is caused by

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counted for; and, as we have already explained, no secular changes of obliquity can influence the right ascension, except such as arise, not from a change in the position of the plane of the ecliptic, but from a change in the position of the plane of the equator.

But we must, in this case, have recourse to Physical Astronomy. The secular equation, which we are in search of, is so small that observations are unable to indicate it. Let I denote the mean obliquity of the ecliptic at the beginning of the year 1750, which ecliptic, for distinction's sake, is called the *Fixed* Ecliptic; let t denote the number of years reckoned from 1750: then, by the results of Physical Astronomy, (see Laplace, tom. III. p. 158.)

$$I = 23° \; 28' \; 18'' + t^2 \times 0''.000009842.$$

Hence, $Æ$, L, I, &c. being supposed functions of the time, or $d\,Æ$, $d\,L$, $d\,I$, in the formula of p. 399. standing for

$$\frac{d\,Æ}{dt}\,dt, \quad \frac{dL}{dt}\,dt, \quad \frac{dI}{dt}\,dt,$$

we have

$$\frac{dI}{dt} = 2t \times 0''.000009842 = t \times 0''.000019684,$$

and accordingly, the formula of p. 399. l. 27. for expressing that increment of the right ascension, which is to be added to the mean right ascension of 1750, in order to obtain the mean right ascension at any other epoch distant from 1750 by t years, will be

$$d\,Æ = {}^{*}\,50''.239055\,(\cos. I + \sin. I . \sin. Æ . \cot. \delta) - 0''.201633\,t$$
$$- \; 0''.000019684 . \cos. Æ . \cot. \delta \times t,$$

which is a formula like that which M. Zach has given at p. 12, of his *Supplement aux Nouvelles Tables d'Aberration.*

If we substitute the value of dI, just obtained, in the expression for the variation in north polar distance, we have

$$d\delta = - \, t \times 50''.239055 \sin. I - 0''.0000196844 \times t.$$

In these expressions, it is to be observed, that allowance is made for all

* According to Laplace this coefficient equals 50″.2875.

the action of the planets. Hence, if we estimate the precession from observations, made on the observed differences of right

all the inequalities that affect the precession in right ascension and north polar distance: for instance, the whole effect of the luni-solar precession in right ascension is diminished by $0''.201633$ (the effect of the planets in causing the equinoctial point to progress) and by

$$0''.0000196844 \cos. Æ \cot. \delta,$$

which is the variation in the right ascension produced by a change of obliquity: which last correction it is scarcely ever necessary to make account of.

Having now spoken of the change produced, in the right ascensions and declinations of stars, by the obliquity varying from a change in the position of the plane of the equator, we will consider what effect on the positions of stars will be produced by the obliquity varying from a change in the ecliptic itself.

A mere oscillation of the plane of the ecliptic, round an axis passing through the two equinoctial points, will affect neither the declinations nor the right ascensions of stars: but the latter quantities will be affected, if, as is the case (see Chap. xxii. Vol. II. of *Astronomy*) the change of obliquity is accompanied by a *progression* of the equinoctial points: both inequalities, indeed, arise from the same cause. The declination, however, will remain constant; and, accordingly, in deducing the equations (*c*), (*d*), of p. 399, we supposed δ to remain constant. Those equations, therefore, will represent the variations of the longitudes and latitudes of stars due to any change of the obliquity, provided such change arise from the displacement of the ecliptic itself. But there are no periodical changes of the ecliptic: the sole change to which it is subject is a *secular variation* produced by the action of the planets: the annual value of which is about $0''.5$ (see Vol. II. *Astronomy*, p. 461.) Accordingly, the expressions (*c*), (*d*), become

$$dL = 0''.201633\,t\,(\cos. I - \tan. \lambda \sin. I \sin. L) - 0''.5\,t \cos. L \tan. \lambda,$$
$$\text{and } d\lambda = -t\,(0''.201633 \cos. L \sin. I + 0''.5 \sin. L)$$

the former of which expressions represents solely the change produced in the longitude of a star by the action of the planets on the plane of the Earth's orbit, and makes no account of the effect of precession.

In deducing the above values of dL and $d\lambda$ we have supposed the annual variation of the inclination produced by a change of the ecliptic to be

ascensions at different epochs, we determine a quantity with which the above-mentioned progression is blended. But the

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be $0''.5$: its value, unlike the other of which we have deduced (see p. 401. l. 19.) may be obtained from the comparison of observations: but (see *Astronomy*, Vol. II. Chap. xxii.) it may be also derived from theory, which, besides the term involving t and the coefficient $0''.5$, furnishes another term involving t^2: thus, according to Laplace, (*Mec. Celest.* tom. III. p. 153.) I' representing the *true* ecliptic,

$$I' = 23° \; 28' \; 18'' - t \times 0''.52114 - t^2 \times 0''.000002723,$$

consequently,

$$\frac{dI'}{dt} = - 0''.52114 - t \times 0''.00000545,$$

and, accordingly, the former expressions of dL and $d\lambda$, in order to be more correct, ought to be increased by the term $- t \times 0''.00000545$: but the practical correctness thence ensuing, is, as it is plain, of very little moment.

In the preceding investigations, account has been made solely of the variations to which the mean inclination is subject: whether such mean inclination be the inclination of the plane of the true ecliptic, or of the *fixed* ecliptic of 1750. But the true ecliptic, besides its secular diminution, is subject to periodical variations; one the solar, the other the lunar nutation. In order then to represent the true obliquity at any time of the year, let I', determined by the above equations of l. 9. be the mean value of the true ecliptic at the beginning of the year: let n be the number of days elapsed from the beginning, then the *true* value of the *true* ecliptic is

$$I' - \frac{n}{365.25} \times 0''.52114 + 0''.435 \cos. 2 \odot + 9''.64 \cos. \Omega.$$

We have already quoted from Laplace the values of the obliquity, &c. we subjoin, from the same Author, the values of some other quantities connected with this subject of enquiry,

the precession (ψ) on the fixed ecliptic $= 50''.2876\,t - 0''.0001217945\,t^2$,

(ψ') on the true ecliptic $= 50''.09915 + 0''.000122148\,t^2$.

Hence making $t = 1$,

$$\psi - \psi' = 0''.18848, \text{ nearly,}$$

which was, in the year 1750, the *progression* of the equinoctial points in longitude

declination of a star is not affected by such progression; consequently, the precession determined from the annual precession in north polar distance, will be different from that which is determined from the annual precession in right ascension. The difference, perhaps, is too minute to be detected by observation; but, if the results of Physical Astronomy be relied on, it exists as really as the precession itself.

Thus, if the precession in north polar distance, of a star situated in the equinoctial colure, should be $20''.04$, then the precession in longitude $= \dfrac{20''.04}{\sin. I} = 50''.324$,

and the precession in Æ would $= 20''.04 \,.\, \cot. I = 46''.162$.

But the precession in right ascension obtained by computing the right ascensions of the equinoctial point, at two different epochs, would be $46''.162 - 0''.202 = 45''.96$.

Having now ascertained the causes which affect the precession, we will explain, by means of an instance, the method of detecting the *proper* motions of stars.

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longitude occasioned by the displacement of the ecliptic. The progression, therefore, of the equinoctial points in right ascension

$$= 0''.18848 \times \sec. 23° \, 28' \, 18'' = 0''.20415,$$

and, for a time t, $= 0''.20415\,t$. The access of the equinoctial point in the direction of latitude towards the south pole of the ecliptic of 1750

$$= t \times 0''.18848 \times \tan. 23° \, 28' \, 18'' = 0''.081\,t, \text{ nearly.}$$

The expression of p. 403. is for the mean *precession*: but, as we have seen in Chap. XIV. there is an inequality arising from the unequal actions of the Sun and Moon: if, therefore, ψ' deduced from p. 403. l. 31. be the precession from 1750 to the beginning of a year distant from 1750 by the time t, and if n be the number of days elapsed from the beginning of this latter year, we have, at the end of these number of days, the true *retrogradation* of the equinoctial point, or the true precession equal to

$$\psi' + \frac{n}{365.25} \times p - 1''.\sin. 2 \odot - 16''.544 \,.\, \sin. \Omega,$$

p being the annual precession in the proposed year.

It is required to determine, from the observations of 1755, and 1802, the proper motion of Arcturus *.

The *proper motion in right ascension* will be the difference between the mean right ascension in 1755 increased by the precession in right ascension due to the interval of forty-seven years, and the mean right ascension of 1802.

Now, as it has been explained in pp. 343, &c. the annual precession, whether in right ascension or north polar distance, depending on the star's right ascension and north polar distance, must be different according to the epoch for which it is computed. Its values, therefore, in 1755 and 1802 will be different, although in a small degree. Suppose the precession of Arcturus to be that which would result from the *mean* value of its right ascension and north polar distance: then, since

according to Bradley in 1755, its right ascension	$=7^s$	1°	$7'$	$25''.155$
and according to Maskelyne in 1802	$=7$	1	39	27.6
its mean right ascension for the middle time	$=7$	1	23	26.378

Again,

north polar distance in 1755...............	$=69^\circ$	$31'$	$54''$
—————— in 1802.................	69	46	49.8
its mean north polar distance for middle time. .	$=69$	39	21.9

We must now find the precession in right ascension from the formula of p. 399. but, previously, we must determine the value of the *luni-solar precession* to be used in that formula.

In 1750 its value was......................	$50''.239055$
† the prop^l. part of its secular equat^n. for 28.5 years. .	0 .006693
∴ the value for the mean time of 1778.5	50.245748

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* This instance is taken from the *Supplement aux Nouvelles Tables d'Aberration et de Nutation.* By the Baron de Zach, Marseilles, 1813.

† According to M. Zach (*Supplement aux Nouvelles Tables d'Aberration*, p. 13.) the year 1750 being the epoch,

the luni-solar precession $= 50''.239055 \pm 0''.00023485\, t.$

which is the value of dL to be used in the present instance (see p. 400.)

$-$log. r	-10
log. 50″.245748	1.7010994
log. cos. obliquity	9.9625002
	1.6635996 (= log. 46″.08924).

Again,

—log. r^3	—30
log. 50″.245748	1.7010994
log. sin. obliquity	9.6001570
log. sin. Æ	9.7167296
log. cot. δ	9.5691196
	0,5871056 = (log. —3″.8641)
	42.22463

But this value (42″.22463) is (see p. 398.) the value of the luni-solar precession in right ascension. In order, then, to find that value, which the observations give, we must diminish it by the *progression* of the equinoctial points: consequently, such value must equal to

42″.22463 − 0″.20172 = 42″.02291 *.

This last quantity, then, is the value of the precession in right ascension deduced from those values of the *luni-solar precession*, and of the right ascension and north polar distance of Arcturus corresponding to an epoch, which is the mean of 2756 and 1802. Hence, the mean right ascension of Arcturus, *computed* from such precession and its mean right ascension in 1755, is equal to

	s	°	′	″
	7	1	7	25.155
(+ 47 × 42″.02291 =)	0	0	32	55.07677
	7	1	40	20.23177
but the *observed* right ascen^n. of ✱ in 1802 =	7	1	39	27.6
the unaccounted for differ^e ∴ in 47 years is	0	0	0	52.63177.

* The epoch being 1750, the progression caused by the action of the planets is 0″.20168 + 0″.0000012 t.

This difference, for want of an explanatory theory, or from ignorance of its cause, is attributed to the star's *proper motion:* and its *annual* proper motion, thence computed, is $\frac{52''.63177}{47} = 1''.1196$. Now, by reason of this proper motion, the *computed* right ascension (see p. 406, l. 36.) is greater than the observed right ascension. In order to make the two right ascensions agree, therefore, the proper motion must be applied, with a negative sign, or must be made to diminish the precession, or must be thus written $-1''.1196$. The annual precession being then (see p. 406.) $42''.02291$, the *annual variation* (which is the term given in catalogues and in the Nautical Almanack to the *sum* of the precession and of proper motion) is

$$42''.02291 - 1''.1196 = 40''.90331 =, \text{ in time, } 2^s.727.$$

In order to compute the proper motion of the same star in north polar distance, we have (see pp. 348.)

$-$ log. r^2	-20
log. $50''.245748$	1.7010994
log. sin. obliquity	9.6001570
log. cos. right ascension	9.9312726 $-$
	1.2325290 (= log. $17''.08162$).

The mean north polar distance of Arcturus then, computed from such precession and its north polar distance in 1755, is equal to

	°	′	″
	69	31	54
$+ 17''.08162 \times 47$	0	13	22.83614
	69	45	16.83614
but the observed north polar dist. in 1802. .	69	46	49.8
the unaccounted for diff. ∴ in 47 years is. . . .	0	1	32.96386

The *proper annual motion*, therefore, in north polar distance is equal to $\frac{1'\ 32''.96386}{47} = 1.97795$. And as this proper motion makes the computed north polar distance less than the observed,

it must be added to the precession in north polar distance, and written 1″.97795. The precession therefore, being 17″.08162
and the proper motion . . . . 1 .97795

the annual *variation* in north polar distance . . . . 19.05957 *

In like manner the proper motions of other stars are to be determined: and Dr. Maskelyne computed, in the first Volume of the Greenwich Observations, the proper motions of Sirius, Castor, Procyon, Pollux, Regulus and α Aquilæ. The list has subsequently been much increased (see Greenwich Observations Vol. I. tab. 9.)

It is plain, from what has been said, and from the preceding computation, that the proper motions of the stars are determinable by no formulæ. They are ascertained solely by observation. We are ignorant of their causes and laws. We cannot even presume that the proper motions, determined by the comparison of observations made at different epochs, were the same in the preceding, or will be the same in future periods. All that can be said is, that, if such a presumption were made, the error consequent on it will be very small, inasmuch as the proper motions themselves, as far as they have been hitherto ascertained, are very small. But, notwithstanding our ignorance of the causes of these proper motions, still it is essential to know their quantities, since they affect observations precisely, as any other inequality does, and lessen or augment the right ascensions and declinations of stars. Their effects, therefore, are now, as we have said, regularly combined with the results from precession, and then inserted in catalogues.

It is the excellence of modern instruments and observations that causes the proper motions of stars to be known. They were formerly blended with the effects of other inequalities, and not distinguishable; principally for this reason, that their quantities were far less than the probable errors of observations. They are not even now easily made out: for, as it appears by the instance of p. 405, &c. they are not determined by single observstions, or by

* Dr. Maskelyne's explanation and Use of the Tables, p. 4. makes the proper motion of Arcturus in right ascension = − 1″.395, and in north polar distance = + 2″.01.

the comparison of observations made during short intervals, but by the comparison of observations made in former times with present observations. Now, as Dr. Maskelyne remarks (p. 9. Explanation and Use of the Greenwich Tables); 'we are in want of good antient observations.' But, even if we did possess obervations of the latter character, the question concerning *proper motions*, would be one of considerable difficulty; or, rather, the nature of these motions is, as it must always be, ambiguous. The great Astronomer, whose words we have just quoted, says a little farther on 'the other stars of the Table do not appear to have any proper motions.' It should rather have been said that they do not appear to have proper motions according to the *method* used for determining them. For it is easy to feign a case in which a star should have a proper motion, which should not be indicated by the method used in detecting it. For instance, since the precession, as determined by the differences of the longitudes of stars at different epochs, or of their right ascensions, is the mean of the precessions due to the several stars diminished or augmented by the proper motions of those stars, such mean may be exactly equal to the precession plus or minus the star's proper motion: in which case, the star would appear to have no proper motion. If we would state the case symbolically, let P be the precession, α, β, γ, δ, &c. (either positive or negative) the proper motions in longitude, of the stars which are used in determining the precession, then the precession, determined according to the method we have described, is

$$\frac{P + \alpha + P + \beta + P + \gamma + \&c.}{m},$$

m being the number of stars; let ρ be the proper motion of the star the proper motion of which is sought, then it equals

$$\frac{mP + \alpha + \beta + \gamma + \&c.}{m} - (P + \rho)$$

$$= \frac{\alpha + \beta + \gamma + \&c.}{m} - \rho.$$

which may become equal to 0 by a variety of ways, that is, by variously adjusting the proportional values of α, β, γ, δ, &c.

There will be an error or uncertainty of a like nature whatever the star be, the proper motion of which should be required: that is, the result of its proper motion will be an ambiguous result, whether the star be or be not one of those stars that are used in determining the precession. If the former be its predicament, we then, in deducing the star's proper motion, are arguing (which is a common case in Astronomy) in a *vicious circle;* since the quantity to be determined is already implicated in the quantities from which it is to be determined.

Dr. Maskelyne mentions only a few stars as having proper motions. But M. Bessel, by examining 2959 stars of Piazzi's Catalogue, finds 425 that have an annual motion not less than $0''.2$. As there is no law dependent on the places of these stars regulating their proper motions, so there is no connexion subsisting between the magnitudes of stars and the quantities of such motions. The only circumstance worthy of note seems to be that, amongst the stars apparently endowed with considerable proper motions, there are many double stars. α Cassiopeæ and α Geminorum are two instances, the proper motion

of the first being, in Æ $= 1''.85$, in N. P. D. $= 0''.47$,
of the latter being, in Æ $= 0.58$, in N. P. D. $= 0.64$.

But the stars with the largest proper motions are 40 D of Eridanus and 61 * of Cygnus: that of the former, in north polar distance, being $4''$, of the latter $-3''.3$. So that, according to M. Zach's method of illustrating the subject, if we were to determine the latitude of Greenwich by means both of one star and the other, and the two determinations should exactly agree in 1821, then, in 1822, they would differ by $7''$, if in the process of correcting the observations, we made no account of, or were ignorant of, these proper motions.

The preceding discussions relate to very minute changes in the positions of stars: of which minute changes there are two kinds: one of the points or planes from which a star's place is measured: (as for instance, the changes of intersection of the equator and ecliptic and the plane of the ecliptic from the action of the planets:) the other of the position of the star from some

* 61 Cygni in Flamstead's Catalogue.

unexplained motion of the star itself. The former change, like the other changes from precession and nutation, leaves to the star unimpaired its character of being fixed: the latter takes away from the propriety of that denomination.

But the estimation of these minute changes, whether they be those of the observer, or of the star itself, is, as it must have appeared, a matter of considerable nicety. We must compare present observations made with large instruments with tolerably good catalogues. It is needless to say that, the better the catalogue the more exact will be our operations: but, it is to be noted that, beyond Bradley's time, there are no catalogues of sufficient exactness for determining quantities so small as those of the proper motions of stars. We are thus deprived of the means of remedying, the almost inevitable errors of particular observations, by the comparison of observations distant from each other by very large intervals of time: and, in truth, the fixing of the laws and quantities of those minute variations, which have just been the subjects of discussion, is a point reserved for times to come. It is a matter not to be questioned that our present instruments, and our present means of forming catalogues of stars (which at the epochs of their construction are the most faithful registers of the *mean* places of stars) are better now than they were seventy years ago.

We speak of the catalogues made, for particular epochs, by Flamstead, Bradley, Mayer, &c.: but, in fact, in a modern Observatory the business of making, or of improving, the catalogue of stars is an operation continually going on. We will, for a short time, turn our attention to that point, and first notice the practice of the Greenwich Observatory.

Observations of north polar distances are now made at Greenwich by means of a *Mural Circle*, of which a short description has been given in pages 109, &c. The accuracy of the divisions of the instrument is examined by other means than Astronomical. It is, in fact, presumed to be a perfect instrument: an imperfect instrument, used with a perfect catalogue, (admitting, for an instant, the possibility of the latter circumstance) would necessarily tend to make the catalogue erroneous.

The first operation with the mural circle is, (the telescope occupying that position in which the star was seen bisected by the centre of the cross-wires) to *read off* the six microscopes, and, see pp. 112, 113, to take the mean of these as the instrumental polar distance of the star. This distance is next to be corrected for refraction, precession, aberration, and nutation, and *reduced* to the first day of the year on which the observation was made (see Chapters IX, to XV.) Such reduced distance is the *mean* distance, and it ought, supposing the observation to be truly made, to agree, allowing for the index error, with the tabulated mean polar distance: or, the mean polar distance of the catalogue. If it does not agree, the error is held to be in the catalogue.

Thus, suppose Polaris to have been observed on June 11, 1812, both above and below the pole, and the *reduced* north polar distance above the pole to be .... $1^{\circ}\ 41'\ 55''.98$
.... below.................... $358\ \ 18\ \ 31.79$

then, were it not for the index error, the sum of these ought to equal 360°: if the sum differs, it differs by twice the index error. In the present instance, then, since the sum is

$$360^{\circ}\ 0'\ 27''.77,$$

the index error is $13''.885$: consequently, in order to obtain the true north polar distance above and below the pole, we must diminish the former by $13''.885$, and increase the latter by the same quantity. Reversely, the observed distance minus or plus the true distance must give the index error. Now, if the north polar distance of the catalogue were the true north polar distance, the observed instrumental north polar distance, minus or plus the north polar distance of the catalogue, ought, if the latter were correct, to give the same index error. If it does not, the error is the error of the catalogue. Thus, in the instance before us,

the north polar distance being	$1^{\circ}\ 41'\ 55''.98$
and the index error	$0\ \ 0\ \ 13.885$
the true north polar distance	$1\ \ 41\ \ 42.095$

but, if the north polar distance of the catalogue were $1^{\circ}\ 41'\ 41''.30$ the difference between it and $1^{\circ}\ 41'\ 55''.98$, would be equal

to 14″.68, instead of 13″.885 : the difference, therefore, of these two differences, that is, 14″.68 − 13″.885 = .795, must be the error of the *catalogued* north polar distance. The principle of illustration used in this instance extends to all other like instances. Consider, therefore, the mean north polar distances of the catalogue to be the true mean distances, subtract them from the observed, and the results are the index errors : their mean is the mean index error. Subtract this index error from the observed north polar distance of a circumpolar star above the pole, and add it to the north polar distance of the same star below the pole : the sum, as it has been shewn, ought to equal 360°, if the *catalogued* be the correct distances : if not, the error is that of the catalogue, which, from the defect from 360°, become known. The thing will be made more clear by examples.

Observations made with the mural circle from June 11, to June 18, 1812, the position of the telescope being O, (see p. 115.)

	No. of Obser.	Reduced Observa. by Instruments.	N. P. D. by Catalogue.	Difference.	Diff. × No. of Observations.
Polaris. . . .	2	1° 41′ 55″.98	41″.30	14″.68	29″.36
S.P.	7	358 18 31.79	18.70	13.09	91.63
β Ursæ Min.	6	15 4 46.88	34.23	12.65	75.90
α Cassiopeæ.	1	34 29 54.20	42.48	11.72	11.72
γ Draconis. .	3	38 29 15.88	2.85	13.03	39.09
η Ursæ maj.	6	39 44 52.09	39.61	12.48	74.88
Capella. . . .	1	44 12 37.76	25.03	12.73	12.73
α Cor. Bor. .	5	62 38 55	42.96	12.04	60.20
Arcturus. . .	5	69 50 11.81	0. 0	11.81	59.05
β Leonis. . .	3	74 21 49.18	37.11	12.07	36.21
Regulus . . .	3	77 7 19.26	5.29	13.97	41.91
α Serpentis. .	3	82 58 38.52	27.50	11.02	33.06
Total.	45				sum 565.74

$$\frac{565''.74}{45} = 12''.57$$ equation for north polar distance subtractive.

In the above process, that operation is made with eleven stars which was (see p. 412.) illustrated by means of one. The result is now a mean result. If we went no farther than we are allowed to go by these observations made, during seven days, and on eleven stars, we should have a quantity 12″.57 representing the index error, and which it is necessary to subtract from the observed distances, in order to obtain distances which would be the true distances, were the catalogues correct. The test of that correctness is to be found, as we have already shewn, in the sum of the two polar distances of a circumpolar star. Taking Polaris, then, as such a star, we have

	Observed N. P. D.	Equation to N. P. D.	Corrected N. P. D.
above pole	1° 41′ 55″.98	− 12″.57	1° 41′ 43″.41
below pole	358 18 31 .79	− 12 . 57	358 18 19 . 22
			360 0 2 . 63

The catalogue, therefore, (see p. 413.) cannot be right; the mean subtractive equation of north polar distance ought to be greater than 12″.57 by ½ (2″.63), or 1″.315: but, in order to obtain a greater index error, or subtractive equation of north polar distance, we must lessen the mean north polar distances of the catalogue: consequently, the correction of the catalogue would be − 1″.315: and, that being made, the sum of the two north polar distances would be, as it ought to be, exactly 360°.

But, in a matter of such astronomical importance as the correction of a catalogue, it would be unsafe to trust to the observations of a few stars, made during a short period. If the instrument were, with regard to its divisions, a perfect one, it would still not be exempt from the effects of partial expansion. To annul these effects and those of the inequalities of graduation, (for in degree, at least, such must be supposed to exist) it is necessary to multiply observations and (we are speaking of the Greenwich mural circle) to change the position of the telescope, and to observe the same stars (see p. 413.) when it shall occupy the positions 0°, 30°, &c. Thus, as a specimen of the operations when the two former positions are employed:

1812.	Position.	Equation to N. P. D.	Above Pole uncorrected N. P. D.	Corrected.	Below Pole uncorrected N. P. D.	Corrected.
			° ′ ″	° ′ ″	° ′ ″	° ′ ″
June 11 to 18.	0°	− 12″.57	1 41 55.98	1 41 43.41	358 18 31.79	358 18 19.22
19 to 20.	0	− 10.08	0 0 51.90	0 0 41.82	0 0 29.26	0 0 19.18
21 to July 1.	30	− 23.57	1 42 5.39	0 0 41.82	0 0 41.57	0 0 18.00

Reductions, similar to those indicated in the above schedule, are made of observations for the year 1812. If, in order to obtain greater accuracy, we employ the observations of 1813, we must reduce the mean of the latter to the mean of the former, by adding, to the north polar distance of Polaris, the quantity $\mp 19''.45$, which expresses its annual precession in north polar distance.

In the pages of the Volume of the Greenwich Observations (for 1815) from which the preceding extracts have been made, the observations of 1812 are combined with those of 1813: and

$$\begin{array}{lr} \text{the mean of the N.P.D's of Polaris above the pole} = & 1^\circ\ 41'\ 21''.50 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{ below} \ldots\ldots = & 358\ \ 18\ \ 38.32 \\ \hline & 359\ \ 59\ \ 59.82 \end{array}$$

The mean result, then, of the observations of two years differs from that (see p. 414, l. 13, &c.) of the observations of seven days: and, since the sum of the two north polar distances is less than 360 by $0''.18$, the correction of the catalogue becomes additive and equal to $0''.09$.

By these means a new catalogue is made, which is, by like operations, again to be reformed.

The above method will, generally speaking, render more correct the mean polar distances of stars; it may, indeed, by its peculiar nature, render the mean distances of some less correct than they were before the process of correction. For, in finding the index error, a great variety of stars are used, some of which are considerably distant from the zenith of Greenwich. Now, a previous operation in finding the index error is the reduction of the several observed distances; the accuracy, therefore, of the

index error so to be found, depends on the *reductions* of all the stars being operations of like certainty. One of these reductions is the correction for refraction: which, as Dr. Brinkley has observed in his Memoir (*Irish Transactions*, 1815.) on the parallaxes of stars, is a correction of considerable uncertainty (see pp. 233, &c.) If the correction used for refraction should be wrong, the index error cannot be right. The like may be said of the other corrections. In short the index error, and the consequent correction of the catalogue must, in degree at least, partake of that uncertainty to which the reductions of any of the stars used in finding the index error are liable.

This method, then, of correcting the mean declination of stars requires, as Dr. Brinkley notes, great attention in all enquiries concerning (what indeed modern Astronomy is now conversant about) minute changes in the places of stars.

But the excellent Astronomer, whom we have just quoted, uses a different instrument and method for determining the mean places of stars. The method may indeed be said to be *essentially* different, since it has no concern with the *index error*, which, as we have seen, plays so great a part in the uses of the mural circle. We will now speak briefly of the description of the *Circle* of the Observatory of Trinity College, Dublin, and more fully of its uses and application.

The *Circle* planned and partly executed by Ramsden, is eight feet in diameter. What is peculiar to it, being so large an instrument, is its capability of being turned round a vertical axis: so that the same face of the instrument may be turned both to the east and west. In this principle of its construction it resembles a zenith sector, and those small quadrants and declination circles that are furnished with azimuthal motions (see pp. 65, &c. also *Phil. Trans.* 1806, pp. 406, &c.): and its maker intended that it should derive from that principle, the same advantage which the smaller instruments possess, namely, that of determining the true zenith distance of a star, independently of the line of collimation (see pp. 67, &c.)

A plumb-line is used in the present instrument, not for determining the zenith point on the instrument, but for adjusting

the vertical axis. The divisions of the limb are read off by means of three microscopes: one at the bottom, opposite to the lowest part of the circle: the other two respectively opposite the left and right extremities of the horizontal diameter. This short description is sufficient for our purpose: a fuller description has been given by Dr. Brinkley in the *Irish Transactions* for 1815.

It was the original intention of the maker of the instrument that meridional observations should be made with it. And the instrument can readily be placed in the plane of the meridian: but, in that case, only one observation of the same star can be made with it on the same day. Such (see pp. 67, &c.) is to be reckoned only half an observation: we must wait twenty-four hours at least before we reverse the instrument and complete the observation. If the weather should be unfavourable we may be obliged to wait several days. But, even in the interval of one day, the temperature may have altered and affected the instrument. To prevent this evil, or to obviate the objection that may be founded on its supposed existence, Dr. Brinkley observes the star with the face of the instrument to the east, once or twice before it reaches the meridian, and then, as often, with the face of the instrument to the west, after the star has passed the meridian. Thus the two essential parts of an observation are made within the space of ten or twelve minutes. But the observations thus made are, in a certain sense, imperfect ones, since they are not observations of meridional zenith distances. They may, however, by the aid of calculation, be made to become, or be *reduced* to, such observations. The main condition necessary to be known is the time of the observation, or, rather, the interval of time between the observation of the zenith distance and the star's transit over the meridian. This is easily had in an Observatory. What else remains is a matter altogether of calculation, which, as on like occasions, will furnish us with a formula and rule of solution.

We will now direct our attention to the formula, which is to express the difference of the meridional zenith distance of a star, and of its zenith distance observed very near to the

meridian, in terms of the interval between the times of observation and the star's transit, and of certain given quantities.

Let L denote the latitude of the place of observation,

D the star's polar distance,

z, z', two zenith distances,

h, h', the corresponding hour-angles;

then, if we form two spherical triangles ZPs, ZPs', Z being the zenith, P the pole, and s, s' two positions of the star, we have (see *Trigonometry*, Chap. IX.)

$$\cos. h = \frac{\cos. z - \sin. L . \cos. D}{\cos. L . \sin. D},$$

$$\cos. h' = \frac{\cos. z' - \sin. L . \cos. D}{\cos. L . \sin. D};$$

consequently, $\cos. h' - \cos. h = \dfrac{\cos. z' - \cos. z}{\cos. L . \sin. D}$.

or (see *Trigonometry*, p. 33.)

$$\sin. \frac{h'+h}{2} . \sin. \frac{h'-h}{2} = \sin. \frac{z'+z}{2} . \sin. \frac{z'-z}{2} \times \frac{1}{\cos. L . \sin. D}.$$

Let one observation (that to which h, z belong) be made on the meridian, then, since $h = 0$,

$$\sin.^2 \frac{h'}{2} = \sin. \frac{z'+z}{2} . \sin. \frac{z'-z}{2} \times \frac{1}{\cos L . \sin. D.}.$$

Now z', z are nearly equal: let δ denote their difference, then

$$\frac{z'+z}{2} = z + \frac{\delta}{2},$$

$$\frac{z'-z}{2} = \frac{\delta}{2};$$

$$\therefore \sin. \left(z + \frac{\delta}{2}\right) . \sin. \frac{\delta}{2} = \cos. L . \sin. D . \sin.^2 \frac{h'}{2},$$

but, δ being very small,

$$\sin. \frac{\delta}{2} = \frac{\delta}{2}. \sin. 1'', \text{ nearly},$$

$$\sin. \left(z + \frac{\delta}{2}\right) = \sin. z + \frac{\delta}{2} \sin. 1''. \cos. z;$$

$$\therefore \frac{\delta}{2} \sin. 1'' \times \left\{\sin. z + \frac{\delta}{2} \sin. 1''. \cos. z\right\} = \cos. L. \sin. D. \sin.^2 \frac{h'}{2},$$

$$\text{and } \frac{\delta}{2} = \sin.^2 \frac{h'}{2}. \frac{\cos. L. \sin. D}{\sin. 1'' \left(\sin. z + \frac{\delta}{2} \sin. 1''. \cos. z\right)}$$

$$= \sin.^2 \frac{h'}{2}. \frac{\cos. L. \sin. D}{\sin. 1''. \sin. z}. \left\{1 - \frac{\delta}{2} \sin. 1''. \cot. z\right\}, \text{ nearly.}$$

If no great accuracy be required we may reject the second term, in which case, we have

$$\frac{\delta}{2} = \sin.^2 \frac{h'}{2}. \frac{\cos. L. \sin. D}{\sin. 1''. \sin. z}.$$

Substitute this value in the preceding expression, and we shall have a second approximate value of $\frac{\delta}{2}$, in the expression

$$\frac{\delta}{2} = \sin^2. \frac{h'}{2}. \frac{\cos. L. \sin. D}{\sin. 1''. \sin. z} - \frac{\sin.^4 \frac{h'}{2}}{\sin. 1''}. \left(\frac{\cos. L. \sin. D}{\sin. z}\right)^2. \cot. z;$$

which, since $z = L - 90° - D$, is Delambre's expression, and from which the correction δ, or *the reduction to the meridian*, may be computed.

Dr. Brinkley, however, very rightly prefers another formula (or rather a transformation of the above formula) in which the arc $\frac{h'}{2}$ and its powers, should be involved instead of the powers of its sine. Thus, since

$$\sin. \frac{h'}{2} = \frac{h'}{2} \sin. 1'' - \frac{1}{6} \left(\frac{h'}{2}\right)^3. \sin.^3 1'', \text{ nearly},$$

$$\sin.^2 \frac{h'}{2} = \left(\frac{h'}{2}\right)^2 \sin.^2 1'' - \left(\frac{h'}{2}\right)^4. \frac{\sin.^4 1''}{12}, \text{ nearly};$$

writing, therefore, in the above formula, cosec. z instead of $\frac{1}{\sin. z}$, and $15h'$ instead of h', in order to convert h', expressed in parts of space; into time,

$$\delta = \frac{\sin. 1''}{2} . 15^2 . \cos. L . \sin. D . \text{cosec.}\, z \times (h')^2$$

$$- \frac{\sin.^3 1''}{24} . 15^4 . \cos. L . \sin. D . \text{cosec.}\, z \times (h')^4$$

$$- \frac{\sin.^3 1''}{8} . 15^4 . (\cos. L . \sin. D . \text{cosec.}\, z)^2 . \cot. z \times (h')^4,$$

from which δ may be computed. We may, however, for the purposes of computation, express δ more commodiously.

Let the first term (C), the first correction, $= A \sin. D . \text{cosec.}\, z . (h')^2$

then the 2d term, or cor^n. $(C') = - \frac{\sin.^2 1''}{12} . 15^2 . A \sin. D . \text{cosec.}\, z . (h')^4$

and the third term, or correction $(C'') = - C^2 . \frac{\sin. 1''}{2} . \cot. z.$

Hence, since $A = \frac{\sin. 1''}{2} . 15^2 . \cos. L$, we have

$\log. A = \log. \sin. 1'' + 2 \log. 15 + \log. \cos. L + \text{ar. com.}\, 2 - 20,$

$\log. C = \log. A + \log. \sin. D + \log. \text{cosec.}\, z - 20 + 2 \log. h',$

$\log. C' = \log. A + 2 \log. \sin. 1'' + 2 \log. 15 + \text{ar. com.}\, 12$

$\qquad + \log. \sin. D + \log. \text{cosec.}\, z - 30 + 4 \log. h',$

$\log. C'' = 2 \log. C + \log. \sin. 1'' + \text{ar. com.}\, 2 + \log. \cot. z - 20.$

When the observations are made at the same place, the log. cos. L, which is a given quantity, may be added to the other constant quantities: for instance, the latitude of the Dublin Observatory being 53° 23′ 13″.5, its log. cos. = 9.77552, which being combined with the logarithms of the three first terms of the expression for log. C, the result is 16.51225. Hence

$$\log. C = 6.51225 + \log. \sin. D + \log. \operatorname{cosec}. z - 20 + 2 \log. h',$$
$$\log. C' = 7.15638 + \log. \sin. D + \log. \operatorname{cosec}. z - 20 + 4 \log. h',$$
$$\log. C'' = 4.38454 + 2 \log. C + \log. \cot. z - 10.$$

These are the formulæ of computation for any star, the latitude of the place being equal to that of the Observatory of Trinity College, Dublin. But, as it is necessary to make a great number of observations of the same star, it is convenient to possess peculiar formulæ of computation adapted to the several stars. For instance, if the star should be Arcturus, the sum of the second and third terms of log. C is a constant quantity ($=20.23344$); the third term of log. C'' is also a constant quantity ($=10.1831$): and accordingly, the three formulæ for Arcturus, observed at the Observatory of Trinity College, Dublin, become

$$\dagger \log. C = 6.74569 + 2 \log. h',$$
$$\log. C' = 7.3898 + 4 \log. h',$$
$$\log. C'' = 8.05902 + 4 \log. h'.$$

Above the pole $z' > z$, and $z' - z = \delta$; $\therefore z = z' - \delta$. Hence

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Constant Number in log. C computed.

* Log. cos. lat.	9.77554
log. sin. 1″	4.68557
2 log. 15	2.35218
Arith. comp. 2	9.69896
(log. A)	26.51225

Constant Number in log. C' computed.

log. A	6.51225
2 log. sin. 1″	9.37114
2 log. 15	2.35218
ar. comp. 12	8.92081
	7.15638

Constant Number in log. C'' computed.

log. sin. 1″ .... =	4.68557
log. 2 .... =	.30103
	4.38454

† Log. sin. N. P. D. ($=69° 52′ 46″$) 9.97265
log. cosec. z ($=33\ 16$) 10.26079

	20.23344	.23344
	6.51225	7.15638
	6.74569	7.38982
	2	
	3.49138	
	4.38454	
	1831	
	8.05902	

(see p. 418,) in order to reduce the observations, we have this formula,

meridional zen. dist. = observed zen. dist. $-\ (C - C' - C'')$,

and below the pole

meridional zen. dist. = observed zen. dist. $+\ C - C' + C''$.

The following instance of the star Arcturus observed, May 12, 1820, at the Dublin Observatory, contains the application of the preceding formulæ,

Latitude of the Observatory	$53^{0}\ 23'\ 13''.46$
mean N. P. D. of Arcturus for 1820	69 52 31.89
mean Æ	211 51 51. 6
place of Moon's node ....11^{s}	29 26 0

Time by Clock.	Left Micros.	Bottom Microscopes.	Right Micros.	Mean of the three Microscopes.	Refraction.
$13^{h}\ 56^{m}\ 28^{s}$	$49''.7$	$33^{0}\ 19'\ 50''.5$ E	$4''.3$	$33^{\circ}\ 19'\ 54''.83$	$37''.82$
14 0 28	31.7	33 17 32.6 E	47.1	0 17 37.13	37.77
14 9 51	50.6	33 14 54.5 W	45.0	0 14 50.03	37.74
14 14 52	38.0	33 16 41.0 W	31.7	0 16 36.90	37.77

Barometer 29.67. Thermometer Int. 52.5. Thermometer Ext. 48.

Time of ✱'s passage by clock	$14^h\ 7^m\ 3^s.3$	$14^h\ 7^m\ 3^s.3$	$14^h\ 7^m\ 3^s.3$	$14^h\ 7^m\ 3^s.3$
time of obser^n.	13 56 28	14 0 28	14 9 51	14 14 52
values of h' . . .	0 10 35.3	0 6 35.3	0 2 47.7	0 7 48.7
values of h' in seconds	635.3	395.3	167.7	468.7
logarithms of h'	2.80298	2.59693	2.22453	2.67089
2 logarithms of h'	5.60596	5.19386	4.44906	5.34178
constant quantity	6.74569	6.74569	6.74569	6.74569
log. C	2.35165	1.93955	1.19475	2.08747
C	224.72	87.008	15.66	122.31
4 log. h'	1.21192			
	7.3898			
log C'	8.60172			
C'	0.0399			
4 log. h'	1.21192	. . . .	. . . .	0.68356
constant quantity	8.05902	. . . .	. . . .	8.05902
log. C''	9.27094			8.74258
C''	0.186			0.055
C'	0.0399			
C	224.72			122.31
$(C - C' - C'')$ . .	3′ 44″.49	87″.008	15″.66	2′ 2″.25
refraction	37.82	37.77	37.74	37.77
	3 6.67	49.23	22.08	1 24.54
mean of 3 mic^s.	33 19 54.83 E	17 37.3 E	50.03	16 36.90
	33 16 48.16 E	16 48.07	12.11	15 12.36 W
	48.07		12.36	
	33 16 48.11 E		33 15 12.23 W.	
	33 15 12.23 W			
	33 16 0.17			
aber. prec^n. nut. . .	13.53			

33 15 46.64 mean zen. dist. *January*, 1820.

The above is the whole of the process necessary for reducing each of the four observed zenith distances to the meridional zenith distance. In the left hand column, since the interval (h') between the time of observation and the transit was $10^m\ 35^s$, all the three corrections C, C', C'', were computed: but in the second and third columns, when the values of h' are only $6^m\ 35^s$, $2^m\ 47^s$, the

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* The value of h' is made the difference between the times of observation by the clock and of the star's transit by the clock. The most ready way of determining it, is to observe the star's transit by the transit instrument, and to note its time by the clock. The difference of that time and of the time of observation by the same clock is the value of h'. If it be not convenient to observe the star's passage, we must compute its Æ and thence, and from the error and rate of the clock, compute h'.

The special object of the example in the text is the illustration of the method of finding the meridional zenith distance of a star by means of zenith distances observed before and after the star's transit. But the example may be made to serve another purpose: it is a kind of practical proof that the duties of an Observatory, are laborious duties. The computation, as it stands in the text, is a long one; yet the whole of it is not given: for instance, the computations of the four refractions and of the inequalities of aberration, nutation, and precession are omitted. Again, we have considered only one star: but, if ten or more stars be observed, they will all require reductions similar to the preceding reduction. Observations, then, by means of a circle, such as we have been speaking of, and so used, are considerably more operose than those made by a mural circle or quadrant.

The above method of deducing zenith distance is peculiar to the Observatory of Trinity College, Dublin. It renders the duties more laborious than when the meridional zenith distance is observed by means of mural quadrants or circles. The other parts of the daily business in an Observatory, the observations and computations of right ascensions, occultations, eclipses of satellites, are nearly the same at Greenwich, Paris, and Dublin. Bradley's theories, and instruments like Bird's, make one Observer quite unequal to the proper discharge of the duties of an Observatory.

We

values of C', C'' (they are to the values first obtained nearly as the squares of the times) are too small to be taken account of. In the fourth column C' is too inconsiderable to be computed.

The catalogues of mean right ascensions stand, also, in need of continual corrections: and such corrections are effected on grounds not altogether unlike those that have been used in correcting north polar distances.

We have endeavoured to draw the attention of the reader to the respective constructions and uses of the two instruments of Greenwich and Dublin for measuring north polar distances and zenith distances. The former, as we have seen, does not determine the north polar distance of a star, except by the intervention of several other stars, used for determining the index error. The latter is capable of determining the zenith distance of a star, if such star should be a solitary one in the heavens. It is capable of determining, within so short an interval as fifteen minutes, the zenith distance of a star: and, on that account, is admirably adapted to note (should there be any such) the peculiar motions of a star. On the mere footing of theory, no instrument is better adapted for discovering (should it be capable of being discovered) parallax. But various objections are made to it. 1stly, The great mass of metal that forms the frame of the instrument, and revolves with it, and likely, from its derangements from heat, &c. to derange the instrument: 2dly, the unfixedness of its microscropes, and their position: 3dly, the uncertainty of the permanence of position of the plumb-line, by which, at each observation, the instrument is adjusted. These objections are certainly deserving of attention, and ought, (as much as they can be,) to be examined by the observations which the instrument itself furnishes, in the same way as the recorded observations by the mural circle are perhaps sufficient to determine its precision in settling the mean place of any proposed star.

In the mural circle the derangements from unequal temperature can arise only from the circle itself being affected. Its six microscopes are fixed: the position of its telescope may be varied. These are its great excellencies. The accuracy of its division (of which, however, the Astronomer Royal has given the fullest testimony) is without the present question, which regards not an individual instrument, but a class of instruments and the principles of their construction.

With

The method of finding the right ascensions of stars has been explained in Chap. VII. of this Treatise. It depends on the finding the time of the Sun's entering the equator. The same process determines the right ascensions of the Sun and of the stars employed in that process. If by that, and like processes, we obtain, for a certain epoch, a correct catalogue of the mean right ascensions of stars, we are able, by a knowledge of the several inequalities to which the places of stars are subject, to determine their right ascensions for any other epoch, and thence to regulate the Astronomical Clock. We could thence determine the right ascension of the Sun. If, therefore, by any means, other than those of the right ascensions of stars, we are able to determine, at the latter epoch, the Sun's right ascension, such determination, compared with the former, would be a test of its accuracy; and, consequently, of the computed right ascensions of stars. Now we possess such means of determining the Sun's right ascension in a knowledge of his declination, which can be observed, and of the obliquity of the ecliptic which can be computed. An instance has been given of this method in pages 151, &c. The Sun's right ascension, then, is when he is near to the equinoxes, to be computed from the clock; in other words, from the right ascensions of stars, and from his observed north polar distances. The differences of the two results, then, would (supposing the latter computations to be exact) be the errors of the catalogue. The following Table will exemplify the method.

With regard to the mural circle, it ought farther to be noted, that, although it uses no plumb-line, and cannot be reversed as the Dublin circle can, it is not destitute of the means of determining the zenith point. Such means are found in an artificial horizon (a basin of oil or of quicksilver); and then certain stars are observed both by reflection and by immediate vision. This operation is now practised at the Greenwich Observatory.

Observations of the SUN *about the* EQUINOXES.

1812.	Sun's R. A. by Stars.	Sun's R. A. by Dec.		1817.	Sun's R. A. by Stars.	Sun's R. A. by Dec.	Diff.	Sum.
	° ′ ″	′ ″	″		° ′ ″	′ ″	″	″
Aug. 8,	138 11 41.1	11 50.0	+ 8.9	May 5,	42 6 41.8	6 34.2	− 7.6	+ 1.3
13,	142 55 56.4	55 49.6	− 6.8	Apʳ. 30,	37 19 53.2	20 7.8	+14.6	+ 7.8
21,	150 23 50.7	24 3.2	+12.5	22,	29 47 31.0	47 45.0	+14.0	+26.5
26,	155 0 5.5	59 57.5	− 8.0	16,	24 12 52.8	13 6.8	+14.0	+ 6.0
Sept. 2,	161 22 40.9	22 43.1	+ 2.2	10,	18 41 35.2	41 48.5	+13.3	+15.5
4,	163 11 21.0	11 19.6	− 1.4	8,	16 51 45.6	52 6.7	+21.1	+19.7
5,	164 5 27.9	5 28.2	+ 0.3	7,	15 57 0.0	57 8.3	+ 8.3	+ 8.6
12,	170 23 32.1	23 23.3	− 8.8	Mar. 31,	9 34 40.8	34 45.3	+ 4.5	− 4.3
14,	172 11 17.8	11 11.0	− 6.8	30,	8 40 10.0	40 22.0	+12.0	+ 5.2
20,	177 34 33.1	34 33.2	+ 0.1	23,	2 18 54.0	19 4.8	+10.8	+10.7
23,	180 16 19.9	16 23.4	+ 3.5	20,	359 35 10.8	35 23.4	+12.6	+16.1
25,	182 4 36.7	4 25.6	−11.1	18,	357 45 53.1	45 59.8	+ 6.7	− 4.4
Oct. 15,	200 20 15.7	19 57.0	−18.7	Feb. 26,	339 17 14.2	17 31.4	+17.2	− 1.5
26,	210 44 47.0	44 40.2	− 6.8	16,	329 43 58.5	44 19.3	+20.8	+14.8
27,	211 42 37.5	42 30.0	− 7.5	15,	328 45 43.7	46 3.3	+19.6	+12.1
Nov. 3,	218 32 24.3	32 7.4	−16.9	8,	321 52 36.4	52 45.5	+ 9.1	− 7.8
7,	222 31 3.4	30 52.0	−11.4	5,	318 52 49.7	53 1.5	+11.8	+ 0.4
10,	225 32 14.7	32 2.5	−12.2	1,	314 49 56.1	50 17.3	+21.2	+ 9.0
11,	226 33 1.8	32 37.0	−24.8	Jan. 31,	313 48 53.5	49 12.0	+18.5	− 6.3

The sum of the positive results in the last column is 153″.7 : of the negative 24″.3, their difference, therefore, is 129″.4. There are thirty-eight comparisons, and $\frac{129''.4}{38}$: which, in time, is $0^s.28$, is the correction of the catalogue: or, is the common quantity by which the mean right ascensions of the catalogue are to be increased.

By a repetition of such processes and the use of improved instruments, the catalogues of the right ascensions will continue to be improved.

We have now given, not enough indeed for all purposes, but sufficiently for the plan of the Treatise, the theory of the *fixed* stars. It, we may say, necessarily precedes that of *wandering stars* or *planets*. The right ascensions and declinations of the

fixed stars are used in determining those of the planets. They have many things in common. Both are subject to the inequalities of refraction, aberration, precession and nutation. But, without going far into the circumstances of distinction between the fixed stars and the planets, it is obvious that there must be peculiarities belonging to the latter from their relative proximity to the observer, and their continual change of place. The former circumstance renders them subject to parallax, and the latter (to mention one instance) modifies the quantity of aberration: for, in the time of the transmission of light from the planet to the Earth, the former has changed its place.

But these are only slight circumstances of distinction. The planets' distances, velocities, the forms of their orbits must be investigated: subjects of enquiry to which there is nothing like in the preceding discussions, and depending on principles not yet laid down. Our attention will be directed to these points in the succeeding part of this Volume: and as, amongst the planets, the Earth claims the chief consideration, its theory shall be first discussed.

END OF THE FIRST PART OF THE FIRST VOLUME.

A TREATISE ON ASTRONOMY

THEORETICAL AND PRACTICAL.

BY

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PART II. VOL. I.

CONTAINING THE

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CHAP. XVII.

ON THE SOLAR THEORY.

Inequable Motions of the Sun in Right Ascension and Longitude.—The Obliquity of the Ecliptic determined from Observations made near to the Solstices.—The Reduction of Zenith Distances near to the Solstices, to the Solstitial Zenith Distance.—Formula of such Reduction.—Its Application.—Investigation of the Form of the Solar Orbit.—Kepler's Discoveries.—The Computation of the relative Values of the Sun's Distances and of the Angles described round the Earth.—The Solar Orbit an Ellipse.—The Objects of the Elliptical Theory.

In giving a denomination to the preceding part of this Volume, we have stated it to contain the Theories of the fixed Stars; such theories are, indeed, its essential subjects; but they are not exclusively so. In several parts we have been obliged to encroach on, or to borrow from, *the Solar Theory;* and, in so doing, have been obliged to establish certain points in that theory, or to act as if they had been established.

To go no farther than the terms Right Ascension, Latitude, and Longitude. The right ascension of a star is measured from the *first point of Aries,* which is the technical denomination of the intersection of the equator and ecliptic, the latter term designating the plane of the *Sun's orbit:* the latitude of a star is its angular distance from the last mentioned plane; and the longitude of a star is its distance from the first point of Aries measured along the ecliptic.

The fact then is plain, that the theories of the fixed stars have not been laid down independently of other theories: and it is scarcely worth the while to consider whether or not, for the

sake of a purer arrangement, it would have been better to have postponed certain parts of their theories till the theory of the Sun's orbit, and of his motion therein, should have been estalished.

According to our present plan, indeed, (a plan almost always adopted by Astronomical writers) we shall be obliged to go over ground already trodden on. But we shall go over it more carefully and particularly. In those parts of the solar theory which it was necessary to introduce, either for the convenient or the perspicuous treating of the sidereal, we went little beyond approximate results and the description of general methods. For instance, in pages 137, 138, it is directed, and rightly, to find the obliquity of the ecliptic from the greatest northern and southern declinations of the Sun. But the practical method of finding such extreme declinations was not there laid down; and on that, as on other occasions, much detail, essentially necessary indeed, but which would then have embarrassed the investigation, was, for the time, suppressed.

Such detail is now to be given together with other methods, that belong to the solar theory. But it may be right, previously to enumerate some of the results already arrived at.

In Chapter VI, which was on the Sun's Motion and its Path, it was shewn that the Sun possessed a peculiar motion tending, in its general description, from the west towards the east, almost always oblique to the equator, and inequable in its quantity. These results followed, almost immediately, from certain meridional observations made with the transit instrument and mural circle.

By such observations two motions or changes of the Sun's place are determined; one in the direction of the meridian, the other in a direction perpendicular to the meridian. The oblique motion of the Sun, therefore, is, in strictness, merely an inference from the two former motions: or, if we suppose the real to be an oblique motion, its two resolved parts will be those which the transit instrument and mural circle discover to us; neither of which motions (see p. 126.) is an equable one.

But although the two resolved motions are inequable, it does not at once follow that the oblique or compounded motion must be inequable. For, if it were equable, the resolved parts, namely, the motion in right ascension, and the motion in declination, would be inequable. Some computation, therefore, is necessary to settle this point, and a very slight one is sufficient.

Thus, by observations made in 1817,

July 1, ☉'s Æ	$6^h\ 40^m\ 1^s.7$	Decl.	$23^\circ\ 8'\ 44''$ N.
2,	6 44 9.7		23 4 35
Jan. 1,	18 17 2.2		23 1 26 S.
2,	18 51 27.0		22 56 14

Compute the longitudes of the Sun by means of this formula,

1 × sin. ☉'s long. = cos. ☉'s dec. × cos. ☉'s Æ,

and we have

		Difference.
July 1, ☉'s long.	$3^s\ 9^\circ\ 11'\ 39''$	$0^\circ\ 57'\ 11''$,
2,	3 10 8 50	
Jan. 1,	9 10 48 38	1 1 10.
2,	9 11 49 48	

The oblique daily motions then, instead of being equal, are to one another as 3431 to 3670.

Besides the results relating to the Sun's path and motion, there were obtained, in Chapter VI, other results, such as the obliquity of the ecliptic, and the times of the Sun entering the equator and arriving at the solstitial points. But the methods by which the results were obtained require revision, or rather we should say, that these methods having answered their end, namely, that of forwarding us in the investigation which we were then pursuing, may now be dismissed, and make way for real practical methods.

We will turn our attention, in the first place, to the determination of the obliquity of the ecliptic.

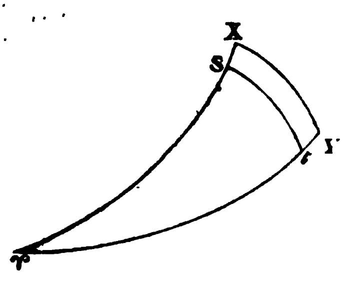
If at an Observatory, the Sun arrived at the solstice exactly when he was on the meridian, the observed declination would

be the measure of the obliquity. But it is highly improbable that such a case should happen : nor is it, indeed, on the grounds of astronomical utility, much to be desired. A solitary observation, under the above-mentioned predicament, would not be sufficient to establish satisfactorily so important an element as that of the obliquity. It would be necessary to combine with it other observations of the Sun's declination, made on several days before and after the day of the greatest declination, to *reduce*, by computation, such less declinations to the greatest, and then to take their *mean* to represent the value of the obliquity. In such a procedure, it is clearly of little or no consequence, whether the middle declination be itself exactly the greatest, or whether, like the declinations on each side of it, it requires to be similarly reduced to the greatest.

The *reduction* of declinations to the greatest, which is the solstitial declination, is an operation of the same nature, and founded on the same principles, as the reduction of zenith distances observed *out* of the meridian to the meridional zenith distance: the formulæ of which latter reduction, together with their demonstration, were given in pages 418, &c. It is convenient, however, on the present occasion, to modify the result of that demonstration, or to express it by a different formula: which we will now proceed to do.

Let then,

d $(= St)$ be the Sun's declination, d' $(= XY)$ the solstitial,



$\odot$ $(= ♈ S)$, $\odot'$ $(= 90^0)$ the corresponding longitudes,

w, the obliquity of the ecliptic,

then, by Naper's Rules, we have

$$\sin. d = \sin. \odot . \sin. w,$$

$$\sin. d' = \sin. \odot' . \sin. w;$$

consequently,

$$\sin. d' - \sin. d = \sin. w\,(\sin. 90^0 - \sin. \odot),$$

or (see *Trigonometry*, pp. 32, 42),

$$\sin. \frac{w-d}{2} . \cos. \frac{w+d}{2} = \sin. w . \sin.^2 \frac{u}{2}, \text{ if } u = 90^0 - \odot.$$

Let $w (= d') = d + \delta$, then,

$$\sin. \frac{\delta}{2} . \cos. \left(w - \frac{\delta}{2}\right) = \sin. w . \sin.^2 \frac{u}{2},$$

$$\text{and, } \sin. \frac{\delta}{2} \left\{\cos. w \cos. \frac{\delta}{2} + \sin. w \sin. \frac{\delta}{2}\right\} = \sin. w . \sin.^2 \frac{u}{2}.$$

Substitute, instead of $\sin. \frac{\delta}{2}$, $\cos. \frac{\delta}{2}$, and $\frac{\delta}{2} - \frac{\delta^3}{48}$, and $1 - \frac{\delta^2}{8}$, respectively, (suppressing for the present $\sin. 1''$, $\sin.^3 1''$, $\sin.^2 1''$, by which δ, δ^3, δ^2, ought, respectively, to be multiplied), and we shall have this approximate expression,

$$\left(\frac{\delta}{2} - \frac{\delta^3}{48}\right)\left\{\cos. w - \cos. w \frac{\delta^2}{8} + \sin. w \frac{\delta}{2}\right\} = \sin. w . \sin.^2 \frac{u}{2},$$

$$\text{whence } \frac{\delta}{2} - \frac{\delta^3}{48} = \frac{\tan. w . \sin.^2 \frac{u}{2}}{1 - \frac{\delta^2}{8} + \frac{\delta}{2} \tan. w};$$

or, nearly,

$$\frac{\delta}{2} = \tan. w . \sin.^2 \frac{u}{2} \left\{1 - \frac{\delta}{2} \tan. w + \frac{1}{2} . \frac{\delta}{2} \frac{\delta}{2} + \frac{\delta}{2} . \frac{\delta}{2} \tan.^2 w\right\}$$
$$+ \frac{1}{6} . \frac{\delta}{2} . \frac{\delta}{2} . \frac{\delta}{2},$$

from which expression, approximate values of $\frac{\delta}{2}$, of sufficient

exactness may be obtained: for instance, to obtain a first approximation, neglect the terms on the right hand side of the equation, that involve δ, and

(1st value) $\frac{\delta}{2} = \tan. w . \sin.^2 \frac{u}{2}$.

Again, retain the terms involving δ and neglect those involving δ^2, and

$$\text{(2nd value)* } \frac{\delta}{2} = \tan. w . \sin.^2 \frac{u}{2} \left\{1 - \tan. w . \sin.^2 \frac{u}{2} . \tan. w\right\}$$

$$= \tan. w . \sin.^2 \frac{u}{2} - \tan.^3 w . \sin.^4 \frac{u}{2}.$$

Again, substitute this new value, and neglect those terms that involve higher dimensions of $\sin. \frac{u}{2}$ than the 6th, and

$$\frac{\delta}{2} = \tan. w . \sin.^2 \frac{u}{2} - \tan.^3 w . \sin.^4 \frac{u}{2}$$

$$+ 2 \tan.^5 w . \sin.^6 \frac{u}{2} + \frac{1}{6} \tan.^3 w . \sin.^6 \frac{u}{2}.$$

$$\text{But } \sin. \frac{u}{2} = \frac{u}{2} - \frac{1}{2.3} . \frac{u^3}{8} + \frac{1}{2.3.4.5} . \frac{u^5}{32} + \&c.$$

From this value find $\sin.^2 \frac{u}{2}$, $\sin.^4 \frac{u}{2}$, &c. and substitute in the preceding expression, and then

$$\dagger\ \delta = \tan. w . \frac{u^2}{2} - \frac{1}{24} . \tan. w (1 + 3 \tan.^2 w) u^4$$

$$+ \frac{1}{720} . \tan. w (1 + 30 \tan.^2 w + 45 \tan.^4 w) u^6,$$

which is sufficiently exact for all practical purposes, since u rarely exceeds 10^0.

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* This is the expression which Biot uses in his *Astronomie*, pp. 31, 336.

† This is the same expression as Delambre's, Tom. II. p. 244, but is differently obtained.

For the purpose of avoiding multiplicity of symbols, the powers of sin. $1''$ (see p. 5. l. 12, &c.) were omitted in the preceding investigation. These, however, must be restored in order to render the above expression for δ fit for application. This is easily effected: δ being very small, δ has been written instead of sin. δ: whereas δ . sin. $1''$ should have been written; on the right hand of the side, instead of u^2, u^4, u^6, &c. $u^2 . \sin.^2 1''$, $u^4 . \sin.^4 1''$, $u^6 . \sin.^6 1''$, &c. should have been written: supplying then the omitted symbols, and dividing each side of the equation by sin. $1''$, we have

$$\delta = \frac{\tan. w}{2} . u^2 . \sin. 1'' - \frac{\tan. w}{24}(1 + 3 \tan.^2 w) u^4 . \sin.^3 1''$$

$$+ \frac{\tan. w}{720}(1 + 30 \tan.^2 . w + 45 \tan.^4 w) u^6 . \sin.^5 1'';$$

u is the difference betwen 90^0 (the longitude of the Sun at the solstice), and the Sun's longitude at the time of observation. If the place of observation be Greenwich, u is known by the Nautical Almanack, and from the value therein given, may easily be computed for any other place of observation. Suppose, for instance, the Sun's meridional distance either from the north pole, or from the zenith to have been observed at Greenwich, on June 18, 1812. By the Nautical Almanack,

$$\odot = 2^s\ 27^0\ 0'\ 4''; \therefore u = 2^0\ 59'\ 56'' = 10796''.$$

In this case the reduction to the solstice (δ) will be expressed with sufficient exactness by the first term. w, then, being taken $= 23^0\ 27'\ 54''$, we have

$$\delta \doteq \frac{1}{2} \tan. 23^0\ 27'\ 54'' . \sin. 1'' \times (10796)^2 = 2'\ 2''.6\ *.$$

---

* Computation.

Log. tan. $23^\circ\ 27'\ 54''$	$= 9.6375760$
arith. comp. of 2	$= 9.6989699$
log. sin. $1''$	$= 4.6855749$
2 log. 10796	$= 8.0665258$
	$2.0886466 = \log. 122''.64.$

If, therefore, the observed meridional zenith distance of the Sun's centre (after being corrected for refraction), were, on the noon of June 18, equal to $28^0\ 3'\ 2''.5$ the reduced zenith *solstitial* distance would be, nearly,

$$28^\circ\ 3'\ 2''.5 - 2'\ 2''.6, \text{ or } 28^\circ\ 0'\ 59''.9.$$

This is an application of the formula to one instance: and like applications to other instances are easily made; with greater length of computation, indeed, if the Sun should be so far from the solstice, as to render it necessary, by reason of the magnitude of u, to compute the second and third terms of the value of δ. Now the obliquity of the ecliptic being an element of great astronomical importance, the finding it by means of the *reduction* is a frequent operation. It becomes worth the while, then, to construct a Table from the preceding expression, and for every ten minutes of the Sun's distance from the solstice. To obtain this latter end, instead of u write $10'\ u = 600''\ u$, and

$$\delta = \frac{\tan.\ w}{2}.\ \sin.\ 1''.\ (600)^2.\ u^2$$

$$-\frac{\tan.\ w}{24}.\ (1 + 3\ \tan.^2 w)\ \sin.^3\ 1''.\ (600)^4\ u^4 + \&c.$$

or, the value of the obliquity being assumed equal to $23^0\ 27'\ 54''$,

$$\delta = 0''.378812\ u^2 - 0''.0000004181\ u^4 + 0''.00000000000006217\ u^6.$$

From this expression a Table may be expeditiously constructed. The values of δ, most easily obtained, are those which belong to u, when its values are, respectively, 1, 2, 3, 4, &c. 10, 20, 30, 100, &c. that is, since the value of the unit of u is $10'$, when the distances from the solstice are $10'$, $20'$, $30'$, $40'$, &c. $1^\circ\ 40'$, $3^\circ\ 20'$, 5°, &c. $16^0\ 40'$, &c.

For instance,

Distance from Solst.	Values of u.	
$0^0\ 10'$	1	$\delta = 0''.3788 \ldots\ldots\ldots\ldots\ldots\ 0^0\ 0'\ 0''.3788$
20	2	$\delta = 0\ .3788 \times 4 \ldots\ldots\ldots\ 0\ \ 0\ \ 1\ .515$
30	3	$\delta = 0\ .3788 \times 9 \ldots\ldots\ldots\ 0\ \ 0\ \ 3\ .409$
&c.		
1 40	10	$\delta = 37''.881 - .00418 \ldots\ 0\ \ 0\ 37\ .876$
3 20	20	$\delta = 151\ .524 - .0668 \ldots\ 0\ \ 2\ 31\ .457$
16 40	100	$\delta = 3788\ .\ 12 - 41.81 + .6217\ 1\ \ 2\ 26\ .\ 93$

This is a sample of a Table, to be constructed from the preceding expression. M. Delambre has given such a Table in p. 269, of the second Volume of his Astronomy. In that Table the expressed numerical values of δ belong to an obliquity $= 23° 28'$.

Our values belong to an obliquity $= 23° 27' 54''$, and, therefore, are somewhat smaller, as they need must be, than Delambre's. But a very slight correction will reduce one set of values to the other. And M. Delambre's Table furnishes the means of effecting this: since it contains, in a separate column, a series of corrections due to a variation of $100''$ in the obliquity, and corresponding to the several values of u.

In order to obtain the algebraical expression of the correction just mentioned we must resume the original value of δ, or, which will be sufficient for the occasion, express it by its first term: now, if

$$\delta = \frac{1}{2} \tan. w . \sin. 1''. u^2,$$

$$\dot{\delta} = \frac{\dot{w}}{2} . \sin.^2 1''. \sec.^2 w . u^2,$$

$\dot{\delta}, \dot{w}$, expressing the corresponding variations of δ and w.

If $\dot{w} = 100''$,

$$\dot{\delta} = .000000001396\, u^2, \text{ the unit of } u \text{ being } 1''.$$

If, as in the former case, we make the unit of u equal to $10'$,

$$\dot{\delta} = .000000001396 \times (600)^2 u^2 = 0''.000502812\, u^2,$$

and from this expression the column of corrections, to which we alluded at l. 11, may be computed.

We will now give an example of the computation of the obliquity of the ecliptic, from observations of the Sun's meridional zenith distances observed during several days on each side of the solstice.

1812, June.	Refraction.	Zenith Distance by Instrument.	Sun's Semidiameter.	Zenith Distance of Sun's Centre.	Reduction.	Solstitial Zen. Dist.
	″	° ′ ″	′ ″	° ′ ″	′ ″	° ′ ″
12	30.3	28 1 58.9 U	15 47.2	28 18 16.4	17 15.5	28 1 0.9
14	29.4	27 55 14.4 U	15 47.2	28 11 31.0	10 32.4	28 0 58.6
18	29.6	27 46 46.1 U	15 46.8	28 3 2.5	2 2.6	28 0 59.9
19	29.7	28 17 13.4 L	15 46.8	23 1 56.3	0 57.0	28 0 59.3
20	29.2	27 45 0.2 U	15 46.8	28 1 16.2	0 16.3	28 0 59.7
23	30.4	28 16 58.74 L	15 46.6	28 1 42.56	0 42.7	28 0 59.8
24	29.8	27 46 26.56 U	15 46.6	28 2 42.96	1 41.1	28 1 1.9
25	30.4	28 19 20.76 L	15 46.6	28 4 4.56	3 4.2	28 1 0.4
27	29.7	27 51 58.76 U	15 46.6	28 8 5.06	7 4.7	28 1 0.7
28	30.7	28 25 56.76 L	15 46.6	28 10 40.86	9 41.4	28 0 59.4
29	30.1	27 57 26.66 U	15 46.6	28 13 43.36	12 43.0	28 1 0.4
30	30.7	28 32 24.76 L	15 46.6	28 17 8.86	16 8.9	28 1 0

The refractions in the second column are computed from the heights of the barometer and thermometer, and the zenith distances of the Sun's limb, according to the Rules of Chapter X, (see pp. 247, &c.) The zenith distances of the Sun's centre in the fifth column, are formed by adding the refractions to the zenith distances of the observed limb, and by adding or subtracting (according as the observed limb is an upper or lower limb) the Sun's semi-diameter. The *reductions* in the sixth column are computed by the formulæ of p. 436 *, or may be taken from a Table constructed from such formulæ: the *solstitial* zenith distances of the Sun's centre in the seventh

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* In computing these reductions, the values of u are known by the Nautical Almanack. Thus, we have from that book,

$$\odot\text{'s long. June } 12, \text{ being } 2^s\ 21^\circ\ 16'\ 22'',\ u = 8^\circ\ 43'\ 38'',$$
$$14 \ldots\ldots 2\ 23\ 10\ 59, \ldots\ldots 6\ 49\ 1,$$
$$20 \ldots\ldots 2\ 28\ 54\ 32, \ldots\ldots 1\ 5\ 28;$$

therefore on the 12th $u = 8^\circ\ 43'\ 38'' = 52.3633$, which being substituted in the value of δ (see p. 436, l. 21,)

$$\delta = 17'\ 15''.46,$$

on the 20th $u = 1^\circ\ 5'\ 28'' = 6.54666$, &c. and $\delta = 16''.23$.

column are formed by subtracting the numbers in the sixth from those in the fifth column: the *decimals* being expressed by the figures that most nearly represent their values *.

The sum of the numbers in the last column, is

$335^\circ 12' 1''$

$$12 \times 28^\circ\, 12'\, 1'',$$

the 12th of which, in the nearest numbers, is

$$28^0\; 1'\; 0''.1,$$

which represents the mean solstitial zenith of the Sun's centre deduced from twelve observations. But such zenith distance has been corrected for refraction only. It is, therefore, for reasons abundantly given in the preceding part, an *apparent* zenith distance, and is affected with nutation, parallax, and another inequality arising from the attraction of the planets, and explained in Chapter XXII, of *Physical Astronomy*. With regard to the first inequality, the nutation (the place of the Moon's node being $5^s\ 2^0\ 9'$) equals (see pp. 375, &c.) $-\ 8''.4$, the parallax also equals $-\ 4''$, and their sum, accordingly, equals $-\ 12''.4$. The value of the third inequality, the *Sun's Latitude*, as it is called, caused by the Sun being drawn from the plane of the ecliptic by the action of the planets, is $+\ 0''.63$.

So that we have (from l. 7,)

Sun's solstitial zenith distance	28^0	$1'$	$0''.1$
nutation and parallax			$-\ 12.4$
Sun's mean solstitial zenith distance....	28	0	47.7
if the co-latitude (ZP) be	38	31	21.5
$\therefore\ ZP + Z\odot$	66	32	9.2
therefore, solstitial declination	23	27	50.8
subtract Sun's latitude			.63
mean obliquity of summer solstice......	23	27	50.17

This is the determination of the obliquity from the summer solstice, and is founded on a knowledge of the latitude of the

* For instance, *decimals* such as .86, .47, &c. would be represented by .9, .5.

place, which knowledge is founded on that of the quantity and law of refraction (see Chapter X.) Now, with regard to this latter point, there is something that remains still to be determined by Astronomers. For, if we suppose the Sun, at the winter solstice, equally distant from the equator as at the summer solstice, the obliquity determined at the former season from the expression,

$$Z P + Z \odot' - 90^0,$$

ought to equal the obliquity determined, as it just has been, from

$$90^0 - \{Z P + Z \odot\};$$

if the theory of refractions were good, and the observations accurately made. Now the fact is, as we have already stated it at p. 138, the two values of the obliquity do not agree, when the respective zenith distances of the Sun are corrected by that formula of refraction which results from a comparison of the observations of circumpolar stars, (see p. 230.)

Let L be the latitude of the place, then, at the summer solstice,

$$\begin{aligned} w &= 90^\circ - \{90^\circ - L + Z\} \\ &= L - Z; \end{aligned}$$

at the winter solstice,

$$\begin{aligned} w &= 90^\circ - L + Z' - 90^\circ \\ &= Z' - L. \end{aligned}$$

In the first case then, (supposing Z, Z', the solstitial zenith distances to be correct)

$$dw = dL, \text{ in the second } dw = -dL.$$

If we suppose then an error in the value of the latitude of the place of observation, the obliquity, determined from the summer solstitial distance, will be increased by it, and, if determined from the winter, equally diminished. If, therefore, we add the two values of the obliquity together, their half sum, or *mean*, may, in a certain sense, be said to be free from the error of latitude; but the *mean*, thus determined, will not necessarily

be the true value of the obliquity, since the zenith distances (Z, Z') are corrected by the formula of refraction, and partake of its uncertainties.

To illustrate the formula of the *reduction to the solstice*, and the method of finding the obliquity of the ecliptic, an example was taken of observations made at Greenwich with the mural circle. Like observations made with a mural quadrant, would have answered precisely the same end: and so, indeed, would observations made, as they are made (see pp. 417, &c.) at the Observatory of Trinity College, Dublin, with Ramsden's circle, or by the repeating circle, according to the practice of the French Astronomers. These latter observations, being made *out* of the plane of the meridian, require, in order to be made to bear on the point in question, a previous *reduction to the meridian*, founded, as we have already shewn, (see pp. 418, 432,) on the same principle as the *reduction to the solstice*, and to which the latter, as well as the observations made in the meridian, are equally subject.

There is indeed a peculiarity, belonging to observations made on the Sun with the repeating circle, and instruments so used, which is this. In the interval between the observation and the meridional transit of the Sun, the Sun changes his declination: whereas, in the investigation of the formula of reduction to the meridian, the declination of the observed body is supposed to suffer no change. This change of condition, then, requires some slight correction. Suppose the observations to be made before the Sun has reached the solstice, then, in the interval (h), between the observation and the Sun's meridional transit, the Sun's north polar distance is diminished. The Sun's *real* meridional zenith distance, then, is less than the reduced. Let e be the change of declination answering to one minute of time, then, if such change be uniform, the change in a time h equals he. Consequently, if Z be the zenith distance observed out of the meridian, R the computed reduction (see p. 418, &c.) the meridional zenith distance equals

$$Z - R - he,$$

if Z', R', h', &c. be other zenith distances corresponding re-

ductions and hour angles, the corresponding meridional zenith distances will be

$$Z' - R' - h'e,$$

$$Z'' - R'' - h''e,$$

&c.

After the Sun has passed the meridian, the contrary effect, with regard to the correction for the change of declination, will take place. The *reduced* zenith distance will be less than the real meridional zenith, because, after the passage of the meridian, the Sun's north polar distance (the Sun not having attained the solstice) has decreased. If, therefore, $Z_{\prime}$, $R_{\prime}$, $h_{\prime}$ be the corresponding zenith distances, reduction and hour-angles, the corresponding meridional zenith distance will be

$$Z_{\prime} - R_{\prime} + h_{\prime}e.$$

Hence, if n be the number of observations, the mean meridional zenith distance will be

$$\frac{1}{n}\left\{\begin{array}{c} Z' + Z'' + \&c. - (R' + R'' + \&c.) + Z_{\prime} + Z_{\prime\prime} + \&c. - (R_{\prime} + R_{\prime\prime} + \&c.) \\ -(h' + h'' + \&c.) + (h_{\prime} + h_{\prime\prime} + \&c.)\, e, \end{array}\right\}$$

and, consequently, the last correction of which we have been treating, will be

$$\frac{1}{n}(W - E)\, e,$$

W being the sum of the hour-angles to the west of the meridian, and E of angles to the east; and e being the change of declination in one minute of time.

For instance, suppose the Sun's zenith distances to have been observed on June 15, 1809, eleven times before it reached the meridian, and seventeen times after it had passed, and the sum of the hour-angles of the eleven observations to have been $75^{m}.6$, of the seventeen, $187^{m}.12$. Now, by the Tables, or the Nautical Almanack, it appears that e very nearly equals $0''.1$: consequently,

$$\begin{aligned} \text{since, } W &= 187^{m}.12 \\ E &= \underline{\ 75.60\ } \\ W - E &= 111.52 \qquad (W-E)\,e = 11''.15, \end{aligned}$$

$$\text{and } \frac{(W-E)\,e}{n} = \frac{11''.15}{28} = 0''.3982.$$

In the preceding matter we have described the method, such as is practised in Observatories, of finding the obliquity of the ecliptic. The parts of that method are founded, all save one, on observation, or, rather we should say, on results that can be deduced from observation. Such a result, for instance, is the quantity of nutation. The *excepted* part of the process of page 439, is the correction for the Sun's latitude, which (see *Physical Astronomy*, Chap. VI, and XXII.) is known from Physical Astronomy.

But this is far from being a solitary instance of the aid of this latter science. The solar theory is mainly founded on it: at least it may be said that the solar Tables are indebted, for their accuracy, to the computed results of planetary perturbation.

Before, however, our attention is called to these results, there are others of much less difficult enquiry, that must be considered. The Sun, as we have seen (pp. 431, &c.) moves in some orbit, the plane of which is inclined to that of the equator, and does not move equably in that orbit. To find the laws of its *inequable* motion, it would seem to be necessary, previously to investigate its form, or the nature of its curvilinear path. And this, in fact, is the enquiry which, two hundred years ago, Kepler instituted, and after many years of incessant study brought to an happy issue. The orbit of the planet Mars was the object of his researches: their result was the *planet Mars moves in an ellipse round the Sun placed in the focus of the ellipse*.

If this result be extended to the other planets, of which the Earth is one, then the Earth moves round the Sun in an ellipse,

the Sun being placed in its focus: or, to use the common Astronomical language, the solar orbit is elliptical *.

The elliptical form of a planet's orbit was a truth not easily arrived at. In endeavouring to reach it, Kepler had to strive against, and to overcome, his own prejudices, which were also those of the age. From some vague notions of simplicity the antient Astronomers fancied that the motions of the heavenly bodies must, of necessity, be performed in the most simple curves, and that, for such a reason, a planet must move in a circle. After Kepler had found, by his reasonings on observations, that the orbit of Mars could not be a circle of which the Sun occupied the centre, he did not altogether abandon his former opinions, but tried whether the observations of the planet were consistent with its movements in a circle, the Sun occupying a point within the circle, but not in its centre. This conjecture, like his former ones, proving fallacious, Kepler, at last, hit upon the right one, or found the observed places of Mars consistent with its description of an ellipse of certain dimensions.

This, like many other astronomical results, is now so familiar to us, that we do not properly appreciate Kepler's merit in discovering it. If we view, however, the state of Science, and Kepler's means and the inherent difficulty of the investigation, we must consider it to have been a great discovery. And even now, availing ourselves of all the facilities of modern science, it is not easy, briefly to shew, from a comparison of the observations of the Sun, that the solar orbit is an ellipse.

The two kinds of observations, to be used for the above purpose, are those of distances and angles: the former to be known, as far as their relative values are concerned, from observations of the Sun's diameter: the latter from the Sun's longitudes to be computed from the observed right ascensions of the Sun and the obliquity of the ecliptic.

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* The Earth moves round the Sun, but an observer sees the Sun to move, and to describe a curve similar to that which would be seen if we imagine the observer transferred to the Sun.

With these data we might from a centre set off a series of distances, *Radii Vectores* as they are called, and draw a curve through their extremities, which, being of an oval form, might be guessed to be an ellipse, and would, on trial, be verified as such. This, in fact, was Kepler's way, and modern mathematicians have no other, except they ground their speculation on Physical Astronomy, and shew, on mechanical principles, the necessity of the description of an elliptical orbit.

It has just been said that the relative distances of the Sun from the Earth may be known from the observed diameters of the Sun: for, the Sun being supposed to remain unaltered, the visual angle of his disk will be less, the greater his distance, and in that proportion. But there exists a better method of determining the same thing, founded on a discovery of Kepler's, and which, in time, was antecedent to that of the elliptical form of Mars' orbit. The discovery was, that at the aphelion of the orbit, the area comprehended within the arc described, and two radii vectores, drawn from the extremities of the arc to the Sun, was equal to a similar area at the Perihelion, supposing the two arcs to be described in equal times. A like fact has since been proved to be generally true: that is, areas comprehended, respectively, within their arcs and two radii vectores, are equal, provided the arcs are of such a magnitude as to be described in equal times. Now this fact, or law, as it is now called, enables us easily to compute the relative distances of the Sun from the Earth. For by observing (see Chapter VII.) the transits of the Sun and stars, the right ascension of the former may be determined; from which and the obliquity of the ecliptic the Sun's longitude may be computed. The difference of the Sun's longitudes on two successive noons is the angle described by the Sun in twenty-four hours of apparent solar time, from which (as we shall soon shew) the angle described in twenty-four hours of *mean* solar time (which twenty-four hours represent an invariable quantity) may be computed. Let v represent this latter angle: then the small circular arc which, at the distance r, measures the same angle, is rv, and the corresponding small area will be, nearly, $rv \times \frac{r}{2}$, or $\frac{r^2 v}{2}$. Suppose one of the values

of r to be 1, and A to be the corresponding value of v: then the area $= 1 \times \frac{A}{2}$: and from Kepler's Law of the equal description of areas

$$\frac{r^2 v}{2} = \frac{A}{2},$$

$$\text{whence, } r = \sqrt{\frac{A}{v}};$$

and consequently, in order to compute r, we must be able to determine A and v.

A is the angle corresponding to the mean distance 1, and, therefore, in an ellipse of very small eccentricity (and such an ellipse is the solar orbit) is nearly, the *mean* of the greatest and least angular velocities, or has for its measure half the sum of the angles respectively described, in twenty-four hours, at the perigean and apogean distances: which angles, as it has been already explained, are the daily increases of the Sun's longitudes. Now, by examining the longitudes, it will be found that their greatest daily difference takes place at the end of December: their least at the beginning of July: the value of the former is

	$1^{\circ}\ 1'\ 9''.94$
of the latter	$57\ \ 11.48$
so that their mean is	$59\ \ 10.7$

and, if we take this latter angle to represent the value of A, we have

$$r = \sqrt{\left(\frac{59'\ 10''.7}{v}\right)}.$$

In order to determine v for any particular day, we must first take the difference of the Sun's longitudes on the noon of that day, and on that of the day succeeding, and if (which will almost ever be the case) the interval between the two noons be greater or less than twenty-four mean solar hours, we must, in computing v, allow for such excess: for instance, let d represent the difference of two longitudes of the Sun on two successive noons, and let $24 \pm x$ represent the time elapsed, then, very nearly,

$$d : v :: 24 \pm x : 24;$$

$$\therefore v = \frac{24\, d}{24 \pm x},$$

or, if we wish to express (and it is sometimes convenient so to express it) the time in parts of sidereal time,

$$v = \frac{24^h.0657}{24^h.0657 \pm x} . d,$$

and accordingly,

$$r = \sqrt{\left(\frac{59'\ 10''.7}{d} \times \frac{24.0657 \pm x}{24.0657}\right)};$$

or, using mean solar time,

$$r = \sqrt{\left(\frac{59'\ 10''.7}{d} \times \frac{24 \pm x}{24}\right)}.$$

It only remains to shew the method of exhibiting the numerical values of r: suppose, then, such values were required on January 12, and April 1775. In order to find the values of d and x on those days, we must have recourse to recorded observations. In those of Greenwich we find, on January 12, the transits of the Sun's first and second limb, and of the stars α Ceti Rigel, β Tauri, α Orionis, α Lyræ: from which (see pp. 102, 103, &c. Chap. VII.) the right ascension of the Sun's centre may be computed: if computed, it will be found to be

$19^h\ 36^m\ 2^s.7936$, or, in degrees,
$9^s\ 24^0\ 0'\ 41''.9$.

If then we take the obliquity, as it is expressed in the Nautical Almanack, to be equal to $23^0\ 27'\ 58''.5$, we shall from this expression,

$$\tan. \odot . \cos. w = \tan. Æ,$$

($\odot$ being the Sun's longitude and w the obliquity),

find ($\odot$), the longitude equal to $9^s\ 22^0\ 13'\ 35''$.

Institute a like process for the next day, January 13, that is, from the observed transits of the Sun and the fixed stars, and the Catalogues and Tables belonging to the latter, deduce

(see pp. 102, 103,) the clock's error and rate, and then the Sun's right ascension: which right ascension, in the case we are treating of, would be $9^s\ 25^\circ\ 5'\ 29''.9$: from which the longitude deduced as before (see p. 447,) will be

$$9^s\ 23^\circ\ 14'\ 42'',$$

the difference between which and the Sun's longitude on the 12th (see p. 447, l. 26,) is $1^\circ\ 1'\ 7''$, which accordingly is the value of d. Again, since the difference of the Sun's right ascensions on the 13th and 12th

$$\text{is}\quad 9^s\ 25^\circ\ 5'\ 29''.9 - 9^s\ 24^\circ\ 0'\ 41''.9,$$

$$\text{or}\quad 1^\circ\ 4'\ 48'',\quad \text{or in time,}\quad 4^m\ 19^s.2\,;$$

consequently, the interval, in sidereal time, of the two transits on the 12th and 13th is $24^h\ 4^m\ 19^s.2$ ($= 24^h.072$) and, accordingly, (see p. 447, l. 7,)

$$r = \sqrt{\left(\frac{59'\ 10''.7}{61'\ 7''} \times \frac{24.072}{24.0657}\right)}$$

$$= .98418.$$

In like manner if we investigate the Sun's right ascensions on April 28, and April 29, and thence compute his longitudes and take their difference, it will be found to be equal to $58'\ 14''.34$, whilst the interval between the transits, in sidereal time, is only $24^h\ 3^m\ 47^s.66$ ($= 24^h.06324$), and therefore less than a mean solar day. In this case then

$$r = \sqrt{\left(\frac{59'\ 10''.7}{58'\ 14''.34} \times \frac{24.06324}{24.0657}\right)}$$

$$= 1.00798.$$

We might thus compute the distance for every pair of successive observations made during the year. The value of r that results from the computation should be made to belong to the *mean* of the two successive longitudes from which it is computed. Thus, the Sun's longitudes being

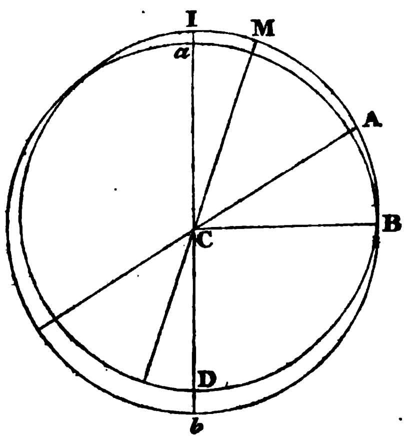
on January 12, ...	9^s	22°	$13'$	$35''$
on January 13, ...	9	23	14	42
their mean is	9	22	44	8.5

to which $r = .98418$ belongs; and if we apply this rule, and computations like to the preceding, to certain of the Sun's

longitudes computed by M. Delambre from Maskelyne's Observations (of 1775), and inserted by the former Astronomer in the Berlin Acts for 1785, (pp. 206, &c.) we shall have the following results which may be arranged in a Table:

Times of Observation.	Longitudes of Sun.	Distances from the Earth.
Jan. 12 to 13	$9^s\ 22^\circ\ 44'\ 8''.5$	.98418
Feb. 17 to 18	10 29 13 59.7	.98950
March 14 to 15	11 24 15 37.5	.99622
April 28 to 29	1 8 26 20.7	1.00798
May 15 to 16	1 24 51 45.9	1.01234
June 17 to 18	2 26 27 43.4	1.01654
July 1 to 3	3 10 17 38.7	1.01658
August 26 to 27	5 3 27 46.6	1.01042
Sept. 22 to 23	5 29 44 22.7	1.00283
Oct. 24 to 25	7 2 24 24.2	.99303
Nov. 18 to 20	7 28 2 46.4	.98746
Dec. 17 to 18	8 25 58 47.8	.98415

The above Table contains twelve longitudes and twelve corresponding distances. Assume a centre C, and with a radius $= 1$ describe a circle Bab. From a point B in this circle begin



to reckon the longitudes, and then, through the extremities of the

arcs proportional to such longitudes draw radii and set them off proportional to their values. Thus, if the angles *BCA*, *BCM*, *BCI* be proportional to

$$1^s\ 8^0\ 26'\ 20'',\quad 1^s\ 24^\circ\ 51'\ 46'',\quad 2^s\ 26^0\ 27'\ 43'',$$

CA, *CM*, *CI* must be made proportional to 1.00798, 1.01234, 1.01654, and accordingly the points *A*, *M*, *I* will fall a little without the circle described with the radius *CB*.

If the remainder of the figure be formed in a like manner, the points belonging to November, December, January, will fall a little within the circle, so that a curve drawn through all the points will be (very little differing, however from a circle) an oval, most drawn in about *D*, most going out near *I*: in other words, in the oval representing the solar orbit, the apogean distance will be near to *I*, the perigean near to *D*.

The distances (see the Table of p. 449.) for November 18, December 17, January 12, being .98746, .98415, .98418, the least or perigean distance is evidently between the first and third dates. So, the apogean distance is between June 17, and August 26. In order to discover whether the perigean distance is between June 17, and July 2, or between July 2, and August 26, we must have recourse to the original observations which have already been used in forming the preceding Table; and amongst these we find the following * :

	Sun's Right Ascen.	Sun's Longitude.	Diff. of R. A.	Diff. of Long.
June 30,	$6^h\ 36^m\ 32^s.6$	$3^s\ 8^\circ\ 23'\ 29''.3$	$4^m\ 7^s.9$	$57'\ 6''.2$
July 1,	6 40 40.5	3 9 20 35.5		

* This is not strictly correct. The right ascensions and longitudes of the text are *not* expressed in the Greenwich Observations, but are deduced from them. We cannot do better, considering the object of this Work, which is to teach the very methods of Astronomical Science, than to subjoin the original observations, and the means of reducing them to those forms under which they appear in the text.

1775.

which are the Sun's right ascensions and longitudes reduced, according to the processes of the subjoined note, from the original observations.

1775.	I.	II.	III.	IV.	V.	*Stars.*
June 29,		$0^m\ 51^s$	$4^h\ 20^m\ 22^s.8$	$0^m\ 54^s.5$		Aldebaran.
June 30,		32 10.6 34 28	6 32 43.9 6 35 1.4	38 17 35 34.4		☉ 1 L 2 L
July 1,		36 17.8 38 35	6 36 51 6 39 8.3	37 23.7 39 41.4		☉ 1 L 2 L
July 2,	16.5 16	0 48 0 46.8	4 20 20 5 40 17.5	0 51.5 0 48	23.3 19.1	Aldebaran. α Orionis.

If the intervals of the wires were all equal we could immediately take the *means* of the times, as is done in pages 86, 87, &c.: which *means* would denote the transits of the stars and Sun by the clocks. But we find from Dr. Maskelyne's Introduction to these Observations (see p. iv,) that in the year 1775, the *equatoreal* intervals (see p. 91, of this Work) between the several wires of the Greenwich transit instrument were

$$30^s.40 \mid 30^s.54 \mid 30^s.36 \mid 30^s.55 \mid$$

consequently, (see p. 90,) the intervals of a star, the north polar distance of which is Δ, would be the above intervals multiplied, respectively, into cosec. Δ : and, if t were the time at the middle wire, $t-a$, $t-b$, $t+c$, $t+d$ the times of an equatoreal star at the second, first, fourth, and fifth wire, t, $t-a.\text{cosec.}\ \Delta$, $t-b.\text{cosec.}\ \Delta$, &c. would be the times of a star distant from the pole by Δ : hence, the mean transit would be

$$t - \tfrac{1}{5}(a+b-c-d)\ \text{cosec.}\ \Delta = m\ (\text{suppose})$$

$$\text{consequently, } t = m + \tfrac{1}{5}\{(a-d)+(b-c)\}\text{cosec.}\ \Delta;$$

or, the correction to be applied to m the mean of the times, is

$$\tfrac{1}{5}(a-d+b-c)\ \text{cosec.}\ \Delta.$$

In

Hence, since $4^m\ 7^s.9 = 0^h.06887$, we have from the formula of p. 447,

$$r = \sqrt{\left(\frac{59'\ 10''.7}{57'\ 6''.2} \times \frac{24.06887}{24.0657}\right)} = 1.018.$$

In the case before us $a = 30.40 + 30.54 = 60.94$

$$\begin{array}{rl} d = & 30.36 + 30.55 = 60.91 \\ \hline a - d = & \qquad\qquad\qquad\quad .03 \\ b = & 30.40 \\ c = & 30.36 \\ \hline b - c = & \quad .04 \end{array}$$

therefore the correction, or $\frac{1}{5}(a - d + b - c) = .014$.

In the case of Aldebaran $\Delta = 74^\circ$ nearly, and cosec. $74 = 1.04$
of Orion $\Delta = 82$..............cosec. $= 1.009$
of $\odot$ at solstice $\Delta = 6632$...........cosec. $= 1.09$,

and therefore the three corrections are $+\ 0^s.0145,\ 0^s.0141,\ 0^s.0153$.

Hence, the corrected transit of Aldebaran on June 30, is $4^\circ\ 20'\ 22''.8$
but (pp. 351, 372,) its Æ by the Catalogue and Tables is $4\ \ 23\ \ 1.74$

clock too slow...... $0\ \ 2\ \ 38.94$

Again, transit of Aldebaran by the clock on July 2, is $4^\circ\ 20'\ 19''.9$
by the Catalogue ....$4\ \ 23\ \ 1.82$

clock too slow...... $0\ \ 2\ \ 41.92$

Again, transit of Orion by clock on July 2, is .... $5^h\ 40^m\ 17^s.53$
by Catalogue .... $5\ \ 42\ \ 59.36$

clock too slow ...... $0\ \ 2\ \ 41.83$

Hence, by a *mean* of Aldebaran and Orion, the clock was too slow on July 2, at five hours, by .... $2^m\ 41^s.88$
but on the June 30, it was too slow by .... $2\ \ 38.94$

(see pp. 103, &c.) clock's loss in two days $23^h\ 20^m$ .... $0\ \ 2.94$
and its daily rate was nearly .... $-\ 0.98$

Having now ascertained the *error* and *rate* of the clock we can determine the Sun's transit or right ascension.

June 30,

Hence, since the distances June 17, June 30, July 2, August 26, are

1.01654, 1.018, 1.01658, 1.01042,

it is plain that the Sun must arrive at his apogean distance before July 2, and very nearly at that time. In like manner, if we examine the observations and reduce them, we shall find that the Sun's increase of longitude between December 30, and December 31, is 1° 1′ 15″.1 and the difference, in sidereal time, between the two transits, is $24^h.07397$, we have, therefore, (as before, in pp. 447, &c.)

$$r = \sqrt{\left(\frac{59'\ 10''.7}{61'\ 15''.1} \times \frac{24.07397}{24.0657}\right)} = .98309,$$

which is, very nearly, the least or perigean distance.

If we take the *means* of the longitudes of June 30, and July 1, and of December 30, and December 31, we shall have

June 30, transit of Sun's centre by clock	$6^h\ 33^m\ 52^s.6$
Error of clock	0 2 40
Sun's right ascension by observation	6 36 32.6
Again, July 1, transit of Sun	$6^h\ 37^m\ 59^s.5$
Error of clock	0 2 40.99
Sun's right ascension by observation	6 40 40.5 nearly,

which right ascensions are those which are specified in page 450, at the bottom line.

In order to compute the longitudes, we have the above right ascensions, and an obliquity = 23° 27′ 59″.5, from which, and by means of the equation tan. L. cos. w = tan. right ascension, or by the formula or Table of reduction to the ecliptic, the longitudes in the text (see p. 450,) may be computed.

The above process may appear somewhat long; but it is given, on the grounds already assigned in p. 424, &c. because it is the real and practical process by which original observations are *reduced* and made to become results fit for the illustration or establishment of Astronomical Science.

	Mean Longitude.	Distance from Earth.
June 30, July 1,	$3^s\ 8^\circ\ 55'\ 2''.4$,	1.018
Dec. 30, Dec. 31,	$9^s\ 9^\circ\ 14'\ 11''.3$,	0.98309.

The difference of the longitudes is $6^s\ 0^\circ\ 19'\ 8''.9$, differing from 6^s by $19'\ 8''.9$, so that the two distances, which are, nearly, the greatest and least, lie, very nearly, in the same straight line: and consequently there arises a presumption, that the longitudes of the apogean and perigean distances, if exactly found, would exactly differ by 6^s.

Now this is a property of an ellipse. Two lines drawn, respectively, from the *focus* of an ellipse, to the extremities of the axis major are the greatest and least of all lines that can be drawn from the focus to the curve. The solar orbit then having a general resemblance to an ellipse, and one of its properties, may have all: and, on such a presumption, an ellipse would be assumed and compared with the solar orbit.

The dimensions of the ellipse, so to be made trial of, would be assigned by the preceding results. Its eccentricity, which is half the difference of the greatest and least distances, would be equal to $\frac{1}{2}(1.018 - .98309)$, or .01745. The next step would be to compute, from the properties of the ellipse, or by means of analytical * expressions expounding those properties, the relative values of the *Radii Vectores* as they are called, and the angles included between those radii and a fixed line, the axis major, for example. If the relations between these angles and radii should be found to be the same, as the relations which have just been made (see p. 449.), there would be established a proof of the Earth's orbit being an ellipse, the Sun occupying its focus.

Kepler's investigations were directed not towards the Earth's but Mars' orbit. His proof of that orbit being an ellipse rests,

* The analytical expression between the angle (v) and the radius (r)

$$\text{is } r = \frac{a.(1 - e^2)}{1 + \cos. v}.$$

in fact, on the same principle as the preceding: which is, the agreement of the computed places in an assumed ellipse with the places computed from observations. The process by which Kepler established this proof is very long, and no process, even taking the most simple case, namely, that of the solar orbit, can be very short. Of which assertion, what has just preceded, is some sort of proof.

The proof of the solar orbit being elliptical has been founded on the equable description of areas: and, historically, this latter fact, or Law, as it is called, (only partially established, however, by Kepler,) preceded the former. To the equable description of areas, and the elliptical forms of planetary orbits, Kepler added a third law, according to which the cubes of the greater axes varied as the squares of the periodic times.

We must now consider the astronomical uses of these discoveries. In the first place it is evident, that, since we know the nature of the solar orbit, and one law regulating the motion in that orbit, we have made some approach towards a knowledge of the Sun's real motion in the ecliptic. If the latter motion should be known, the Sun's right ascension and declination would thence be determinable by the Rules of Spherical Trigonometry. The law of a body's motion in an elliptical orbit is the first and essential thing to be determined. Let the body begin to move from one of the apsides of the ellipse, and let the time be reckoned from the beginning of such motion, then, the problem to be solved, is the assigning of the body's place in the ellipse after a certain elapsed time. . This, in fact, is *Kepler's* Problem, as it has been called for distinction's sake. And, by its solution, that great Astronomer, laid the first ground-work of Solar Tables.

The enquiry, then, in the next Chapter, concerning the best method of solving Kepler's problem, will be purely a mathematical enquiry. A result being attained, the next step will be to apply it. If we begin our reckonings for an apside, we must know where the apsides of the Sun's orbit (which, in other words, are the apogee and perigee) are situated. That is,

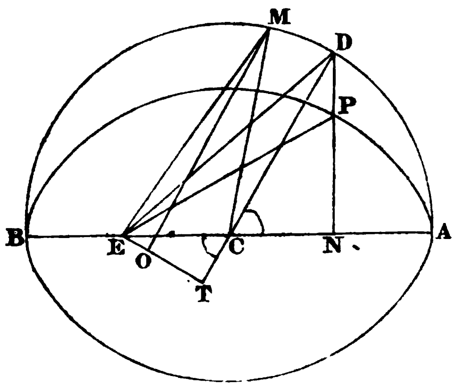
we must know the longitudes of those points. We indeed, by what has preceded, already know them to a certain degree of exactness, since in page 454, the longitude of the apogee was found to be nearly $3^s\ 8^\circ\ 55'\ 2''.4$. After we have discussed Kepler's problem, we will devise more exact methods for determining the place of the apogee. The place of the apogee being determined, there will arise a question concerning the permanency of that place in the Heavens. In the preceding instance (see p. 447.) the longitude of the apogee was found for the year 1775. Will it be the same for any other epoch? The obvious method of solving this question will be to find, for two different epochs, by the same process, the longitudes of the apogee. The results will shew whether the apogee be stationary, progressive, or regressive.

The place of the apogee being known for any given epoch, and the law of its translation, the place may be determined for any other epoch; and thence, since Kepler's problem determines the body's place in the ellipse, we shall be able to determine the Sun's place or longitude for any assigned epoch. This it is the object of Solar Tables to effect. If their elements be correct, they enable us to assign the Sun's longitude for years that are to come. But the elements of the Tables stand in need of frequent revision: for, the dimensions of the solar ellipse, from the action of the planets, are continually varying, and, which is a reason of a different sort, our means of determining the dimensions become, from the advancement of science and art, progressively better. If, therefore, the construction of solar and planetary Tables be our first object, their correction will be the second.

CHAP. XVIII.

On the Solution of Kepler's Problem, by which a Body's Place is found in an Elliptical Orbit.—Definition of the Anomalies.

LET APB be an ellipse, E the focus occupied by the Sun, round which P the Earth or any other planet is supposed to revolve. Let the time and planet's motion be dated from the



apside or aphelion A. The *condition given,* is the time elapsed from the planet's quitting A; the *result sought* is the place P; to be determined either by finding the value of the angle AEP, or by cutting off, from the whole ellipse, an area AEP bearing the same proportion to the area of the ellipse which the given time bears to the periodic time.

There are some technical terms used in this problem which we will now explain.

Let a circle AMB be described on AB as its diameter, and suppose a point to describe this circle uniformly, and the whole of it, in the same time, as the planet describes the ellipse in: let

also t denote the time elapsed during P's motion from A to P: then if $AM = \frac{t}{\text{period}} \times 2\,AMB$, M will be the place of the point that moves uniformly, whilst P is that of the planet's; the angle ACM is called the *Mean Anomaly*, and the angle AEP is called the *True Anomaly*.

Hence, since the time (t) being given, the angle ACM can always be immediately found (see l. 2.) we may vary the enunciation of Kepler's problem, and state its object to be, *the finding of the true anomaly in terms of the mean.*

Besides the mean and true anomalies, there is a third called the *Eccentric Anomaly*, which is expounded by the angle DCA, and which is always to be found (geometrically) by producing the ordinate NP of the ellipse to the circumference of the circle. This eccentric anomaly has been devised by mathematicians for the purposes of expediting calculation. It holds a mean place between the two other anomalies, and mathematically connects them. There is one equation by which the mean anomaly is expressed in terms of the eccentric: and another equation by which the true anomaly is expressed in terms of the eccentric.

We will now deduce the two equations by which the *eccentric* is expressed, respectively, in terms of the *true* and *mean* anomalies.

Let t = time of describing AP,
P = periodic time in the ellipse,
$a = CA$,
$ae = EC$,
$v = \angle\, PEA$,
$u = \angle\, DCA$; ($\therefore ET$, perpendicular to DT, $= EC \times \sin. u$),
$\rho = PE$,
$\pi = 3.14159$, &c.;

then, by Kepler's law of the equable description of areas,

$$t = P \times \frac{\text{area } PEA}{\text{area of ellip.}} = {}^{*}P \times \frac{\text{area } DEA}{\text{area } \odot} = \frac{P}{\pi a^2}(DEC + DCA)$$

* Vince's *Conics*, p. 15. 4th Ed.

$$= \frac{P}{\pi a^2}\left(\frac{ET.DC}{2} + \frac{AD.DC}{2}\right) = \frac{Pa}{2\pi a^2}(EC.\sin. u + DC.u)$$

$= \frac{P}{2\pi}(e \sin. u + u)$: hence, if we put $\frac{P}{2\pi} = \frac{1}{n}$,

we have

$$nt = e.\sin. u + u\ldots\ldots(a),$$

an equation connecting the mean anomaly nt, and the eccentric u.

In order to find the other equation, that subsists between the true and eccentric anomaly, we must investigate, and equate, two values of the radius vector ρ, or EP.

First value of ρ, in terms of v the true anomaly;

$$\rho = \frac{a.(1 - e^2)}{1 + e.\cos. v} * \ldots\ldots(1).$$

Second, in terms of u the eccentric anomaly,

$$\begin{aligned}
\rho &= a(1 + e.\cos. u)\ldots\ldots\ldots\ldots(2).\\
\text{For, } \rho^2 &= EN^2 + PN^2\\
&= EN^2 + DN^2 \times (1 - e^2)\\
&= (ae + a.\cos. u)^2 + a^2 \sin.^2 u.(1 - e^2)\\
&= a^2\{e^2 + 2e.\cos. u + \cos.^2 u\} + a^2.(1 - e^2)\sin.^2 u\\
&= a^2\{1 + 2e.\cos. u + e^2 \cos.^2 u\}.
\end{aligned}$$

Hence, extracting the square root,

$$\rho = a(1 + e.\cos. u).$$

Equating the expressions (1), (2), we have

$(1 - e^2) = (1 - e.\cos. v).(1 + e \cos. u)$, whence,

$\cos. v = \frac{e + \cos. u}{1 + e.\cos. u}$, an expression for v in terms of u; but, in order to obtain a formula fitted to logarithmic computation, we must find an expression for $\tan. \frac{v}{2}$: now, (see *Trig.* p. 40.)

* Ibid. p. 23. Bridge, p. 93.

$$(b)\ \tan.\frac{v}{2} = \sqrt{\left(\frac{1-\cos.v}{1+\cos.v}\right)} = \sqrt{\left(\frac{(1-e)(1-\cos.u)}{(1+e)(1+\cos.u)}\right)}$$
$$= \sqrt{\left(\frac{1-e}{1+e}\right)}\tan.\frac{u}{2}.$$

These two expressions (a) and (b), that is,

$$nt = e.\sin.u + u,$$
$$\tan.\frac{v}{2} = \sqrt{\left(\frac{1-e}{1+e}\right)}.\tan.\frac{u}{2},$$

analytically resolve the problem, and, from such expressions, by certain formulæ belonging to the higher branches of analysis, may v be expressed in the terms of a series involving nt *.

Instead, however, of this exact but operose and abstruse method of solution, we shall now give an approximate method of expressing the true anomaly in terms of the mean.

MO is drawn parallel to *DC*. (1.) Find the half difference of the angles at the base of the triangle *ECM*, from this expression,

$$\tan.\tfrac{1}{2}(CEM - CME) = \tan.\tfrac{1}{2}(CEM + CME) \times \frac{1-e}{1+e},$$

(see *Trig.* p. 27.) in which, $CEM + CME = ACM$, the mean anomaly.

(2.) Find CEM by adding $\frac{1}{2}(CEM+CME)$ and $\frac{1}{2}(CEM-CME)$ and use this angle as an approximate value to the eccentric anomaly DCA, from which, however, it really differs by $\angle EMO$.

---

* The following is the series for v in terms of nt;

$$v = nt -$$
$$\left(2e - \frac{1}{4}e^3 + \frac{5}{96}e^5\right).\sin.nt + \left(\frac{5}{4}e^2 - \frac{11}{24}.e^4 + \frac{17}{192}.e^6\right)\sin.2nt$$
$$-\left(\frac{13}{12}.e^3 - \frac{43}{64}.e^5\right).\sin.3nt + \left(\frac{103}{96}.e^4 - \frac{451}{480}.e^6\right).\sin.4nt$$
$$-\frac{1097}{960}e^5.\sin.5nt + \frac{1223}{960}e^6\sin.6nt,$$ in which the approximation is carried to quantities of the order e^6.

(3.) Use this approximate value of $\angle DCA = \angle ECT$ in computing ET which equals the arc DM: for, since (see p. 458,)

$$t = \frac{P}{\text{area} \odot} \times DEA, \text{ and (the body being supposed to revolve in the}$$

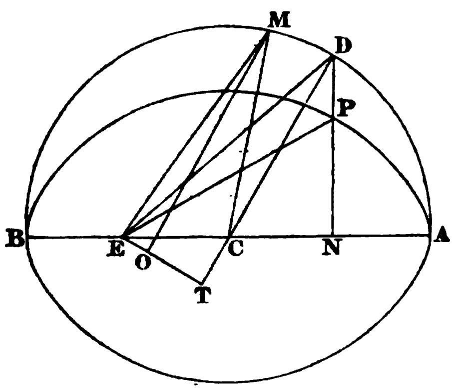
$$\text{circle } ADM) = \frac{P}{\text{area} \odot} \times ACM; \therefore \text{area } AED = \text{area } ACM,$$

or, the area DEC + area ACD = area DCM + area ACD;
consequently, the area DEC = the area DCM,
and, expressing their values,

$$\frac{ET \times DC}{2} = \frac{DM \times DC}{2} \text{ and } \therefore ET = DM.$$

Having then computed $ET = DM$, find the sine of the resulting arc DM, which sine $= OT$: the difference of the arc and sine $(ET - OT)$ gives EO.

(4.) Use EO in computing the angle EMO, the real difference, between the eccentric anomaly DCA, and the $\angle MEC$: add



the computed $\angle EMO$ to $\angle MEC$, in order to obtain $\angle DCA$. The result, however, is not the exact value of $\angle DCA$, since $\angle EMO$ has been computed only approximately; that is, by a process which commenced by assuming $\angle MEC$, for the value of the $\angle DCA$.

For the purpose of finding the eccentric anomaly, this is the entire description of the process; which, if greater accuracy be

required, must be repeated; that is, from the last found value of $\angle DCA = \angle ECT, ET, EO,$ and $\angle EMO$ must be again computed.

(5.) A sufficiently correct value of the eccentric anomaly (u) being found, investigate the true (v), from the formula (b) of p. 460, that is,

$$\tan. \frac{v}{2} = \sqrt{\left(\frac{1-e}{1+e}\right)} \cdot \tan. \frac{u}{2}.$$

EXAMPLE I.

The Eccentricity of the Earth's Orbit being .01691, *and the Mean Anomaly* = 30^0, *it is required to find the Eccentric and the true Anomalies,*

(1.) log. tan. 15 . . . . $\bar{9}.4280525$
log. $(1-e)$, or log. .98309 . . . $\bar{1}.9925933$
arith. comp. $\overline{1+e}$, or of 1.01691 $\bar{1}.9927218$

log. tan. $\frac{1}{2}(CEM - CME)$ . . $9.4133676 = $ log. tan. $14^0\ 31'\ 22''$.

(2.) $\frac{1}{2}(CEM - CME) = 14^0\ 31'\ 22''$
$\frac{1}{2}(CEM + CME) = 15 \quad 0 \quad 0$

$CEM \qquad = 29 \quad 31 \quad 22.$ 1st approx^e. value of CDA.

(3.) log. sin. $29^0\ 31'\ 22''$ . . . . $\bar{9}.6926438$
log. .01691 . . . . $\bar{2}.2281436$
+ log. (arc = rad^s.) 5.3144251
log. DM in seconds . . . . 3.2352125 = log. 1718.7.

$DM = 28'\ 38''.7$, and its sine expressed in seconds differs from the arc DM by less than half a second.

(4.) The operation prescribed in this number (see p. 461, l. 19, &c.) is, in this case, needless, since the correction for the angle EMC is so small, that the first approximate value of the eccentric anomaly may be stated at $29^0\ 31'\ 22''$.

(5.) log. tan. $\frac{u}{2}$, or log. tan. $14^0\ 45'\ 41''$ 9.4207651
$\frac{1}{2}$ log. $(1-e)$, or $\frac{1}{2}$ log. .98309 . . . . 4.9962966
$\frac{1}{2}$ log. $(1+e)$, or $\frac{1}{2}$ log. 1.01691 . . . . 4.9963608

log. tan. $\frac{v}{2}$ 9.4134225

$$= \text{log. tan. } 14^0\ 31'\ 28'';$$

$$\therefore \text{ the true anomaly} = 29^0\ \ 2'\ 56''.$$

The difference of the mean and true anomalies, or, as it is called, the *Equation of the Centre*, equals $57'\ 4''$.

If the eccentricity had been assumed $= .016813$, or $.016791$, the equation of the centre would have resulted $= 56'\ 46''.4$, or $= 56'\ 41''.4$, respectively.

EXAMPLE II.

Instead of .01691, *suppose the Eccentricity of the Earth's Orbit be taken at* .016803*, *and the Mean Anomaly, reckoning from Perigee, according to the Plan in the new Solar Tables, be* $10^s\ 12^\circ\ 22'\ 12''.4$.

Taking out 6 signs, we have the mean angular distance from apogee $= 4^s\ 12^0\ 22'\ 12''.4$.

(1.)	log. tan. $66^\circ\ 11'\ 6''.2$	10.3552029
	log. .983197	$\bar{1}$.9926406
	arith. comp. 1.016803	$\bar{1}$.9927645
		10.3406080 = log. tan. $65^\circ\ 27'\ 56''.4$.

(2.)	$\frac{1}{2}(CEM - CME)$	$65^0\ 27'\ 56''.4$
	$\frac{1}{2}(CEM + CME)$	66 11 6.2
		131 39 2.6 approx^e. value of CDA (u)

(3.)	log. tan. $\frac{u}{2}$, or log. tan. $65^0.49'\ 31''.3$	10.3478640
	$\frac{1}{2}$ log. .983197	4.9963203
	$\frac{1}{2}$ arith. comp. 1.06803	4.9963816
	log. tan. $\frac{v}{2}$	10.3405659;

$$\therefore\ \frac{v}{2} = 65^0\ 27'\ 49''.2, \text{ and } v = 4^s\ 10^\circ\ 55'\ 38''.4;$$

* In 1750, the eccentricity was 0.016814, and, the secular variation being .000045572, in 1800, it was 0.016791, and in 1810, (for which epoch Delambre's Tables are constructed) .0167866.

∴ the true anomaly, reckoning from perigee, $= 10^s\ 10^\circ\ 55'\ 38''.4$, and difference of the mean and true anomaly $=\ 1^\circ\ 26'\ 34''$.

This difference, or *Equation of the Centre,* is stated, for 1800, in Lalande's Tables, Vol. I. *Astron.* ed. 3. p. 23, at $1^\circ\ 26'\ 38''.6$; but, in the new Tables, Vince, Vol. III. p. 38, at $11^s\ 28^\circ\ 32'\ 44''.4$. Now the difference of this, and of 12 signs, is $1^\circ\ 27'\ 15''.6$, which is still greater than Lalande's result by $45''$. But, it is purposely made greater; for these $45''$ are the sum of the maxima of several very small equations. (See the explanation in Delambre's Introduction, and in Vince's, p. 6.)

In the two preceding Examples, it appears that, by reason of the small eccentricity of the Earth's orbit, the true anomaly and equation of the centre are found by an easy and short process; no second approximation being found necessary. It appears also, by the results, that a small change in the eccentricity makes a variation of several seconds in the equation of the centre. Thus, arranging the results in the preceding Examples:

Mean Anomaly.	Eccentricity.	Equation of Centre.
$30^\circ\ 0'\ 0''$	.016910	$0^\circ\ 57'\ 4$
30 0 0	.016813	0 56 46.4
30 0 0	.016791	0 56 41.4

Now, by observation and theory, it appears, that the eccentricity of the Earth's orbit is diminishing. Hence, Tables of the equation of the Earth's orbit, computed for one epoch, will not immediately suit another: but, they may be made to suit, by appropriating a column to the *secular variation* of the equation of the centre. Thus, in Lalande's Tables, tom. I. ed. 3. p. 18, the equation of the centre is stated to be $56'\ 41''.2$, and in a column by the side, the corresponding secular diminution to be $9''.36$. Now Lalande's Tables were computed for 1800* : (when the eccentricity of the Earth's orbit was .016791) consequently, for the *preceding* epochs of 1750, 1500, the equations of the

* Delambre states, that Lalande's Tables answer better to the epoch of 1809, or 1810, than to 1800. See Introduction to his new Tables.

centre would be $56' \; 41''.2 + 4''.68$, and $56' \; 41''.2 + 23''.44$, that is, $56' \; 45''.9$, and $57' \; 4''.6$ respectively. These are nearly the results previously obtained in p. 463, which they ought to be, since, the secular diminution of the eccentricity being .000045572, the eccentricities corresponding to 1750 and 1560 will be, nearly, .016813 and .016910.

By this mode we may also reconcile the two results in Example 2, in p. 453; for, the equation of the orbit in Lalande's Tables is $1^\circ \; 26' \; 30''$, (that is, for an eccentricity, .016791) therefore, for 1760, when the eccentricity was .016803, the equation will be, the secular diminution being $13''.9$, equal to

$$1^\circ \; 26' \; 30''.6 + 3''.4, \text{ that is, } 1^\circ \; 26' \; 34''.$$

Example III.

The Eccentricity of the Orbit (that of Pallas) *being* 0.259, *the Mean Anomaly* $= 45^\circ$: *it is required to find the Eccentric and true Anomalies.*

(1.) log. tan. $22^\circ \; 30'$ ...... $\bar{9}.6172243$
log. tan. 741 .... $\bar{1}.8698182$
arith. comp. 1.259 .... 9.8999743
log. tan. $\frac{1}{2}(CEM - CME)$ 9.3870168 = log. tan. $13^\circ \; 42' \; 3''.3$.

$$\begin{aligned}
(2.)\; \tfrac{1}{2}(CEM - CME) &= 13^\circ \; 42' \; 3''.3 \\
\tfrac{1}{2}(CEM + CME) &= 22 \;\; 30 \;\; 0 \\
\hline
\therefore\; CEM &= 36 \;\; 12 \;\; 3\;.3 = \text{1st approx}^e.\text{value of } \angle\, CDA, \\
\text{and } CME &= 8 \;\; 47 \;\; 56.7
\end{aligned}$$

(3.) log. sin. $36^\circ \; 12' \; 3''.3$ .... $\bar{9}.7713071$
log. .259 .... $\bar{1}.4132998$
log. (arc = radius).......... 5.3144251
log. DM in seconds .... 4.4990320 = log. 31552.4 ;
$\therefore DM = 31552''.4 = 8^\circ \; 45' \; 52''.4$;
$\therefore$ log. sin. 9.1829067
log. (arc = rad.) 5.3144251
4.4973318 = log. 31429 ;

$$\begin{aligned}
\therefore \text{ since } DM &= 31552.4 \\
\text{and sin. } DM &= 31429 \\
\hline
EO &= \quad 123.4
\end{aligned}$$

(4.) (*a*) log. .259	$\bar{1}.4132998$
log. sin. 45°	9.8494850
	9.2627848
log. sin. 8° 47′ 56″.7	9.1845968
	.0781880
	5.3144251
	5.3926131
log. r	10
log. 123.4	2.0913152
	12.0913152
(*a*) log. (arc = radius) + log. EM	5.3926131
log. sin. EMO	6.6987021

$$\therefore EMO = 1' \; 43''.1.$$

Hence, since $CDA = 36° \; 12' \; 3''.3$
and $EMO = \;\; 0 \quad 1 \;\; 43.1$

corrected value of $CDA = 36 \quad 13 \;\; 46.4$, the eccentric anomaly.

log. tan. 18° 6′ 53″.2	9.5147282
$\frac{1}{2}$ log. .741	4.9349091
$\frac{1}{2}$ arith. comp. 1.259	4.9499871

log. tan. $\frac{v}{2}$ .... 9.3996244 = log. tan. 14° 5′ 19″;

$\therefore$ the true anomaly is 28° 10′ 38″.

The eccentric and true anomalies being determined, the radius vector ρ may be computed from either of the two expressions, (1) (2) p. 459, but most conveniently from the latter.

Example IV.

Required the Earth's Distance from the Sun, the Mean Anomaly (reckoning from Aphelion) being $4^s \; 12° \; 22' \; 12''.4$, *and the Eccentricity* = .016803. See Ex. 2. p. 463.

$$\rho = 1 + e \, . \cos. u, \text{ if } a = 1,$$

and $u = 131° \; 39' \; 2''.6$.

log. cos. 131° 39′ 2″.6	$\bar{9}$.8225523
log. .016803	$\bar{2}$.2253868
log. .011167	8.0479391

(since cos. is —), $\rho = 1 - .011167 = .988833$.

Example V.

Required the Distance of Pallas *from the Sun, in the conditions of* Ex. III.

log. cos. 36° 13′ 46″.4	9.9066881
log. 0.259	$\bar{1}$.4132998
log. .208923	9.3199879

$\therefore$ distance = 1.208923

and log. distance = 0.823979.

The knowledge of these distances is useful*, as we shall hereafter see, in computations of the heliocentric longitudes and latitudes of planets. But, in such computations, the *logarithms* of the distances are required. Those can, indeed, be immediately found from the computed distances, by means of the common Tables; with more brevity and facility of computation, however, by taking out, during the process of finding the true anomaly, when the log. sine is taken out, the log. cosine of the eccentric anomaly.

Assume then, $e\,.\cos.\,u = \cos.\,\theta$, or, $\log.\cos.\,\theta = \log.\,e + \log.\cos.\,u$; thence θ is known: and, lastly,

$$\log.\,\rho = \log.\,(1 + e\,.\cos.\,u) - 10 = \log.\,(1 + \cos.\,\theta) - 10$$

$$= \log\ 2.\ \cos.^2\frac{\theta}{2} - 20$$

$$= \log.\,2 + 2\log.\cos.\,\frac{\theta}{2} - 20 = 2\log.\cos.\,\frac{\theta}{2} - 19.6989700.$$

The sole object of this latter method, is compendium of calculation.

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* The Nautical Almanack expresses the logarithm of the Sun's distance for every 6th day of the year.

By means of the preceding rule, (see pp. 460, 461,) the true anomaly (as in the Examples) may always be computed from the mean, which is known, by a single proportion from the time. The difference of the true and mean anomalies, is the equation of the centre, or the equation of the orbit. And, the Solar Tables assign to the mean anomaly, as the *argument*, this latter quantity, instead of the true anomaly. It serves then as a *correction* or *equation* to the mean anomaly, by means of which the inequality between the *mean* and *true* places of a planet, at any assigned time, may be compensated. It is additive or subtractive, according as the mean is less or greater than the true anomaly: subtractive, therefore, whilst the body P moves, from A the aphelion to B the perihelion, or, through the first 6 signs of mean anomaly, (reckoning anomaly from the aphelion) and additive, whilst P moves, from B to A, or, through the last 6 signs of mean anomaly.

These circumstances, Lalande's Tables (ed. 3.) used to express, in the common way, by the algebraical signs $-$ and $+$. But the new Solar Tables, (see Delambre's Tables, and Vince's *Astronomy*, Vol. III.) adapted to the operation of addition only, when the mean anomaly exceeds the true, express not the *equation of the centre*, but its *supplement to* 12 *signs* (360°). The 12 signs, therefore, must be subsequently struck out of the result. This is not the sole difference in the construction of the Tables. In Delambre's last*, the mean anomaly is reckoned from the perihelion, and the equations of the centre are increased by 45″, the sum of several small inequalities: an arrangement made for the same purpose as the former, l. 20; that of avoiding the operation of subtraction.

The *greatest equation of the centre*, it is plain, can mean nothing else than the greatest difference between the true and mean anomalies; which must happen when the body P moves with its mean angular velocity. For, if we conceive a body to move uniformly in a circle round E as a centre, with an angular velocity, the mean between the least of P at A, and its greatest at B,

* Both Tables were constructed by Delambre.

and such, that departing with P from AB the line of the apsides, it shall, in the same time, again arrive at it, together with P; then, it is plain, at the commencement of the motion, the first day, for instance, P moving with its least angular velocity, describes round E a less angle than the fictitious body does: the next day, a greater angle than on the first, but still less than the angle described by the fictitious body: similarly for the third, fourth day, &c.: so that, at the end of any assigned time, the two angular distances of the two bodies from the aphelion, will differ by the accumulation of the daily excesses, of the angular velocity of the fictitious body, above that of the body P. And this accumulation must continue, until P, (always moving, till it reaches B, with an increasing angular velocity), attain its mean angular velocity, or, that velocity with which the body moves in the circle; then, this latter body can, in its daily rate, no longer gain on P; and, past this term, it must lose: exactly at that term, then, the difference of its angular distance from A, or from the line of the apsides, must be the greatest.

The difference of the mean and true anomalies is technically called the *Equation of the Centre.* If we date the planet's motion from the aphelion, then, at the beginning of that motion, the planet moves with its least angular velocity, and consequently the imaginary point, or body that describes the circle with a mean uniform velocity, precedes the planet. The true anomaly then is less than the mean, and consequently the *true* anomaly is equal to the *mean* minus the *equation* of the centre. If the planet's motion had been dated from the perihelion (as it is now the custom in the construction of Tables), then, in a similar position of P, we should have had the true anomaly equal to the mean plus the equation of the centre.

In order to determine this term, or the point in the ellipse, at which the body is moving with the mean velocity, conceive a circle to be described round E as a centre, and to cut the ellipse in some point P, of the figure of p. 457, then such circle will cut the line EA in some point between E and A. Consequently, if the angular velocities be inversely as the squares of the distances from E, the angular velocity in the ellipse from

A to *P* will be, in every intermediate point, less than the angular velocity of the body in the circle, in all points between *EA* and *P*. Now the angular velocities are inversely as the squares of the distances, if *the areas described, respectively, by the body in the ellipse and the body in the circle, be equal**. This last condition enables us to determine the value of *TP*, or the value of the radius of the intersecting circle. For, if the small areas be equal, the whole areas of the circle and ellipse must be equal, since the whole area $= \frac{\text{area in a given time} \times \text{period}}{\text{given time}}$, and the period, by hypothesis is the same in the ellipse and circle.

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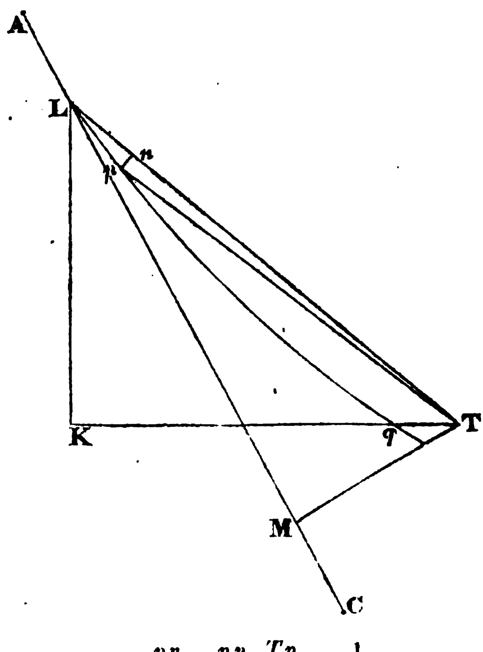
* The angle LTp, which expounds the angular velocity, is measured by $\frac{pn}{Tp}$,



and $\frac{pn}{Tp} = \frac{pn \,.\, Tp}{Tp^2} \propto \frac{1}{Tp^2}$,

if $pn \,.\, Tp$, which is twice the small area LTp, be given.

If, then, x be the sought for value of SP, $2a$ the axis major and ae the eccentricity of the ellipse, we have, by equating the values of the two areas,

$$3.14159 . x^2 = 3.14159 \times a \times a \sqrt{(1 - e^2)};$$

whence,

$$\begin{aligned} x &= a . (1 - e^2)^{\frac{1}{4}} \\ &= a \left(1 - \frac{e^2}{4} - \frac{3}{32} . e^4\right), \text{ nearly,} \\ &= a \times .99992942, \text{ nearly,} \end{aligned}$$

in the solar orbit.

From the above value of the radius vector, the true and eccentric anomalies, at the time of the greatest equation, may be computed, and by the expressions (1), (2), p. 459, viz.,

$$\rho = \frac{a . (1 - e^2)}{1 + e . \cos. v}, \quad \rho = a (1 + e . \cos. u).$$

Hence, the mean anomaly (nt) is known by the expression

$$nt = u + e . \sin. u,$$

and finally there results the greatest equation of the centre $=$

$$\pm (v - nt.)$$

Example.

In the Earth's orbit, e being very small $(= .016814)$,

$$\text{since } (1 - e^2)^{\frac{1}{4}} = 1 + e . \cos. u,$$

$$1 - \frac{e^2}{4} = 1 + e . \cos. u; \therefore \cos. u = -\frac{e}{4},$$

$$\text{and } 1 - \frac{e^2}{4} = (1 - e^2)(1 + e \cos. v); \therefore \cos. v = \frac{3}{4} e;$$

$\therefore$ by the series for the arc in terms of the cosine, and by neglecting the powers of e,

$$nt = \text{quadrant} + \frac{e}{4} + e,$$

$$v = \text{quadrant} - \frac{3}{4} e;$$

$\therefore nt - v$, (the greatest equation) $= \frac{8e}{4} = 2e$, and consequently, in the Earth's orbit, the eccentricity $= \frac{1}{2}$ the greatest equation.

This is one method of computing the greatest equation; but it is usually determined from observations. For that purpose we must observe the longitude of the body, when its angular velocity is equal to its mean angular velocity; thus, according to Lacaille,

		s	°	′	″
1751. Oct. 7, ☉'s longitude		6	13	47	13.7
1752. Mar. 28,		0	8	9	25.5
difference of the two longitudes		5	24	22	11.8
The mean motion proportional to the interval of time was		5	20	31	43.2
the diff. or the double of the greatest equation		0	3	50	28.6

Hence, the greatest equation of the centre in the Earth's orbit is $1° 55' 14''.3$: and more nearly, by correcting the above calculation, $1° 55' 33''$.

The difference of the longitudes of the two points in the orbit, at which the real motion nearly equals the mean, is equal to $5^s 24° 22' 11''$, or $174° 22' 11''$. This is a very obtuse angle formed by two lines drawn from the above two points to the focus of the solar ellipse. The two points then are not very remote from the extremities of the axis minor; they would be exactly there, if the angle were $178° 4' 28''$. Hence, the greatest equation happens when the body is nearly at its mean distance.

In the Example that has preceded, the Sun's longitude was taken on October 7, and March 28; because, at those times, his daily motions or increases of longitude were equal to his mean motion. That circumstance was ascertained by first taking the Sun's longitudes on two successive days, and then their difference, which is his angular motion. The mean angular motion is nearly $59' 8''.3$: the greatest, about the beginning of January, being $1° 1' 10''$; the least, about the beginning of July, being $57' 11''$.

We shall perceive the use of the equation of the centre, when we treat of the equation of time. Astronomers have used its greatest value in determining the eccentricity of the orbit *. If E be the greatest equation, and $\frac{E}{57^{\circ}.2957795}$ be put $= K$, then the eccentricity, or

$$e = \frac{K}{2} - \frac{11\,K^3}{3.2^8} - \frac{587\,K^5}{3.5.2^{16}} - \text{\&c.}\dagger$$

Hence in the case of the Earth's orbit, the eccentricity of which is very small, we have, retaining only the first term of the series, and taking $E = 1^{\circ}\ 55'\ 33''$,

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* See Lacaille, *Mem. Acad.* 1757, p. 123.

† This series was invented by Lambert. The reverse series for the greatest equation is

$$2\,e + \frac{11}{48}\,e^3 + \frac{599}{5120}\,e^5 + \text{\&c.}$$

and according to M. Oriani, *Ephes. de Milan.* 1805.

$$\begin{aligned} E = &- \left(2\,e - \frac{1}{2^2}\,e^3 + \frac{5}{2^5.3}\,e^5 + \text{\&c.}\right) \text{sin. } z \\ &+ \left(\frac{5}{2^2}\,e^2 - \frac{11}{2^3.3}\,e^4 + \text{\&c.}\right) \text{sin. } 2\,z, \\ &- \left(\frac{13}{2^3.3}\,e^3 - \frac{43}{2^6}\,e^5 + \text{\&c.}\right) \text{sin. } 3\,z, \\ &\left(\frac{103}{2^5.3} - e^4\right) \text{sin. } 4\,z \\ &- \frac{1097}{2^6.3.5}\,e^5 \text{ sin. } 5\,z, \end{aligned}$$

not extending the series beyond terms containing e^6.

In a Note to page 460, we gave the series expressing the true anomaly in terms of the mean and the eccentricity. The following is Delambre's expression for the equation of the centre, for the year 1810, in terms of the greatest equation and of the mean anomaly z reckoned from the perigee:

$$1^{\circ}\ 55'\ 26''.352 \text{ sin. } z + 1'\ 12''.679 \text{ sin. } 2\,z + 1''.0575 \text{ sin. } 3\,z$$
$$+ 0''.018 \text{ sin. } 4\,z.$$

$$e = \frac{K}{2} = \frac{1^\circ\ 55'\ 33''}{2 \times 57^\circ.2957795} = .016807.$$

If E be taken $= 1^0\ 55'\ 36''.5$, (the greatest equation in 1750),

$$e = .016814.$$

If E be taken $= 1^\circ\ 55'\ 26''.8$, (the greatest equation in 1800),

$$e = .016791.$$

From these two Examples, the diminution of the greatest equation for 50 years appears to be $9''.7$: and, consequently the *secular diminution* would be $19''.4$. Lalande, in his Tables, states it to be $18''.8$. Delambre, $17''.18$.

In the case of the orbit of *Saturn*, $E = 6^0\ 26'\ 42''$

$$= 6^0.445\ ;\ \therefore K = \frac{6.445}{57.2957795} = .112486,$$

$$\text{and } e = .056243 - 000031 = .056212.$$

We have, in the preceding pages, given only one solution of Kepler's Problem *: which solution is Cassini's, and is an indirect one. But there are several other solutions of the same kind, besides those which may be called direct solutions, and are derived from the simple consideration of the equations of p. 460. The learned Astronomer of the Dublin Observatory, has considered, in a Memoir of the Irish Transactions, these solutions and appreciated their exactness.

In this subject the first object of investigation was strictly a mathematical one. When we apply the result of that investigation to the solar orbit, we find the Sun's place therein cor-

* The reverse problem, by the solution of which the mean anomaly is found in terms of the true, being of little use, has not been introduced into the text. In order to solve it, find u from v by this expression,

$$\tan.\frac{u}{2} = \sqrt{\left(\frac{1+e}{1-e}\right)}\ .\ \tan.\frac{v}{2},$$

and then the mean anomaly $(n\,t)$ from

$$n\,t = e \sin.\, u + u.$$

responding to a given time: and this, as we have stated, is the first step towards the construction of Solar Tables. But it may be asked cannot the investigations of the Sun's elliptical place (which are investigations of no slight intricacy) be superseded by merely registering each day, his longitude. Will not, at the same distance of time from the equinox, the Sun's longitude be the same in 1800 as it was in 1750? Undoubtedly it would be so if the solar ellipse remained fixed in the heavens and of the same dimensions: and in such a case we could dispense, in the solar theory, at least, with Kepler's problem. But if the two preceding circumstances should not take place, if, for instance, the place of the apogee should not remain fixed, the intersection of the equator and ecliptic would not take place in the same point of the solar ellipse. The angular velocity, therefore, of the Sun, in his real orbit, would be variable at that point. It would not be the same in 1800 as in 1750: and, consequently, the Sun's longitude, after the elapsing of a certain time from his departure from the equinox, would not solely depend on such elapsed time. Predicaments similar to these would happen, if the dimensions of the solar orbit (its eccentricity for instance) should be changed. For the above reasons, then, we cannot rely solely on past observations of the Sun's longitude in predicting his future longitudes. Theoretical calculation must be combined with observation. The former will enable us, as we have seen, to assign a body's place in an ellipse when the time from the apside (the mean anomaly, in fact) and the eccentricity of the orbit are given. But, for the purpose of application, we must know the situation of the axis major, or the longitude of one of the apsides. For such knowledge we may have recourse to observation: not indeed to mere observation, but to observation combined with its appropriate method.

The methods then, of so using observations, that from them we may conveniently and exactly deduce the place and motion of the aphelion of a planet's orbit, and the quantity and variation of its eccentricity, will form the subjects of the ensuing Chapters.

CHAP. XIX.

On the Place and Motion of the Aphelion of an Orbit.—Duration of Seasons.—Application of Kepler's Problem to the determination of the Sun's Place.

It follows from what was remarked in p. 445, that the Sun in his perigee being at his least distance, and in his apogee, at his greatest, his apparent diameter in those positions would be respectively the greatest and least. If, therefore, we could, by means of instruments, measure the Sun's apparent diameter with sufficient nicety, so as to determine when it were the least, the Sun's longitude computed for that time, would, in fact, be the longitude of the apogee *.

Or if, computing, day by day, from the observed right ascension and declination, the Sun's longitude, we could determine when the increments of longitude were the least, the Sun's longitude, computed for that time, would be that of the apogee: for, the Sun's angular motion in that point is the least.

The difference of two longitudes thus observed, after an interval of time (t,) would be the angle described by the apogee in that interval. And if the longitudes were not accurately those of the apogee, still, if they belonged to observations, distant from each other by a considerable interval of time, the motion of the apogee would be deduced with tolerable exactness; since, in such a case, the error would be diffused over a great number of years.

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* *Apogee*, if the Sun be supposed to revolve, *Aphelion*, if the Earth; and, although, in reality, it is the latter body which revolves, yet, since it affects not the *mathematical* theory, we speak sometimes of one revolving, and sometimes of the other; and, with a like disregard of precision, we use the terms apogee and aphelion.

Thus, by the observations of Waltherus,

1496. Longitude of the apogee	$3^s\ 3^\circ\ 57'\ 57''$
In 1749, (by Lacaille)	3 8 39 0
∴ progressive motion in 253 years	0 4 41 3

whence the mean annual *progression** results equal to $1'\ 6''$: differing, however, from the result of better observations and methods by more than $1'\ 2''$.

Thus, in the Berlin Memoirs of 1785, M. Delambre, in treating of the Solar Orbit, compares the places of the apogee given by Waltherus (by Lacaille's Calculations) Cocheon King, La Hire, and Flamstead, with Maskelyne's.

Astronomer.	Year.	Longitude of Apogee.	Progression.
Waltherus	1496	$3^s\ 3^\circ\ 57'\ 57''$	$65''.385$
Cocheon King..	1279	3 0 8 0	64.606
La Hire	1684	3 7 28 0	62.116
Flamstead	1690	3 7 35 0	61.584
			4) 253.691
Mean result			63.423

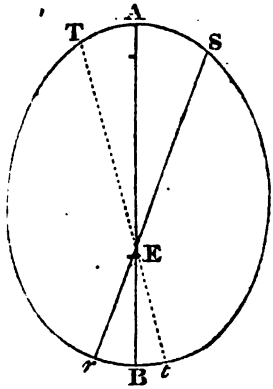
Hence, if the equinoctial year be estimated at $365^d 5^h 49^m 6^s.374$, the anomalistic year, since the time of describing $63''.423$

$$\left(=\frac{63''.423}{59'\ 8''.3}\times 24\right) = 25^m\ 42^s.4, \text{ is } 365^d\ 6^h\ 7^m\ 24^s.307.$$

The more accurate method, however, of determining the progression of the apogee rests upon a very simple principle. Let SEr be a right line, and draw TEt making with the axis major AB of the ellipse, an angle $TEA = SEA$: now, the time through $rBtS$ is less than the time through the remaining arc

* *Progression* is here meant to be used technically: a motion *in consequentia*, or, according to the order of the signs.

$SATr$: for, the equal and similar areas SEt, TEr, are described in equal times, but the area rEt is < area SET; therefore, by



Kepler's law (p. 445,) it is described in less time; therefore $rEt + SEt$, which is equal to the area $SErtS$, is described in less time than $SET + TEr$, which compose the area $SErTS$; therefore the body moves through the arc $rBts$ in less time than through STr. And this property belongs to every line drawn through E, except the line AEB, the major axis, or, the line of the apsides, that line which joins the aphelion and perihelion of the orbit.

Hence it follows, if, on comparing two observations of the Sun at S and at r, (that is, when the difference of the longitudes is 6 signs or 180 degrees) it appears that the time elapsed is not half a year, we may be sure, that the Sun has not been observed in his perigee and apogee. If the interval should be exactly, or nearly, half a year, then we may as certainly conclude, that the Sun was, at the times of observation, exactly, or very nearly, in the line of the apsides.

If the interval of time be nearly half a year, (which is the case that will occur in practice,) then we must find the true position of the apogee by a slight computation, which shall be first algebraically stated, and then exemplified.

The time from r to S = the time from r to B + the time from B to A − the time from S to A;

$\therefore$ time from B to A — time from r to S = ... (a)
time from S to A — time from r to B.

Now the first difference is known, being the difference between half an *anomalistic* year * and the observed interval of observation: and of the second difference, the second term may be expressed by means of the first: thus, let the first term $= t$: then by Kepler's law, (see p. 445,)

$$\text{time from } r \text{ to } B = t \times \frac{\text{area } rEB}{\text{area } SEA}$$

$$= t \times \frac{rB \times EB}{SA \times EA} \ (r \text{ and } S \text{ being near the apsides})$$

$$= t \times \frac{rB}{EB} \times \frac{EA}{SA} \times \frac{EB^2}{EA^2}$$

$$= t \times \frac{EB^2}{EA^2} \left(\text{since } \frac{rB}{EB} = \angle rEB = \angle S\overset{E}{B}A = \frac{SA}{EA}\right)$$

$$= t \times \frac{\text{angular velocity at } A}{\text{angular velocity at } B} \ (\text{see p. 470.})$$

Now, the angular velocities at A and B, or the increments of the Sun's longitudes at the apogee and perigee, being known from observation (see p. 431,) and the time from r to B being expressed in terms of those velocities and of t, the quantity t is the only unknown quantity in the equation (a) l. 1, and accordingly may be determined from it. But t being obtained, we can thence determine the exact time when the Sun (S) is at the apogee A: and his longitude, computed for that time, is the longitude of the apogee.

EXAMPLE.

1743.	Dec. 30, $0^h\ 3^m\ 7^s$ $\odot$'s longitude . . .	$9^s\ 8^0\ 29'\ 12''.5$
1744.	June 30, 0 3 0	3 8 51 1.5
	$\therefore$ difference of 2d and 1st longitudes	6 0 21 49

therefore at the 2d observation June 30th, the Sun was past S.

* The time from the Sun's leaving the apogee to his return to the same.

Third Operation—*Conversion of the Right Ascension in Time into Space.*

By Zach's Tables, Tab. XXIX, or Vince's, vol. II. p. 297,

6^h	3^s	0^0	$0'$	$0''$
39^m	0	9	45	0
39^s	0	0	9	45
$.3^s$	0	0	0	4.5
	3	9	54	49.5 Sun's right ascension in space.

The obliquity was $23^0\ 28'\ 4''$, from that and the right ascension find the Sun's longitude by Naper's Rule, or thus, by the *Tables of Reduction* to the ecliptic.

Fourth Operation—*Reduction of Equator to the Ecliptic* *.

See Zach's Table XXI, in his *Tabulæ Motuum Solis*, or Vince's Table, *Astronomy*, vol. II, p. 352.

Add 3^s.

	Reduction.	Difference for 1'.	
$6^s\ 9^0\ 50'\ 0''.0$	$0^s\ 0^0\ 47'\ 57''.45$	$4''.703$	
0 0 4 49.5	0 0 0 22.69	4	
(obliquity being $23^0\ 28'$)	0 0 48 20.14	18.812	for $4'\ 0''.0$
add for $4''$	0 0 0 0.27	3.88	0 49.5
	0 0 48 20.41	22.69	4 49.5
Sun's right ascension..	3 9 54 49.5		
Sun's longitude......	3 9 6 29.1		

and this is the whole of the process for the actual finding of the Sun's longitude from his observed right ascension.

By a similar process performed on Maskelyne's observation of the Sun's transit on the December 31, we have

$$\odot\text{'s longitude} = 9^s\ 10^0\ 31'\ 7''.6.$$

Fifth Operation—*Difference of Sun's Longitude found.*

The above are the Sun's longitudes when his centre was on

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* See Chapter XXI.

the meridian: they belong, therefore, to *apparent* noon: if, therefore, we add the equations of time (which are $3^m\ 13^s$, $3^m\ 53^s$, respectively) we shall have,

1776, June 30, $0^h\ 3^m\ 13^s$, ☉'s longitude	=	3^s	9^0	$6'$	$29''.1$
Dec. 31, 0 3 53	=	9	10	31	7.6
difference of Sun's longitudes		6	1	24	38.5

If we take from this $33'$, the half yearly progression of the apogee, we have the difference of the Sun's longitudes equal to

$$6^s\ 1^0\ 24'\ 5''.5\,;$$

consequently, by reason of the excess $1^0\ 24'\ 5''.5$ above 6^s, or 180^0, the Sun at the times of the two mentioned observations could not occupy, respectively, the extremities of a line drawn through the focus of the orbit. If t were his position on Dec. 31, at $0^h\ 3^m\ 53^s$, T could not have been his position on June 30, at $0^h\ 3^m\ 13^s$: or, if s were his position at the former time, S could not have been his position at the latter.

Suppose a to be the place of the Sun at the former time, then the difference between the longitudes of T and being 6^s, aT will be equal to $1^0\ 24'\ 5''.5$: in order to find the time of describing it, we have from the Solar Tables, or Nautical Almanack, or by the reduction of observations made on the noons of June 30, and July 1,

June 30, Sun's longitude	3^s	8^0	$23'$	$27''$
July 1, ...	3	9	20	40
	0	0	57	13

Hence, in 24 hours, nearly, the Sun moved through $57'\ 13''$, consequently, he described

$$1^\circ\ 24'\ 5''.5 \text{ in } 35^h\ 18^m\ 1^s \left(= 24\,\frac{1^0\ 24'\ 5''.5}{57'\ 13''}\right);$$

and consequently, he was at T on July 1, at $11^h\ 21^m\ 14^s$.

But, the two opposite positions of the Sun, instead of being, as we have supposed them to be, at T and t, might have been at A and B, or at S and s. In order to ascertain this point, we have the difference of the two times (Dec. 31, $0^h\ 3^m\ 53^s$, and

July 1, $11^h\,21^m\,14^s$) equal to $182^d\,12^h\,42^m\,39^s$. Now (see p. 480,) the half of an anomalistic year is $182^d\,15^h\,7^m\,1^s$ *: consequently, the time from t to T is less than the time from A to B, which it ought to be, since (as in p. 478,) the time from T to t = time from A to B − time from A to T + time from B to t = time from A to B − some quantity, whereas, if S and r had been the points, we should have had the time from S to T = time from A to B + time from S to A − time from r to B, = time from A to B + some quantity.

The Sun, therefore, must have been at some such opposite points as T and t, or, in other words, must, on July 1, $11^h\,21^m\,14^s$, have already passed the apogee.

What remains, then, to be done is the computation of the times of describing AT, Bt.

Sixth Operation—*Corrections of the Times of the Sun's passing the Apsides.*

Let t, t', be the times of describing them,

$$\text{then } t = \frac{t' \,.\, \text{area } AET}{\text{area } BEt} \quad \text{(see p. 479,)}$$

$$= \frac{t' \,.\, AT \,.\, AE}{Bt \,.\, BE} \quad \text{(the points } T, t', \text{ being near to the apsides)}$$

$$= \frac{t' \,.\, AE^2}{BE^2} = t' \,.\, \frac{(1+e)^2}{(1-e)^2},$$

e being the eccentricity.

$$\text{Hence, } t - t' = t' \,.\, \frac{4e}{(1-e)^2}, \text{ or } = t\frac{4e}{(1+e)^2};$$

$$\text{consequently, } \quad t = (t - t') \,.\, \frac{(1+e)^2}{4e},$$

$$t' = (t - t') \,.\, \frac{(1-e)^2}{4e},$$

and $t - t'$ = half the anomalistic year − the time from T to t in the case before us = $2^h\,25^m\,3^s$.

* Or more exactly $182^d\,15^h\,6^m\,59^s.4$.

Hence, if e be the eccentricity for 1776,

$$\log. 2^h\ 25^m\ 3^s = 3.939669 \ldots\ldots\ldots\ldots 3.939669$$

$$\log. \frac{(1+e)^2}{4e} = 1.187047 \quad \log. \frac{(1-e)^2}{4e} = 1.157859$$

$$(\log.\ 133880.5)\quad 5.126716 \quad (\log.\ 125718)\quad 5.097528$$

Hence, since

time at T is July 1, $11^h\ 21^m\ 14^s$ .... at t Dec. 31, $0^h\ 3^m\ 53^s$
$t = 37\ 11\ 20.5$ .... and $t' = 34\ 46\ 18$

∴ time at A June 29, 22 9 53.5 time at B Dec. 29, 13 17 35

which are, respectively, the times of the Sun's passing the apogee and perigee.

The interval of these two times, or the half of an anomalistic year is,

$$182^d\ 15^h\ 7^m\ 41^s.5.$$

The above methods* of determining the place of the apogee are due to Lacaille. That author, on the grounds of simplicity and uniformity, suggested the propriety of reckoning the anomalies from the perihelia of orbits, since, in the case of Comets, they are necessarily reckoned from those points. In the new Solar Tables of Delambre this suggestion is adopted, (see Introduction: also Vince's *Astronomy*, vol. III. Introduction, p. 2.)

In these new Tables the progression of the perigee, and consequently that of the apogee, is made to be about $61''.9$; and the mean longitudes of the perigee for 1750, 1800, 1810, are respectively stated at $9^s\ 8^0\ 37'\ 28''$; $9^s\ 9^0\ 29'\ 3''$; $9^s\ 9^0\ 39'\ 22''$.

The longitude of the winter solstice is 9^s; therefore in 1810 the perigee was $9^0\ 39'\ 22''$ beyond it; at this time, the daily motion of the Sun was $61'\ 11''$; therefore, the solstice happening on December 22, the Sun would be in his perigee about nine days after, or about December 31.

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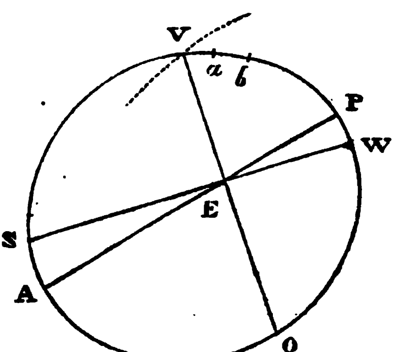
* The method is explained, with singular clearness, by Dalembert, in the historical part (L'Histoire) of the *Memoirs of the Academy of Sciences* for 1742.

From the longitude for any given epoch, and its annual progression, the position of the apogee and of the axis of the solar ellipse, may, by simple proportions, be found for any other epoch. Suppose, for instance, it were enquired when the axis of the solar ellipse was perpendicular to the line of the equinoxes? This, in other words, would be to enquire, when the longitude of the perigee was 270^0, or 9^s. Now, its longitude, in 1750, was $9^s\ 8^0\ 37'\ 28''$: the number of years therefore requisite to describe the difference, or $8^\circ\ 37'\ 28''$, taking the annual progression at $62''$, equals $\frac{8^0\ 37'\ 28''}{62''}$, or about 500 years; that is, the major axis was perpendicular to the line of the equinoxes in the year 1250.

The major axis coinciding with the line of the equinoxes the longitude of the perigee was 180°, or 6^s. Between that epoch, therefore, and 1250, the whole quantity of the *progression* of the perigee was $9^s\ 8^0\ 37'\ 28'' - 6^s = 3^s\ 8^\circ\ 37'\ 28''$: and the time of describing it since $\frac{3^s\ 8^\circ\ 37'\ 28''}{62''} = 5720$ was 5720 years. The epoch happened then about 4000 years before the Christian Æra, and is a remarkable one, inasmuch as chronologists consider it to be that of the beginning of the world.

The knowledge of the place of the perigee is necessary to determine the durations of seasons; which are perpetually



varying from its progression. If *W*, *S*, in the Figure, represent

the winter and summer solstices, V and O the vernal and autumnal equinoxes, PEA the axis of the solar ellipse; then, in the year 1250, P coincided with W; and, on that account, the time from the autumnal equinox O to the summer solstice W was equal to the time from W to the vernal equinox V. Past that year, P, by reason of its progressive motion, began to separate from W; and in 1800, the separation, measured by the angle PEW, was $9^\circ\ 29'\ 3''$. By means of this separation, those parts of the elliptical orbit in which the Earth's real motion is the quickest, being thrown nearer to V and away from O, the time from the autumnal equinox O to the solstice W, became gradually greater than the time from W to the vernal equinox: and the time from V to S became less than the time from S to O. In 1800, the following were nearly the lengths of the seasons:

V to S	92^d	21^h	44^m	28^s
S to O	93	13	34	47
O to W	89	16	47	20
W to V	89	1	42	23
length of year	365	5	48	58

This motion of the perigee also, as will be shewn in a subsequent Chapter, continually causes to vary the equation of time.

What has been said concerning the duration, and change of duration, of the Seasons, is, in some degree, digressive; the main object of the Chapter being to explain the method of finding the place, that is, the longitude of the perigee, in order that Kepler's problem might be applied to the determination of the Sun's place.

By Kepler's problem, we are enabled, from the mean anomaly, to assign the true anomaly, or true angular distance, reckoning from perigee*. The mean anomaly of the Sun, is his mean angular distance computed from perigee: in the Figure, if b be the Sun's mean place, it is $\angle\ PEb$. Now,

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* The mean anomaly is stated to be reckoned from perigee, since the succeeding extracts are from Delambre's new Solar Tables.

$$\angle\, PEb = \angle\, PEV - \angle\, VEb,$$

and, if V be the first point of *Aries*,

$$\angle\, PEV = 12^s - \text{mean long. perigee,}$$

$$\text{and} \quad \angle\, VE\,b = 12^s - \text{mean long. } \odot .$$

Hence, the mean anomaly is the difference between the mean longitudes of the Sun and of the perigee. And the Solar Tables assign the mean anomaly by assigning these longitudes. And then, in the same Tables, the mean anomaly is used as an *argument* for finding the equation of the centre. The process may be illustrated by specimens from the Tables, and their application to an Example.

From Table I.

Years.	Mean Longitude of the Sun.	Longitude of Sun's Perigee.
1809.	$9^s\ 10^0\ 42'\ 49''.8$	$9^s\ 9^0\ 38'\ 20''$
1810.	9 10 28 30.2	9 9 39 22
1811.	9 10 14 10.5	9 9 40 24

From Table IV.
Motion for Days. *November.*

Years. Com.	Years. Bissex.	Mean Longitude of the Sun.	Perigee.
Days.			
12	11	$10^s\ 10^0\ 28'\ 44''$	$53''.5$
13	12	10 11 27 52.3	53.6
14	13	10 12 27 0.7	53.8

From Table V.
Motion of the Sun for Hours, Minutes, Seconds.

Hours.		Minutes.		Seconds.	
H.	Motion of Sun.	M.	Motion of Sun.	S.	Motion of Sun.
1	2′ 27″.8	1	2″.5	1	0″.0
2	4 55 .7	2	4 . 9	2	0 . 1
3	7 23 . 5	3	7 . 4	3	0 . 1

From Table VII.
Equation of the Sun's Centre for 1810, with the Secular Variation. (S. V.)

Mean Anomaly.	Equation.	Diff. +	S. V.
10ˢ 12° 0′	11ˢ 28° 32′ 14″.7	13″.5	13″.13
10 12 10	11 28 32 28 . 2	13 . 5	13 . 09
10 12 20	11 28 32 41 . 7	13 . 5	13 . 06
10 12 30	11 28 32 55 . 2	13 . 6	13 . 03

Suppose now the Sun's longitude were required for 1810, November 13, 2^h 3^m 2^s.

Table I. 1st, the mean longitude for the beginning of 1810, is	....	9ˢ	10°	28′	30″.2
Table IV. Nov. 13	....	10	11	27	52 . 3
Table V. 2^h	....	0	0	4	55 . 7
Table V. 3^m	....	0	0	0	7 . 4
Table V. 2^s	....	0	0	0	0 . 1
rejecting 12ˢ, mean long. at time required (*a*)		7	22	1	25 . 7

The longitude of the perigee is to be had from the same Tables; thus:

Table I. Long. at beginning of 1810	9^s	9^0	$39'$	$22''.0$
Table IV. Nov. 13.	0	0	0	53.6
longitude of perigee at the time required ...	9	9	40	15.6
subtract this from (a) increased by 12 signs, there results the mean anomaly	10	12	21	10.1

With this mean-anomaly enter Table VII, and there results the equation to the centre	11^s	28^0	$32'$	$42''.2$
add to this the mean longitude (a)	7	22	1	25.7
	7	20	34	7.9

This result, $7^s\ 20^0\ 34'\ 7''.9$, is (if no other corrections are required to be performed) the true longitude reckoned from the *mean* equinox. But, as it has been shewn (pp. 353, &c.), the place of the equinox varies from the inequalities of the Sun's action, and of the Moon's action in causing the precession. Two *equations*, therefore, must be applied to the above longitude, in order to compensate the above inequalities, and so to correct the longitude, that the result shall be the true longitude, reckoned from the *true place of the equinox*. Now, it happens, by mere accident, that, in the above instance, the *lunar* and *solar* nutations are equal to $1''$, but affected with contrary signs. These corrections, therefore, affect not the preceding result. The correction for *aberration* (see p. 307,) has, in fact, been applied; for, since that, in the case of the Sun, must be nearly constant, (and it would be exactly so, if the Sun were always at the same distance from the Earth) the Solar Tables are constructed so as to include, in assigning the mean longitude, the constant aberration ($20''$). The variable part of the aberration (variable on account of the eccentricity of the orbit) is less than the 5th of a second. Let us see then, whether the longitude that has been determined, from a knowledge of the place of the perigee, and from Kepler's problem, expressed by means of Tables, be a true result:

By the Nautical Almanack for 1810; we have

Nov. 13, Sun's longitude	7^s	20^0	$29'$	$8''$
Nov. 14, ...	7	21	29	36
increase in 24 hours	0	1	0	28

Now the Sun's longitude is expressed in the Nautical Almanack for *apparent* time: and the equation of time being $-$ 15^m 33^s, the mean time is 11^h 44^m 27^s. Hence, we must find the increase proportional to 2^h 18^m 35^s, which is about $5'$ $47''$; consequently the Sun's longitude, on November 13, 2^h 3^m 2^s, (mean time) was 7^s 20° $34'$ $55''$, which differs from the preceding result, p. 490, l. 11, by about $47''$; consequently, Kepler's problem is not alone sufficient to determine the Sun's place, but some other corrections are requisite to compensate this error of 47 seconds.

Such corrections are to be derived from a new source of inequality; the perturbation of the Earth caused by the attracting force of the Moon and planets; the nature of which will be briefly explained in the ensuing Chapter.

CHAP. XX.

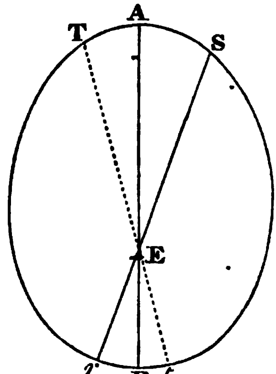
On the Inequalities of the Earth's Orbit and Motion, caused by the Disturbing Forces of the Moon and Planets. On the Methods of determining the Coefficients of the Arguments of the several Equations of Perturbation.

THE discovery of Kepler relative to the form of a Planet's Orbit did not extend beyond the proof of its being an ellipse: and in his *problem* he shewed the method of assigning the planet's place in such an ellipse.

If M be the mean anomaly and E the equation of the centre, then, the planet's elliptical place, or true anomaly is equal to

$$M + E.$$

Newton shewed, on certain conditions and a certain hypothesis, that that must needs take place which Kepler had found to take place. It appears from the third Section of his *Principia*, that if a body, or particle projected, from A perpendicularly to EA,



(E being the place of a body attracting a particle at A, and

elsewhere with a force inversely as the square of the distance from *E*), would describe an ellipse, of which *E* would be the focus.

The revolving particle or body *A*, is supposed to be attracted towards *E*, or to be incessantly urged towards *E*, by a *centripetal* force arising from a number of attracting particles, or from an attractive mass, placed at *E*. The centripetal force being the greater, the greater such mass is, and in that proportion.

If in *EA* produced, we place, at an equal distance from *A*, another body of equal mass, and of equal attractive force with the body at *E*, and again suppose the body at *A* to be projected; then, since it is equally urged to describe an ellipse round the new mass, as round that originally placed at *E*, it can describe an ellipse round neither, but must proceed to move in a direction perpendicular to *EA*.

In this extreme case, the elliptical orbit, and the law of elliptical motion would be entirely destroyed.

If now we suppose the mass of the new body to be diminished, or its distance from *A* to be increased; or, if we suppose both circumstances to take place, then, the derangement, or *perturbation*, of the body that is to revolve round *E*, will still continue, but in a less degree. An orbit, or curvilinear path, concave towards *E* in the commencement of motion, will be described; but, neither elliptical, nor of any other class and denomination.

In this latter case, the new body, being supposed less than the body placed at *E*, may be called the *disturbing* body; disturbing, indeed, by no other force than that of attraction, with which the body at *E* is supposed to be endowed; but which latter, from a difference of circumstance merely, is denominated a *Centripetal* force. In the first supposition, of an inequality of mass and distance in the two bodies, from the similarity of circumstance, either body might be pronounced to be equally attracting or equally disturbing.

The disturbing body, whatever be its mass and distance, will always derange the laws of the equable description of areas, and

of elliptical motion. If its mass be considerable, and its distance not very great, the derangement will be so much as to render the knowledge of those laws useless in determining the real orbit, and law of motion, of the disturbed body. In such case, Kepler's problem would become one of mere curiosity; and the place of the body would be required to be determined by other means.

If, however, the mass of the disturbing body be, with reference to that of the attracting body, inconsiderable, then the derangements, or perturbations, may be so small, that the orbit shall be nearly, though not strictly, elliptical; and the equable description of areas, nearly, though not exactly, true. Under such circumstances, Kepler's problem will not be nugatory. It may be applied to determine the place of the revolving body, supposing it to revolve, which is not the case, but which is nearly so, in an ellipse. The erroneous supposition, and consequently erroneous results, being afterwards *corrected* by supplying certain small *equations*, that shall compensate the inequalities arising from the disturbing body.

In the predicaments just described, are the bodies of the solar system. The mass of the Sun, round which the Earth revolves, is amazingly greater than that of the Moon *, which disturbs the Earth's motion: greater also, than the masses of the planets, which, like the Moon, must cause perturbations. The Earth, therefore, describes very nearly an ellipse round the Sun.

As a first approximation then, and a very near one, we may, as in the last Chapter, determine the Sun's, or Earth's place, by means of Kepler's problem: and subsequently correct such place, by small equations due to the perturbations of the Moon, and of the planets.

But, how are these small corrections to be computed? By finding, for an assigned time, an expression for the place of a

* The Sun is 1300000 times greater than the Earth, and the Earth more than 68 times greater than the Moon.

body, attracted by one body, and disturbed by another; the masses, distances, and positions, of the bodies being given; that is, by solving what, for distinction, has been called the *Problem of the Three Bodies*.

The consideration of *three* bodies is sufficient: for suppose, by the solution of the problem, the equation, or correction, for the Sun's longitude, to be expressed, by means of the Sun's and Earth's masses, distances, &c., and of other terms denoting the mass, distance, &c., of a third body; then, substituting, for these latter terms, the numbers that, in a specific instance, belong to the Moon, the result will express the perturbation due to the Moon. Instead of the Moon, let the third body be *Jupiter:* substitute, as before, the proper quantities, and the result expresses the perturbation due to *Jupiter:* and similarly for the other planets. The sum of all these corrections, separately computed, will be the correction of the longitude arising from the action of all the planets.

The above corrections are what are necessary to complete the process of finding the Sun's longitude, and to supply the deficiency of several seconds, from the true longitude. The number of corrections which it is necessary to consider, and which the latest Solar Tables enable us to assign, is five; arising from the perturbations of the *Moon, Venus, Mars, Jupiter,* and *Saturn.* Those of *Mercury,* the *Georgium Sidus, Ceres, Juno,* and *Pallas,* are disregarded.

The computation of these perturbations has been attempted in another place (see vol. II. on *Physical Astronomy*), by the approximate solution (all that the case admits of) of the *problem of the three bodies.* Even by the little explanation that has already (see p. 494,) been given, it is plain that the results of that solution are essential to the solar theory, and to the construction of Solar Tables. They are equally essential to the planetary theory. In fact, they are as much a part of Newton's System, as the elliptical forms of planetary orbits, and the laws of the periods of planets. The perturbation of the planetary system is as direct a consequence of the principle of universal attraction,

as the regularity of that system would be, on the hypothesis of the abstraction of disturbing forces. The quantities of the perturbations are, indeed, small and not easily discerned: but they are gradually detected as art continues to invent better instruments, and science, better methods, and they so furnish not the most simple proof, perhaps; but the most irrefragable proof of the truth of Newton's Theory.

Observation, it is plain, must furnish numerous results, before the formulæ of perturbations can be numerically exhibited, or, what is the same thing, be reduced into Tables. The positions and distances of the planets must be known: for, without any formal proof, we may perceive, that, according to the position of a planet, the effect of its disturbing force may be to draw the Earth either directly from, or towards, the Sun, or, in some oblique and transverse direction. In fact, the *heliocentric* longitudes of the Earth and the planets form the arguments in the Tables of perturbations.

Having thus explained, in a general way, the theory of perturbations, we will complete the Example of p. 490, by adding certain corrections, computed from that theory, to the Sun's longitude.

By p. 490, ⊙'s longitude	7^s	20°	$34'$	$8''$
correction due to ☽	0	0	0	5.5
to ♀	0	0	0	17.49
to ♂	0	0	0	4.32
to ♃	0	0	0	12.7
to ♄	0	0	0	0.65
∴ Nov. 13, 1810. 2^h 3^m 2^s; ⊙'s true long^e. .	7	20	34	48.86*

* This determination of the Sun's longitude is less by about 7 seconds than the longitude as stated in the Nautical Almanack. But, this latter was computed, (see Preface to the Nautical Almanack) from Lalande's Tables, inserted in the 3d Edition of his *Astronomy:* which differ by a few seconds from Delambre's last Solar Tables (Vince's, vol. III,) and from which the numbers in the text were taken.

By computations like these carried on by the aid of Tables (see pp. 490, &c.), the Sun's longitude is computed for every day in the year, and then registered; in the *Nautical Almanack* of Great Britain, the *Connoissance des Tems* of France, and in the *Ephemerides* of Berlin, and of other cities. The use of registering the Sun's longitude is explained in the Nautical Almanack, at p. 163, &c.

In page 495, l. 5, it was said that the problem of the three bodies was sufficient for the computation of all the inequalities. But this is rather, if we may so express ourselves, practically than metaphysically exact: it is founded on this, that, if v and i should be the perturbations of the Sun's *elliptical* longitude (L) by Venus and Jupiter, the resulting longitude will be

$$L + v + i,$$

whereas i ought, in strictness, to be computed for a longitude $L + v$, and v for a longitude $L + i$. The differences in the two cases are, however, insensible: v and i not exceeding $10''$.

We may add too, some farther limitation to the assertion, that the perturbations of the solar orbit (the variations produced in the Sun's elliptical longitude and distance) are to be computed, by means of the *problem of the three bodies.* Theory alone is not adequate to the above purpose. For, if the Earth be displaced from its elliptical orbit (be made *exorbitant*) by the action of a planet, the *displacement*, in a given position, will be the greater, the greater the mass of the disturbing planet. We must, therefore, know that mass, if we would, *a priori*, compute the displacement. Now, although the masses of Jupiter and Saturn are known from the periods of their satellites, the masses of Venus and Mars and Mercury are not. We can, indeed, setting out from certain effects of their action, indirectly approach, and approximate to, their values (see vol. II, p. 477, &c.). But the method is not a sure one; so that, in computing the perturbations of the Earth's orbit (of which that due to Venus from her proximity to the Earth is probably the greatest) we are obliged to look to other aid than that of mere theory.

The method to be pursued on this occasion is similar to that by which the corrections of the epoch, of the greatest equation, and of the longitude of the apogee, will be investigated in a following Chapter. Thus the true longitude, or L is equal to

$$M + E + P,$$

P being the sum of the perturbations, due to the actions of Mercury, Venus, the Moon, &c.: now the *arguments* of the perturbations are the differences between the longitudes of the disturbing planet and the Earth, or multiples of those differences: thus, if the symbols representing the Moon, Sun, Venus, &c. be made to denote their longitudes, the *argument* for the Moon's perturbation will be ☽ − ☉; for Jupiter's ♃ − ☉, 2(♃ − ☉); for Venus's (♀ − ☉), 2(♀ − ☉), &c.: so that, assuming a, b, c, &c. to be the coefficients of the arguments, the lunar perturbation will be denoted by a. sin. (☽ − ☉); Jupiter's by b. sin. (♃ − ☉) + c. sin. (2♃ − 2☉), &c. and accordingly, the whole perturbation or

$$P = a \,.\, \text{sin.}\,(☽ - ☉) + b \,.\, \text{sin.}\,(♃ - ☉) + c \,.\, \text{sin.}\,(2♃ - 2☉)$$
$$+ d \,.\, \text{sin.}\,(♀ - ☉) + \&c.$$

compute now the Sun's longitude from the elliptical theory, then, (supposing the epoch, greatest equation, &c. to be exact) the computed longitude will differ from the observed by an error C, which error arises from the perturbations of the planets; accordingly,

$$C = a.\text{sin.}\,(☽ - ☉) + b.\text{sin.}\,(♃ - ☉) + \&c.$$
$$+ d.\text{sin.}\,(♀ - ☉) + \&c.$$

in which ☽, ☉, ♃, the longitudes of the Moon, Jupiter, Venus, &c. are known, since C is the difference between two longitudes, one observed at a *given time*, the other computed for the same time. Repeat the operation: or find C', C'', C''', &c. the differences between certain observed and computed longitudes, and there will arise equations similar to the one that has been just deduced; and, it is plain, we may form as many equations as there are indeterminate coefficients a, b, c, &c. from which, by elimination, the values of a, b, c, &c. may be deduced. Or, we may form several groups or sets of equations, on the principle of formation which with be hereafter explained, and obtain, by addition,

equations that shall be, respectively, most favourable for the deductions of the values of *a*, *b*, *c*, &c.*

If the Moon's equation consist of one term, Venus's of two, Jupiter's of two, Mars of two, there will be required, at the least, seven equations for the determination of the seven coefficients. Now the same method, which has been here described for determining these coefficients, will be, in the next Chapter, used for determining the corrections of the *elements* of the solar orbit: which elements are here meant to be, the epoch of the mean longitude, the eccentricity, and the longitude of the apogee. Three equations, therefore, will be required for such purpose: consequently, if, by one and the same operation, we seek to correct the *elements*, and to determine the corrections due to the perturbations of the Moon and the above-mentioned three planets, we must employ, at the least, ten equations. We shall, however, soon see that it is more expedient to employ and to combine one hundred equations, in order to obtain, by virtue of the principle of *mean* results, exact results. No one of the coefficients of the *equations* of perturbations exceeds nine seconds †.

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* The principle is this: if *a* be the coefficient, select those equations in which the values of the term (a sin. A) is the greatest, make them all positive (by changing, if necessary, the signs of all the other terms of the equation) and add them together for the purpose of forming a new equation.

† If v be the longitude and δv be the *error* or *correction* due to the perturbations of the planets,

$$\begin{aligned}\delta v = {} & 8''.9 \sin.(☽ - ☉) + 7''.059 . \sin.(♃ - ☉) - 2''.51 . \sin. 2(♃ - ☉) \\ & + 5''.29 . \sin.(♀ - ☉) - 6''.1 \sin. 2(♀ - ☉) \\ & + 0''.4 \sin.(♂ - ☉) + 3''.5 \sin. 2(♂ - ☉).\end{aligned}$$

See *Physical Astronomy*, p. 311. M. Delambre, (Berlin *Memoirs*, 1785, p. 248), add one more equation for Jupiter, three for Venus, and three for Mars.

CHAP. XXI.

On the Methods of Correcting the Solar Tables. The Formula of the Reduction of the Ecliptic to the Equator, &c.

WE have, in the preceding Chapters, explained and illustrated the method, of finding *a priori*, or by theory and antecedent calculations, the Sun's longitude. The steps of the method are several. The first is to find, from a given epoch and elapsed time, the Sun's mean longitude (L): the next, to find, from the position of the apogee, at a given epoch, and the quantity and law of its *progression*, the longitude (A) of the apogee. The difference of these two angles, or $L - A$ is the mean anomaly (M), which is the third step: the fourth consists in finding (see p. 490,) the equation of the centre (E) corresponding to M. The sixth and last step is to find, at the given time of the required longitude, the sum (P) of the *perturbations* caused by the Moon and planets: the resulting longitude (S) is equal to

$$L - A + E + P,$$

$$\text{or } M + E + P,$$

setting aside the effects of nutation, aberration and parallax.

The results of the preceding methods, (those by which the equation of the centre and the perturbations of the planets are computed,) are registered in *Solar Tables*. From such Tables the national *Ephemerides*, the *Nautical Almanack* of England, the *Connoissance des Temps* of France, are partly computed. The immediate results from the Solar Tables are the Sun's longitudes. The Sun's right ascensions (which occupy the fourth columns of the second page of each month) are deduced from the longitudes and the obliquity; not, in practice, by Naper's Rules, but, (because the thing can be so more conveniently effected) by

the aid of a Table, entitled the *Reduction of the Ecliptic to the Equator.* The construction of such a Table is effected by means of a formula which it is now our business to investigate.

Let Æ, and ⊙ denote the Sun's right ascension and longitude, and let w be the obliquity of the ecliptic, then

$$\tan. Æ = \cos. w . \tan. \odot,$$

$$\text{and } \tan. (\odot - Æ) = \frac{\tan. \odot - \tan. Æ}{1 + \tan. \odot . \tan. Æ} = \frac{\tan. \odot\ (1 - \cos. w)}{1 + \cos. w . \tan.^2 \odot};$$

$$\text{but, (\textit{Trig.} p. 39.) } 1 - \cos. w\,(n) = \frac{2 \tan.^2 \frac{w}{2}}{1 + \tan.^2 \frac{w}{2}} = \frac{2\,t^2}{1 + t^2},$$

making $t = \tan. \frac{w}{2}$.

$$\text{Hence, } \tan. (\odot - Æ) = \frac{1 - n}{1 + n} . \frac{\sin. 2 \odot}{1 + \frac{1 - n}{1 + n} \cos. 2 \odot}$$

$$= \frac{t^2 . \sin. 2 \odot}{1 + t^2 \cos. 2 \odot},$$

$$\text{and thence, } \sec.^2 (\odot - Æ) = \frac{1 + 2\,t^2 \cos. 2 \odot + t^4}{(1 + t^2 \cos. 2 \odot)^2}.$$

$$\text{Now } d\,\{\tan. (\odot - Æ)\} = \frac{t^2 \cos. 2 \odot + t^4}{(1 + t^2 \cos. 2 \odot)^2}\, 2\,\delta \odot;$$

the symbol d denoting the differential,

$$\text{but, generally, } d\,(\text{arc}) = \frac{d\,(\text{tangent})}{(\text{secant})^2};$$

$$\therefore d\,(\odot - Æ) = 2\,\delta \odot \left(\frac{t^2 . (\cos. 2 \odot + t^4)}{1 + 2\,t^2 \cos. 2 \odot + t^4}\right).$$

$$\text{Now, if we assume } 2 \cos. 2 \odot = x + \frac{1}{x},$$

$$1 + 2\,t^2 \cos. 2 \odot + t^4 = (1 + t^2 x)\left(1 + \frac{t^2}{x}\right),$$

$$\text{and} \quad \frac{1}{1 + 2t^2 \cos. 2\odot + t^4} =$$

$$\frac{1}{1 - t^4}\{1 - 2t^2 . \cos. 2\odot + 2t^4 \cos. 4\odot - \&c.\};$$

multiply each side of the equation by $t^2 \cos. 2\odot + t^4$,

$$\text{and} \quad \frac{t^2 \cos. 2\odot + t^4}{1 + 2t^2 \cos. 2\odot + t^4}$$

$$= t^2 \cos. 2\odot - t^4 \cos. 4\odot + t^6 \cos. 6\odot - \&c.$$

$\therefore$ (see p. 501, l. 16,)

$$\int d(\odot - Æ) = t^2 \sin. 2\odot - \frac{t^4 . \sin 4\odot}{2} + \frac{t^6 . \sin. 6\odot}{3} - \&c.$$

$$\text{or}(\odot - Æ)\sin. 1'' = t^2 \sin. 2\odot - \frac{t^4 \sin. 4\odot}{2} + \frac{t^6 \sin. 6\odot}{3} - \&c.$$

or, very nearly, since $2 \sin. 1'' = \sin. 2''$, &c.

$$\odot - Æ = \tan.^2 \frac{w}{2} \frac{\sin. 2\odot}{\sin. 1''} - \tan.^4 \frac{w}{2} \frac{\sin. 4\odot}{\sin. 2''}$$

$$+ \tan.^6 \frac{w}{2} \frac{\sin. 6\odot}{\sin. 3''} - \&c.$$

In order to express the coefficients numerically, we have, assuming the obliquity equal to $23^\circ\ 28'$,

$$\log. \tan. \frac{w}{2}, \text{ or } \log. t = 9.3174299,$$

whence,

2 log. t	= 18.6348598
and log. sin. $1''$	= 4.6855749
log. c	3.9492849
4 log. t	= 37.2697196
log sin. $2''$	= 4.9866049
log. c'	2.2831147
6 log. t	= 55.9045794
log. sin. $3''$	= 5.1626961
log. c''	0.7418833

we have thus the logarithms of the three coefficients c, c', c'', by means of which it is easy to compute $\odot$ — Æ, when $\odot$ is given. The logarithm of the fourth coefficient (log. c''') = 9.24518043.

Hence, the *reduction* $(R) = c\,.\,\text{sin.}\ 2\odot - c'\,.\,\text{sin.}\ 4\odot$
$+ c''\ \text{sin.}\ 6\odot - c'''\ \text{sin.}\ 8\odot + \&\text{c.}$

If $\odot = 3^s$, sin. 2 $\odot$, sin. 4 $\odot$, &c. = 0, and the reduction, as it plainly must, is equal to 0.

If $\odot = 45°$, sin. 2 $\odot$ = 1, sin. 4 $\odot$ = 0, sin. 6 $\odot$ = − 1;
∴ the reduction = 8897″.85 − 5″.519 = 8892″.33
= 2° 28′ 12″.33,

and consequently, the right ascension (Æ = $\odot - R$) = 42° 31′ 47″.67, or, expressed in time, Æ = $2^h\ 50^m\ 7^s.17$.

If $\odot = 10°$, 2 $\odot$ = 20°, 4 $\odot$ = 40°, 6 $\odot$ = 60°, and, accordingly, we have the following computation,

log. sin. 20 . .	9.5340517		
log. c	3.9492849		
	3.4833366,	No. 3043.24	
log. sin. 40 . .	9.8080675		
log. c'	2.2831147		
	2.0911822,	No.	123″.36
log. sin. 60 . .	9.9375306		
log. c''	.7418333		
	.6793639,	No. 4.778	
log. sin. 80 . .	9.9933515		
log. c'''	9.2518043		
	9.2451558,	No.	.175
		3048.018	123.535
		0123.535	
	Reduction	2924.483	

Hence, the reduction $= 48' \; 44''.483$

and consequently, $\text{Æ} = 9^0 \; 11' \; 15''.517$.

In the two former instances the terms of the *reduction* were alternately positive and negative, and the reduction itself subtractive, or the right ascension less than the longitude. The contraries of these circumstances happen in the next instance.

Let $\odot = 9^s \; 5^0 \; 40'$, then

$$
\begin{array}{llll}
2\odot = 18^s \; 11^0 \; 20', & \sin. 2\odot & = & -\sin. 11^0 \; 20' \\
4\odot = 36 \;\; 22 \;\; 40, & \sin. 4\odot & = & \sin. 22 \;\; 40 \\
6\odot = 57 \;\;\; 4 \;\;\; 0, & \sin. 6\odot & = & -\sin. 34 \;\;\; 0 \\
8\odot = 73 \;\; 15 \;\; 20, & \sin. 8\odot & = & \sin. 45 \;\; 20
\end{array}
$$

Now,

sin. $11^\circ \; 20'$	9.2933995	
log. c	3.9492849	
	3.2426844	$1748''.55$
sin. $22^\circ \; 40'$	9.5858771	
log. c'	2.2831147	
	1.8689918	73.96
sin. $34^\circ \; 0'$	9.7475617	
log. c''	.7418833	
	.4894450	3.86
sin. $45^0 \; 20'$	9.8519970	
log. c'''	9.2518043	
	9.1038013	.127
		1826.497

Hence, the *reduction* $(\odot - \text{Æ}) = -\; 30' \; 26''.497$,

and consequently, $\text{Æ} = 9^s \; 5^0 \; 40' + 30' \; 26''.5$, nearly,

$= 9^s \; 6^0 \; 10' \; 26''.5$

and, in time, $= 18^h \; 24^m \; 41^s.7$.

In the Nautical Almanack for 1775, we have very nearly, this result, since,

$$\text{Dec. } 27, \quad \odot = 9^s\ 5^\circ\ 39'\ 59'',$$
$$Æ = 18^h\ 24^m\ 41^s.6;$$

but besides the difference of 1″, between the above longitude and the longitude used in our example, the obliquities are slightly different. On December 27, 1775, the obliquity was 23° 27′ 59″.7, whereas in the preceding instance it was assumed equal to 23° 28′.

The *correction* in the above, and in like instances, corresponding to any change in the obliquity is easily obtained: thus, since

$$\odot - Æ = \tan.^2 \frac{w}{2} \cdot \frac{\sin.\ 2\ \odot}{\sin.\ 1''} - \&c.$$

$$\delta\,(\odot - Æ) = \delta\, w \,.\, \tan.\frac{w}{2} \sec.^2 \frac{w}{2} \,.\, \sin.\ 2\ \odot - \&c.$$

which first term will be sufficient.

The Tables of *reduction* (see Zach's Tab. XXI. of his *Tabulæ Motuum Solis*, and Vince's *Astronomy*, Table XXXVII, vol. II.) contain a column of variations for every ten seconds of variation of obliquity.

A Table of *reductions* of the ecliptic to the equator is wanted, when, in constructing a work like the Nautical Almanack, we deduce from the Solar Tables the Sun's longitude, and from such longitude his right ascension. In examining and correcting Solar Tables, or the longitudes deduced from them, by the test of observations, corrections or *reductions* of a contrary nature are requisite. For, since the Sun's right ascension is observed, we stand in need of an easy process for *reducing* it to the longitude, or, we stand in need of a Table of the reduction of the equator to the ecliptic. We will now explain, by what artifice and rule, the preceding formula (see p. 502,) and a Table constructed from it, may be adapted to this latter purpose, since (see p. 501,)

$$\tan.\ Æ = \cos.\ w \,.\, \tan.\ \odot,$$

$$\tan.\ (90^\circ - \odot) = \cos.\ w \,.\, \tan.\ (90^\circ - Æ),$$

which equation is precisely of the same form as the preceding one of p. 501, l. 6: consequently, a similar formula must result from it, on changing what ought to be changed; that is, by writing $90^0 - \odot$ instead of Æ, and 90° − Æ, instead of $\odot$.

Hence,

$$(90^\circ - Æ) - (90^\circ - \odot) = t^2 . \frac{\sin. (180^0 - 2 Æ)}{\sin. 1''}$$

$$- \frac{t^4 . \sin. (360^\circ - 4 Æ)}{\sin. 2''} + \&c.$$

$$\text{or, } \odot - Æ = t^2 \frac{\sin. 2 Æ}{\sin. 1''} + t^4 . \frac{\sin. 4 Æ}{\sin. 2''} + \&c.$$

which is the formula required, and from which, as in the former case, a Table might be constructed. But it is desirable to avail ourselves of the former Table and to adapt it to this latter purpose. In order to find the means of so adapting it, make

$$Æ = a - 90^\circ,$$

then,

$$\odot - Æ = t^2 . \frac{\sin. (2 a - 180^0)}{\sin. 1''} + t^4 . \frac{\sin. (4 a - 360^0)}{\sin. 2''} + \&c.$$

$$= - t^2 . \frac{\sin. 2 a}{\sin. 1''} + t^4 . \frac{\sin. 4 a}{\sin. 2''} - \&c.$$

$$= - \left(t^2 . \frac{\sin. 2 (Æ + 90^\circ)}{\sin. 1''} - t^4 . \frac{\sin. 4 . (Æ + 90^\circ)}{\sin. 2''} + \&c. \right),$$

but, in the former case, see p. 502,

$$\odot - Æ = t^2 . \frac{\sin. 2 \odot}{\sin. 1''} - t^4 . \frac{\sin. 4 \odot}{\sin. 2''} + \&c.$$

the two series then are similar. If two Tables then were constructed, the numbers in each would be the same, in every case in which Æ + 90^0 and $\odot$ should be of equal values: for instance, the number expounding the *reduction to the equator* when $\odot = 113^\circ\ 4'$, would expound *the reduction to the ecliptic*, when Æ = $23^0\ 4'$. One Table then, would do instead of two. If the Table of the reduction to the equator be already computed, we

may thence deduce the reduction to the ecliptic corresponding to a given right ascension, by this simple rule. *Increase the Æ by 3ˢ and take out from the Table the reduction belonging to the angle 3ˢ + Æ: which reduction, with its proper sign, is the reduction to the ecliptic.*

The above-mentioned Table of the *reduction of the ecliptic to the equator* * is not, it is to be noted, necessary, nor, indeed, does it abridge the work of computation. The Trigonometrical process (rating it by the number of figures,) is shorter. But the *Table* is more convenient because it is inserted, in the same volume, with other Solar Tables, and is alone sufficient to effect its purpose.

If D be the declination of the Sun, then

$$1 \times \sin. D = \sin. \odot . \sin. w,$$

accordingly, from the Sun's longitude computed from Solar Tables, and from the obliquity (the apparent) of the ecliptic, the declination may be computed: and, in point of fact, the Sun's declination inserted in the fifth column of the second page (every month) of the Nautical Almanack is so computed: not necessarily, indeed, by the Trigonometrical formula just given: since, as in the former case of the deduction of the right ascension, the declination may be expressed by a series, and, in practice, may be computed by a Table entitled '*The Declination of the Points of the Ecliptic*'. (See Vince's *Astronomy*, Table XXXVIII, vol. II, and Zach's Tab. XXIII, of his *Tabulæ Motuum Solis*).

We will now return from this digression concerning the *reduction of the ecliptic to the equator*, and similar formulæ of reduction, to the main subject of the Chapter, and which indeed

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* The *reduction of the ecliptic to the equator* has been computed from the formula of page 502. But it is plain that *reductions* of like nature, but of different denominations, may be deduced from the same formula. For instance, the longitude of Venus in her orbit may be reduced to her longitude in the ecliptic: in which case w (see p. 502,) will be expounded by the inclination of Venus's orbit (about 3° 23′), and the series will rapidly converge.

is first announced in its title. The subject is the correction of the Solar Tables: or the method of so applying observations, made either before or after the epoch of the computation of the Tables, or hereafter to be made, as to correct, or to make more exact, the conditions or elements of such computation; and, for the more distinctly handling of the subject, we will recapitulate the steps of the process by which the Sun's longitude is taken from the Solar Tables.

(1.) The mean longitude (M) of the Sun is taken out of the Tables.

(2.) The mean longitude of the perigee (π) is also taken from the Tables.

(3.) The difference of the mean longitude of the Sun, and of the mean longitude of the perigee, is then taken, which gives the mean anomaly (A).

(4.) To the mean anomaly thus obtained the corresponding equation (E) of the centre is sought for in the Tables.

(5.) The equation of the centre thus obtained is, according to the position of the Sun in its orbit, added to or subtracted from the Sun's mean longitude, and the result is the Sun's *elliptical* longitude.

(6.) To the last sum or difference is added the sum (P) of the several perturbations of the Moon and planets.

(7.) Lastly, the preceding result must be corrected for aberration, and the two nutations, if the true apparent longitude of the Sun be required.

Any error or errors, therefore, in the steps of this process must, according to their degrees, vitiate the exact determination of the Sun's longitude.

The mean longitude, which is taken in the first step, is not taken *immediately* from the Tables, but is found by adding to the *Epoch*, as it is called, the mean motion during the interval between the epoch and the assigned time of the required longitude.

The *epoch* (*O*) is the Sun's mean longitude at a certain time. For instance, the *epoch*, or the Sun's mean longitude, on the mean noon of the first of January 1752, is

$$9^s\ 10^\circ\ 31'\ 32''.2,$$

the Sun's mean longitude, therefore, on April 3, 1752, is the above longitude, or epoch, plus the Sun's mean daily motion (59′ 8″.33) multiplied into 93 days, which latter product is

$$3^s\ 1^\circ\ 39'\ 54''.69,$$

so that, the Sun's mean longitude is

$$12^s\ 12^\circ\ 11'\ 26''.89,$$

that is, rejecting the 12 signs,

$$12^\circ\ 11'\ 26''.89,$$

and, if the longitude should be required at any time of the day of April 3, other than its noon, we must add to, or subtract from, the above longitude a proportional part of 59′ 8″.33. Thus, if the time should be April 3, $3^h\ 5^m\ 25^s$, we must add to the former longitude

$$7'\ 35''.9\ \left(= \frac{3^h\ 5^m\ 25^s}{24^h} \times 59'\ 8''.33\right),$$

so that the Sun's mean longitude will be

$$12^\circ\ 19'\ 2''.79^*.$$

We must now consider whether there is likely to be any error in the terms that compose the Sun's mean longitude.

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* The Tables from which the Sun's mean longitude, &c. are taken, are constructed for the meridian of Greenwich, but are easily adapted to any other meridian. Thus the epoch of the Sun's mean longitude for 1822, in Vince's Solar Tables, is $9^s\ 10^\circ\ 33'\ 59''.6$: Dublin Observatory (to take an instance) is $25^m\ 20^s$ west of Greenwich, and the Sun's motion in $25^m\ 20^s$ is equal to $\frac{25^m\ 20^s}{24^h} \times 59'\ 8''.33$, or $1'\ 1''.69$; therefore the epoch for Dublin is $9^s\ 10^\circ\ 35'\ 1''.29$.

The Sun's mean motion is, probably, known to a great degree of accuracy. For, it is determined by comparing together distant observations of the Sun's longitudes and by dividing the difference of the longitudes by the interval of time between them. Any small error, therefore, made in the Sun's longitude will, by reason of the above division, very slightly affect the determination of the Sun's mean motion.

Thus, supposing the mean motion is to be determined by comparing the observations of 1752 and 1802, and the error of Bradley's observations at the former period to have been $5''$, the corresponding error in the difference of the longitudes would amount only to $\frac{5''}{50}$, or $0''.1$.

But the case is somewhat different with the epoch. There is no part of the process in determining it that has an effect, like that we have just described, in lessening its errors. The mean longitude at any epoch, 1752 for example, must depend for its accuracy on individual observations made at that epoch, or, at the most, on the mean of such observations. The Sun's right ascension must be determined (according to the method described in Chapters VII and XVI,) and the Sun's longitude must be thence deduced. The mean longitude, therefore, of the epoch is subject to some uncertainty, and, consequently, the mean longitude of the Sun at the proposed time will be alike subject to the same. Hence, if t be the time elapsed since the epoch, and m be the Sun's mean motion, since

$$M = O + mt,$$

$$dM = dO = z.$$

Suppose, in the next step (see p. 508, l. 9,) the longitude (π) of the perigee to be taken. Now, it is plain, if we revert to pages 477, &c. that there is some uncertainty in that method, or that there may be a probable error of several seconds in the determination of its longitude: such error then will affect the mean anomaly (A), and exactly by its quantity, since

$$A = M - \pi;$$

therefore, if $+ x$ be the error in π, $- x$ will be the corresponding error in A: but (see p. 468,) the equation of the centre (E) depends on A, and, according to the value of A, will be increased or decreased by a given error in A. Now any error in the equation of the centre, will affect, with its exact quantity, the true longitude, since this latter equals $M \pm E$, the effects of planetary perturbation and of the inequalities not being considered.

This is one effect on the longitude produced by an error in the equation of the centre: which error is derived, through the mean anomaly, from the error of the longitude of the perigee. But there is a second source of error of the equation of the centre arising from an uncertainty or error in the determination of the eccentricity, or [since (see p. 473,) the *greatest* equation of the centre is expressed in terms of the eccentricity,] from an error in the greatest equation of the centre. This error, according to the value of the mean anomaly, that is, accordingly as the equation of the centre is to be added to, or subtracted from the mean longitude in the finding of the true longitude, will cause a positive or negative error in the resulting value of the true longitude.

Hence, since the true longitude, or

$$L = M \pm E + P,$$

$$\text{or } = O + mt \pm E + P,$$

$$dL = dO \pm \frac{dE}{d\pi} d\pi \pm \frac{dE}{de} de.$$

Supposing P the sum of the perturbations to be rightly determined, and denoting by $\frac{dE}{d\pi} d\pi$ the error in E, arising from an error ($d\pi$) in the longitude of the perigee, and by $\frac{dE}{de} de$ the error in E, arising from the error in the eccentricity.

What now remains to be done is to find the means of stating these variations (dO, $d\pi$, dE) under a form fitted for arithmetical computation. The error dO may be (see p. 510,) expressed by z, since if z ($5''$ for instance,) be the error in the mean longitude, z

($5''$) will be the corresponding error in the true longitude.

$$\text{Next, } \frac{dE}{d\pi} d\pi, \text{ or } x,$$

affects the longitude by altering, through the mean anomaly, the equation of the centre. Since (see p. 473,) we have an expression for the equation of the centre in terms of the mean anomaly, we can find the error in the former corresponding to a given error in the latter: but it is most convenient, for such purpose, to use the Tables already constructed. Suppose then [for it is necessary (see p. 511,) to take an instance] the mean anomaly to be $6^s\ 18^0$; we find in the Solar Tables,

anomaly	$6^s\ 18^0\ 0'$,	equation of centre	$0^0\ 35'\ 43''.4$
anomaly	$6\ \ 18\ \ 10$,	equation	$0\ \ 36\ \ \ 2.2$
	$0\ \ 0\ \ 10$		$0\ \ 0\ \ 18.8$

Hence, to a variation of $1'$ in the anomaly, there corresponds $1''.88$ in the equation, and, accordingly,

$$60'' : x :: 1''.88 : x \times \frac{18.8}{60} = .0313\, x.$$

We may make a like use of the Solar Tables in finding the numerical value of $\frac{dE}{de} de$. If the eccentricity be changed, the greatest equation of the centre is changed. Now in the Solar Tables the *secular* variation of the greatest equation (when the anomaly is of a certain value) is supposed to be $17''.18$, and corresponding to such a variation, the proportional secular variation of the equation of the centre, corresponding to a mean anomaly $= 6^s\ 18^0$, is $5''.15$.

Hence, if y be the variation or error of the greatest equation,

$$17''.18 : 5''.15 :: y : \frac{5''.15}{17''.18} \times y = .2969\, y,$$

which is the corresponding error in the equation of the centre belonging to an anomaly of $6^s\ 18^0$: we have now then, in this instance,

$$dL = z + .0313\, x - .2969\, y,$$

dL is an error of the *computed* longitude arising from errors in the epoch, the place of the perigee and the value of the greatest

equation. In order to find its value we must compare the computed with the observed longitude (or rather the longitude computed from an observed right ascension and the obliquity of the ecliptic): the difference of the two longitudes, on the supposition of the exactness of the latter, is dL or C, then

$$C = z + .0313\,x - .2969\,y,$$

and in order to determine z, x and y, there is need of two other similar equations.

In page 482, from observations of the Sun's right ascension and the obliquity of the ecliptic, the Sun's longitude was found equal to

$$3^s\ 9^0\ 6'\ 29''.1\,;$$

whereas, in the Nautical Almanack, the computed longitude is

$$3^s\ 9^0\ 6'\ 43''$$

the error of the Tables, then, or C is $13''.9$.

In the instance we have given, the anomaly was assumed equal to $6^s\ 18^\circ$, and the Solar Tables were, on grounds of convenience, made use of to determine the coefficients of y and z. That was effected by merely taking from the Tables the secular variation corresponding to the given anomaly, or to the corresponding equation of the centre, and the difference or variation of the equation of the centre corresponding to a difference of ten minutes in the anomaly. It is plain, then, the coefficient of y will be the greater, the greater is the secular variation, which is the greater the nearer the proposed anomaly is to that anomaly to which the *greatest* equation of the centre corresponds. Now the greatest equation of the centre happens (see p. 472,) in points near to those of the mean distances. The Sun is at his mean distance in March and September. Hence, if we select from observations those made towards the latter ends of those months, and derive equations similar to the above, the coefficients of y will be, nearly, as great as they can be. The contrary will happen, in such observations, to the coefficients of x: since these depend on the *variation* of the equation of the centre corresponding to a given variation of the mean anomaly, they must needs be the smallest when the

former variation is at its least; which happens near to the mean distances, when the equation of the centre is at its maximum. The reverse of this whole case will happen if we select observations made near to the apogee and perigee, the *secular* variation * of the equation of the centre is then the least: but the variation of the equation of the centre, corresponding to a given variation of the anomaly, is the greatest. The coefficients, therefore, of x, in this case, will be as great as they can be, and those of y as small. Hence, if we possess a long series of observations, we have it in our power so to use them, that in the derived equations (such as that of p. 512,) the coefficients of x and y shall be, respectively, as large as possible.

For instance, on March 24, 1775, the Sun's mean anomaly, as it appears by the Tables, was

$$2^s\ 22^\circ\ 42'\ 44''.7.$$

The secular variation is $17''.12$, the *difference* $2''.2$; therefore (see p. 512,) the coefficient of $y = \frac{17.12}{17.18}$ $(= .9965)$, of $x = \frac{2''.2}{60} = .0366$, consequently, if the *error* of the Tables (the difference of the computed and observed longitude) were $-1''.7$, we should have

$$-1''.7 = z + .9965\,y - .0366\,x.$$

Again, (about half a year afterwards, the Sun being again near his mean distance) on September 23, we find anomaly $8^s\ 23^0\ 4'$, secular variation $= 16''.95$, *difference* $= 2''.9$;

* The *secular* variation of the greatest equation of the centre is its variation, (arising from a change in the eccentricity of the orbit) in one hundred years. Its present value is $17''.18$, and whenever the *greatest* equation is changed, every other equation of the centre is changed. If $15''$, or $17''.18$ be the change in the former, there will be, in every case, a proportional and calculable change in the latter. But it is *convenient* to use the change $17''.18$ (denominated for the reasons above specified the *secular*) because, in the Solar Tables, we find the proportional change affixed to every equation of the centre.

therefore the coefficients of y and x are $\frac{16.95}{17.18}$, $\frac{.29}{6}$, and, if the error of the Tables were $5''.4$, we should have this equation

$$5''.4 = z - .9866\, y + .0483\, x,$$

and if we selected fifty observations, half made near to the end of March, the other half near to the end of September, the former would all resemble the first equation, the latter the second; in each the coefficient of y must be large, but in the former the coefficient must be positive, in the latter negative, since, when the mean anomaly is about $2^s\ 20°$, the equation of the centre is additive, when about $8^s\ 20°$, subtractive.

In like manner if we select two observations made near the apsides, on June 25, and December 28, 1784, we have

June 25, anomaly $5^s\ 25^0\ 1'\ 33''$, secular var^n. $1''.44$. *diff.* $19''.6$
Dec. 28, ... 11 28 20 51.6........... 0.5 .... 20.5

and accordingly, the coefficients of x and y are

$$\frac{1''.44}{17.18},\ \frac{0''.5}{17.18} \text{ and } \frac{1''.96}{6},\ \frac{2.05}{6},$$

and the two resulting equations, if the errors of the Tables be, respectively, $-3''.4$, $-1''.5$

$$-3''.4 = z + .0838\, y + .3266\, x$$
$$-1''.5 = z - .02913\, y - .3416\, x,$$

and, in all pairs of equations so derived (from observations made near to the apsides and distant from each other by about six signs) the coefficients of x will be as large, as they well can be, and the coefficients of y, as in the former pairs of equations, will be respectively positive and negative.

Suppose then, we had, in all, one hundred equations, fifty derived from observations near the mean distances, fifty from observations near the apsides, and that we added the one hundred equations together: then the coefficient of z would be one hundred, and the coefficients of y and x would be the excesses of the positive coefficients, in the several equations, above the negative: the equation divided by one hundred would be of this form,

$$A = z - ay + bx \text{ .... (1).}$$

In order to obtain a second equation, take the fifty equations derived from observations near to the mean distances, then twenty-five of these equations (see p. 515,) must be of the form,

$$-1''.7 = z + .9965\,y - .0366\,x,$$

twenty-five of the form $5''.4 = z - .9866\,y + .0483\,x$,

change the signs in every one of the latter twenty-five, then there will be twenty-five equations such as

$$-1''.7 = \quad z + .9965\,y - .0366\,x,$$

twenty-five, such as $-5''.4 = -z + .9866\,y - .0483\,x$.

Add now the whole fifty together and the z's will disappear; the coefficient of y will be the *sum* of such quantities as .9965, .9866, &c. the coefficient of x will be result of combining several positive and negative quantities: the resulting equation divided by the sum of .9965, .9866, &c. will be of the form

$$B = y - mx \,\ldots\ldots\ldots\ldots\, (2).$$

Proceed in like manner with the fifty equations derived from observations made near to the apsides: that is, since the object is to make the coefficient of x, in the resulting equation, as large as possible, make the coefficients of x, in all equations, such as the one of p. 315, l. 20, positive, by thus writing it,

$$1''.5 = -z + .02913\,y + .3416\,x,$$

then, in all the fifty equations, the coefficients of x will be positive: add together the fifty equations, and the coefficient of x will be the sum of fifty quantities such as .3266, .3416, &c. and the coefficients of y and z will be the *differences* of certain quantities: divide by the coefficient of x, and the resulting equation will be of the form

$$C = pz + qy + x \,\ldots\ldots\ldots\ldots\, (3).$$

And it is from these three equations $\{(1), (2), (3),\}$ that the values of x, y, z, are to be derived by elimination.

The principle in the above process of combining sets of equations in order to produce a mean equation is obvious: if x, or y,

or z is to be determined, the larger its coefficient the more exact will be its resulting mean value.

In what has preceded, we have, in substance, followed Delambre's method in the Memoirs of the Academy of Berlin for 1786. In these Memoirs, which are on *the Elements of the Solar Orbit*, one hundred equations are used, fifty from observations of the Sun near his mean distances, fifty from observations of the Sun at his greatest and least distance. The results (see *Mem. Acad. Berlin*, 1786, p. 243,) of M. Delambre, are

correction of the epoch .... = $-$ 0''.4092,
of the longitude of the apogee ..... = $-$ 24''.71,
of the greatest equation of the centre .. + 0''.3227,

which corrections are to be applied to Mayer's Tables, with which Delambre compared Maskelyne's Observations.

By means such as we have described, Mayer's Tables were corrected. The errors of the *corrected* Tables were found not to exceed 9''. The sum of the hundred errors (of the positive and negative together,) amounted to 318''.3, and, therefore, the mean error was 3''.183, which, as the learned author remarks *, is, considering all circumstances, a very small error.

The method of correcting *at one operation* all the elements is what is now generally practised. But, in a preceding volume of the Berlin Memoirs (for 1785,) Delambre corrects the elements individually, by the comparison of particular observations with the results obtained from the Solar Tables. Thus, suppose the longitude of the apogee, or the longitude of the Sun occupying the apogee, to be found, on June 29, at $22^h\ 37^m\ 37^s$ to be

* "Et si l'on se rappele que ces erreurs si peu considerables sont pourtant produites par trois à quatre causes differentes, comme les erreurs des observations, celles des reductions, celles des catalogues d'ètoiles, enfin les quantities negligés ou peu connues dans la theorie, on s'etonnera peut-être que les Geometres et les Astronomes aient pu les renfermer entre des limites aussi etroites, et l'on ne pourra gueres se flatter d'ajouter beaucoup à une parelle precision."

$3^s\ 9^0\ 3'\ 8''$. But, according to Mayer's Tables, the longitude of the apogee was $3^s\ 9^0\ 6'\ 43''$, therefore $-\ 3'\ 35''$ was the correction of such longitude. The same observation corrects also the *mean longitude* of the Tables: for, at the apogee, the mean and true longitudes are the same. The mean longitude, therefore, was $3^s\ 9^0\ 3'\ 8''$: but the Tables gave $3^s\ 9^0\ 3'\ 20''$. The correction, therefore, for the epoch of the Tables, according to the above observation, was $-\ 12''$.

But, whichever be the method employed, it is essential to its accuracy that all the sources of inequality by which the Sun's true longitude is made to differ from its mean, should be known: for, otherwise, the longitude of the apogee, or the equation of the centre, might be wrongly corrected. Before the discoveries of Newton, for instance, those differences of the observed and computed longitudes which are due to planetary perturbation, would, from ignorance of their causes, have been attributed to errors in the epoch, equation of the centre, and longitude of the perigee; and, had such a method of correcting those errors been used as has been already (see pp. 512, &c.) described, its results would have given wrong corrections.

It needs scarcely be observed that the assigning of the laws and quantities of the perturbations caused by the planets is a difficult operation. The *arguments* (see *Physical Astron.* Chap. VII, &c.) may be derived from theory, but their coefficients must be determined from observations. M. Delambre has accomplished these objects, by the comparison of 314 of Maskelyne's observations, and by Laplace's Formulæ. The learned Astronomer in his first correction of the Solar Tables reduced their errors within 15″, whilst the errors of Mayer's Tables sometimes exceeded 23″. But, as he found that the computed and observed longitudes could not be brought nearer to each other, and as their differences did not follow a regular course (in which case they might have been, in part, attributable to the errors of observation) he suspected that the solar theory was in fault, or rather, that part of it which assigns the correction of the Sun's elliptical place on account of the perturbations of the planets. In this emergency he had recourse to Laplace, who, from his Theory, derived two equations due to

Mars' action, the sum of which might amount to $6''.7$: the same great mathematician also assigned $6''$ for the value of the principal term of the lunar equation, and $9''.7$ for the maximum of the equation of Venus.

There are also some other points to be attended to in the correction of the Solar Tables: for instance, the value of the obliquity of the ecliptic. For the *observed longitudes* with which the longitudes derived from the Solar Tables are compared, are, in fact, (see p. 513,) computed from the observed right ascension and the obliquity of the ecliptic, and, therefore, their accuracy depends, in part, on that of the obliquity.

In the deduction of the equations of condition, the coefficients of x and y (see pp. 512, &c.) were obtained by the aid of Solar Tables: an operation, as we then stated, of mere convenience and in nowise essential. If we had not been able to avail ourselves of Tables, we should then have been obliged to have gone back to the very formulæ used in constructing the Tables. And this indeed, but with some loss of expedition, would have been the most scientific proceeding.

We subjoin these formulæ, some of which have been already given.

If e be the eccentricity, and E the greatest equation,

$$e = \frac{1}{2} E \sin. 1'' - \frac{11}{768} E^3 \sin.^3 1'' - \frac{587}{983040} E^5 \sin.^5 1''$$
$$- \frac{40583}{2642411520} E^7 \sin.^7 1'' - \&c.$$

If $E = 1^0\ 55'\ 26''.82$ (its value in 1780) $e = 0.016790543$,

If $Z = nt$, be the mean anomaly, the equation of the centre is equal to

$$-1^\circ\ 55'\ 26''.352 \sin. Z + 1'\ 12''.679 \sin. 2Z - 1''.0575 \sin. 3Z$$
$$+ 0''.018 \sin. 4Z,$$

and the true anomaly (a) is equal to $Z - 1^\circ\ 55'\ 26''.352 \sin. Z + \&c.$, and the *differential* of the true anomaly, or da is equal to

$$dZ - dZ \,.\, \sin. 1'' \times 1^\circ\ 55'\ 26''.352 \,.\, \cos. Z$$
$$+ 2dZ \,.\, \sin. 1'' \times 1'\ 12''.679 \cos 2Z - \&c.$$

let $dZ = \frac{1}{24}(59'\ 8''.2) = 2'\ 27''.8416$ the Sun's * mean horary *anomalistic* motion : da is the Sun's elliptical horary motion, and
$da = 2'\ 27''.8416 - 4''.9645 \cos. Z + 0''.1042 \cos. 2Z - 0.002 \cos. 3Z.$

In order to obtain the horary motion in longitude on the ecliptic, we must, since $\frac{1}{24}(59'\ 8''.33) = 2'\ 27''.8471$, write in the above value of da, this latter quantity instead of $2'\ 27''.8416$.

If v be the Sun's true anomaly, $Z - v$ is the equation of the centre, and the greatest value of $(Z - v)$

$$= \left(2e + \frac{11}{48}e^3 + \frac{599}{5120}e^5 + \frac{17219}{229376}e^7 + \&c.\right)\frac{1}{\sin. 1''},$$

and (v)

$$= 90^0 - \left(\frac{3}{4}e + \frac{21}{128}e^3 + \frac{3409}{40960}e^5 + \frac{99875}{1835008}e^7 + \&c.\right)\frac{1}{\sin. 1''},$$

and the sum of these two equations gives that value (Z) of the mean anomaly to which the greatest equation belongs, and, accordingly,

$$(Z) = 90^\circ + \left(\frac{5}{4}e + \frac{25}{384}e^3 + \frac{1383}{40960}e^5 + \frac{39877}{256 \times 7168}e^7\right)\frac{1}{\sin. 1''}.$$

If we neglect the terms beyond the second, we have

$$(Z) = 90^\circ + \frac{5}{4}\,\frac{e}{\sin. 1''} = 91^\circ\ 12'\ 9''.5\dagger,$$

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* The time and Sun's motion being dated from the perigee, and the perigee being progressive (see p. 486,) at the annual rate of 62", the horary motion is that same portion of 360° which 1 hour is of the time of the Sun's leaving his perigee, to his return to the same : which time is an anomalistic year.

† Log. $5e = 8.9240351$
log. sin. $1'' = 4.6855749$

$4.2384602 = \text{log. } 4329''.5 = \text{log. } 1^\circ\ 12'\ 9''.5.$

Now,

in the solar orbit, in which, at the epoch of 1780,

$$e = .016790543.$$

Since, in the Earth's orbit (e^4, e^5, &c. being extremely small),

$$E = -\left(2e - \frac{e^3}{2^2}\right)\text{ sin. } Z + \frac{5}{2^2}e^2 \text{ sin. } 2Z - \frac{13\,e^3}{2^2.3}\text{ sin. } 3Z,$$

$$dE = -de\left((2 - .75\,e^2)\text{ sin. } Z + 2.5\,e \text{ sin. } 2Z - \frac{13}{4}e^2 \text{ sin. } 3Z\right);$$

therefore, if we make dE to represent the secular variation of the greatest equation of the centre, we have

$$dE \text{ being } = 17''.18,$$

$$de = -\frac{17''.18}{2 \text{ sin. } (Z) - 2\,.\,5\,e \text{ sin. } (2Z) + \frac{13}{4}e^2 \text{ sin. } (3Z)},$$

(Z) being the anomaly (91° 12′ 9″.5) belonging to the greatest equation.

From this equation the secular variation of the eccentricity may be computed.

The variation of the equation of the centre is to be had from the formula of l. 5, and if, in that formula, we substitute for de the secular variation of the eccentricity, the result will be the secular variation † of the equation of the centre corresponding to the anomaly Z. By such an expression, then, we are able to dispense with the Solar Tables, or, which amounts to the same, to compute what is therein computed.

In the preceding pages of this Chapter frequent mention has been made of the *secular* variation of the eccentricity, and (which

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Now,

log.................................. 360° = 2.5563025

anomalistic year = $365^d.25971$ log. = 2.5626017

$$9.9937008 = 0^\circ.9856$$
$$= 59'\ 8''.16,$$
$$\text{and } \frac{1}{24}\text{th} = 2\ 27.84.$$

* Expressed by $17''.177 \text{ sin. } Z - 0''.03606 \text{ sin. } 2Z - 0''.0078 \text{ sin. } 3Z$.

depends upon it) on the secular variation of the greatest equation of the centre. Now these are, as the terms themselves import them to be, the variations effected in one hundred years, and the terms are never applied except to the changes that happen in quantities nearly constant. The method of determining their values, is, in fact, contained in that process (see pp. 511, &c.) by which the elements themselves are determined. Thus, with regard to the greatest equation of the centre, its value ought first to be corrected by comparing the observed longitudes of 1752, for instance, with the computed longitudes. In a second operation, by comparing, for instance, the observed longitudes of 1802, with the computed. The result of each operation would be a corrected value of the greatest equation of the centre. The difference between such values would be the *variation* in fifty years, or would be half the *secular variation*.

There is a method*, other than what has been given, for correcting the elements: it consists in making the sum of the squares of equations like (1), (2), (3), (see p. 515,) a minimum: for instance, using, for illustration, the equations obtained in pp. 515, 516, we should have

$$(1''.7+z+.9965\,y-.0366\,x)^2+(-5''.4+z-.9866\,y+.0483\,x)^2$$
$$+(3''.4+z+.0838\,y+.3266\,x)^2+\&c. = \text{a minimum},$$

and, accordingly, making y to vary,

$$.9965\,(1''.7+z+.9965\,y-.0366\,x)$$
$$-.9866\,(-5''.4+z-.9866\,y+.0483\,x)$$
$$+\&c. = 0.$$

In like manner, make x to vary, and z to vary, and obtain similar equations: then, from the three resulting equations thus obtained, eliminate x, y and z.

We have explained what ought to be understood by the *secular* variation of an element: and there is, what is called, the *secular* motion of the Sun, which is the excess of the Sun's longitude above 36000° in 100 Julian years: a Julian year con-

* Laplace, *Sur les Probabilités*, Chap. IV. Biot, *Phys. Astron.* tom. II. Chap. X.

sisting of 365 ¼ days. Now, by comparing together the Sun's mean longitudes at different epochs, it appears that, in 100 Julian years, or in 36525 years, the Sun's motion = 36000° 45′ 45″, accordingly, in one Julian year of $365^d\ 6^h$, the Sun's motion is 360° 0′ 27″.45, or 12^s 0′ 27″.45; accordingly,

in 1 Julian year of $365^d\ 6^h$ the Sun's motion =	360° 0′ 27″.45
and, in 1 common year of 365 .... =	359 45 40.37
in a Bissextile year of 366 .... =	360 44 48.697

and, accordingly, to find the epochs of the Sun's mean longitude on years succeeding a given epoch, add, for common years, repeatedly, to the epoch, 11^s 29° 45′ 40″.37, and reject the 12^s, or subtract 14′ 19″.63.

When a Bissextile year occurs, add

12^s 44′ 48″.697, or 44′ 48″.697.

Thus, 1781, epoch of Sun's mean longitude	9^s	11°	29′	9″.5
	0	0	14	19.63
epoch for 1782	9	11	14	49.87
	0	0	14	19.63
epoch for 1783	9	11	0	30.24
	0	0	14	19.63
epoch for 1784	9	10	46	10.6
1784, is a Bissextile, therefore add	0	0	44	48.697
epoch in 1785	9	11	30	59.3

Thus the epochs are successively formed: but, if we wish to deduce, at once, the epoch of 1821, for instance, from that of 1781, since in the interval of forty years* thirty-one are common,

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* The year 1800 divisible by four, and, therefore, according to the common rule, a Leap year, is, however not so, but, as a *complementary* year, a common year of 365 days (see the Chapter on the Calendar).

and nine Bissextile years, we must subtract from the epoch of 1781, the difference between

$$31 \times 14' \, 19''.63, \text{ and } 9 \times 44' \, 48''.697, \text{ that is, } 40' \, 50''.23,$$

accordingly, since the epoch of 1781 is ...... $9^s \; 11^0 \; 29' \; 9''.5$

epoch of 1821.......... 9 10 48 19.27

Before we quit this subject we wish to say one word respecting the difference between the French and English Tables of the Sun. The epochs in the former are for the first of January, *mean midnight*, and the meridian of the Paris Observatory: in the latter for the first of January, *mean noon*, and the meridian of Greenwich. Now Paris is $2^0 \; 20' \; 15''$, or in time $9^m \; 21^s$ to the east of Greenwich: consequently, the interval of the two epochs, is $12^h \; 9^m \; 21^s$, in which time, the mean increase of the Sun's longitude ($59' \; 8''.33$ being the increase in a mean solar day,) is $29' \; 57''.2$: consequently, the epochs of the Sun's mean longitudes, for the same years, are greater, in the English Tables, by $29' \; 57''.2$.

The knowledge of the Sun's mean *secular* motion enables us, most correctly, to assign the length of a tropical, or equinoctial year. But this point and others connected with the subject of solar time, will be reserved for the ensuing Chapter.

CHAP. XXII.

On Mean Solar and Apparent Solar Time.—The Methods of mutually converting into each other Solar and Sidereal Time.—The Lengths of the several Kinds of Years deduced.—On the Equation of Time.

It happens with mean solar time, as it does with sidereal time. We cannot obtain their measures immediately from phenomena, but are obliged from phenomena to compute them.

The constant part, the unit, if we may so call it, of sidereal time, is the time of the Earth's rotation round its axis (see pp. 106, &c.): and such time, in our computations respecting portions of sidereal time, or of right ascensions, is supposed to remain unaltered. The phenomena made use of, are the transits of fixed stars over the meridian: but the intervals between successive transits of the sume star, are not (as it has been already explained in pp. 106, &c.) exactly equal: they are, therefore, not sidereal days, if such terms be intended to signify equal portions of absolute time.

Besides the causes that equally affect the fixed stars and the Sun, the proper motion of the latter, inequable from its proper motion in the ecliptic, and inequable by reason of the obliquity of the ecliptic, prevents the intervals between successive transits of the Sun, over the meridian, from being equal portions of solar time. We must consider then, by what means we are able to compute *mean* solar time, and to know whether or not, a clock, going equably, *keeps* mean solar time.

The Sun's motion (see p. 523,) in $365^{d}.25$, is $360^{\circ}\ 0'\ 27''.45$: consequently,

$$\frac{360^{\circ}\ 0'\ 27''.45}{365.25} = 59'\ 8''.33,$$

is the increase of the Sun's mean longitude in one day, consisting of twenty-four mean solar hours. A mean solar day, therefore, must exceed a sidereal day, by the portion of sidereal time consumed in describing $59'\ 8''.33$. Now 360° are described in twenty-four *sidereal* hours;

$$\therefore\ 360^{\circ} : 24^{h} :: 59'\ 8''.33 : 24 \times \frac{59'\ 8''.33}{360}$$

$$= 236^{s}.555 = 3^{m}\ 56^{s}.555 \text{ of sidereal time:}$$

hence, twenty-four mean solar hours are equal to $24^{h}\ 3^{m}\ 56^{s}.555$ of sidereal time: and a clock will be adjusted to mean solar time, if its index hand makes a circuit, whilst that of the sidereal clock makes one circuit and $3^{m}\ 56^{s}.555$ over: or, if each clock beats seconds, the solar clock ought to beat 86400 times whilst the sidereal beats $86636\frac{1}{2}$, nearly.

In order to find the number of solar hours to which a sidereal day of twenty-four hours is equal, we must use this proportion,

$$86636.555 : 24 :: 86400 : 24 \times \frac{86400}{86636.555}$$

$$= 23^{h}.93447 = 23^{h}\ 56^{m}\ 4^{s}.092 \text{ of mean solar time.}$$

The difference between twenty-four hours and the last time, is $3^{m}\ 55^{s}.908$. Hence, subtract from twenty-four hours of sidereal time $3^{m}\ 55^{s}.908$, and the remainder is the number of mean solar hours, minutes, seconds, and decimals of seconds, to which twenty-four hours of sidereal time are equal.

Hence, subtract $1^{m}\ 57^{s}.954$ from twelve sidereal hours, and the remainder is their value in mean solar time; subtract $0^{m}\ 58^{s}.977$ from six sidereal hours, and the remainder is their value in mean solar hours: and these *subtracted* quantities are called the *accelerations* of the stars in mean solar time; a table of which *accelerations* might, as it is plain from what precedes, be easily formed (see Zach's Table XXVI, in his *Nouvelles Tables d'Aberration*, &c.)

By means of these latter results and the Solar Tables, we can now, from the sidereal time, find the mean solar time. Thus, suppose it were required to find the mean solar time at Greenwich, on August 20, 1821, when the corrected sidereal time by the clock was $20^h\ 42^m\ 19^s.4$.

By the Solar Tables,

Sun's epoch for 1821	9^s	10^0	$48'$	$19^s.2$
mean motion to August 20, ...	7	17	41	4.2
mean longitude of Sun on Aug. 20,	16	28	29	23.4

Reject 12^s, and convert the remainder into time, and

$4^s\ 28^0\ 29'\ 23''.4$ .... =	9^h	53^m	$57^s.54$
now equation of equinoxes (see p. 376,)	0	0	0.47
Sun's mean longitude on the meridian at Greenwich on August 20, 1821, ...	9	53	58.01
but true sidereal time	20	42	19.4
diff. of Æ between Sun and the point of* the heavens on the meridian	10	48	21.39
subtract (see p. 526,) the acceleration, or	0	1	46.216
mean solar time when the sidereal time was $20^h\ 42^m\ 19^s.4$	10	46	35.17

Now one use of this operation (the conversion of time shewn by the transit of a star, or by the sidereal clock, into mean solar time) is the correction, or the means of ascertaining the rate, of chronometers. For instance, in the above case, if the chronometer, at the instant the sidereal time was noted, should mark

* The *corrected* time shewn by the sidereal clock, is technically called the *Right Ascension of the Mid-Heaven*. By means of the transits of known stars, the error and rate of the clock (see pp. 104, 105, &c.) are determined. The clock so corrected, must shew at every point of time, during the sidereal day, the right ascension of a star, (should there be any one) or of a point in the heavens then on the meridian.

	$11^h\ 10^m\ 11^s$	of mean solar time, since
(see p. 527, l. 19,)	$10\ \ 46\ \ 35.17$	was the true mean solar time,
	$0\ \ 23\ \ 35.83$	would be the chronometer's error.

If, on the next day, by similar observations and computations,

$11^h\ 12^m\ 13^s$	should be the watch's time,
$10\ \ 48\ \ 38.5$	the true mean solar time,
$0\ \ 23\ \ 34.5$	the error.

Hence, the watch would be $23^m\ 35^s.83$ too fast the first day, $23^m\ 34^s.5$ too fast the second day, accordingly, in the twenty-four mean solar hours the watch would have lost, nearly, $1^s.33$, or, as far as these two observations shewed, its daily rate would be $-\ 1^s.33$.

In illustrating the use of finding, by the Solar Tables and the sidereal clock, the mean solar time, we have supposed the place of observation to be Greenwich, for which our present Solar Tables (those inserted in the third Volume of Vince's *Astronomy*) are constructed. For any other place of observation, (Dublin Observatory, for instance) we must, in computing the Sun's longitude from the Solar Tables, allow for the difference of the longitudes of the two observations of Greenwich and Dublin. That difference, in time, is $25^m\ 20^s$, and the increase of the Sun's longitude in that time is

$$\frac{25^m\ 20^s}{24^h} \times 59'\ 8''.33 = 4^s.15 \text{ in time,}$$

consequently, we must add $4^s.15$ to the Sun's mean longitude expressed in p. 527, l. 13, which will so become

$$9^h\ 54^m\ 2^s.16.$$

The secular motion of the Sun affords, as it was hinted at the end of the last Chapter, a good method of determining the length of the equinoctial year. Thus, in 36500 days the Sun describes $1200^s\ 0^0\ 45'\ 45''$: but in one hundred equinoctial years the Sun describes only 1200^s: consequently,

$$100 \text{ equinoctial years} = \frac{1200^s}{1200^s\ 0^\circ\ 45'\ 45''} \times 36500^d$$

$$= 36524^d.226396593684,$$

consequently,

$$\text{a mean equinoctial year} = 365^d.242264, \text{ nearly},$$
$$= 365^d\ 5^h\ 48^m\ 51^s.6.$$

We may hence deduce a sidereal year. In this year a complete circle of 360^0 is described, whereas, in the equinoctial year, an angle equal to $360^0 - 50''.1$ (supposing $50''.1$ to be the precession) is described.

Hence,

$$359^\circ\ 59'\ 9''.9 : 360^0 :: 365^d\ 5^h\ 48^m\ 51^s.6 : 365^d\ 6^h\ 9^m\ 11^s.5,$$

the length of a sidereal year exceeding the equinoctial by $20^m\ 19^s.9$. This is the kind of year which Kepler's Law speaks of (see p. 455.).

The anomalistic year is the period from apogee to apogee. The *progression* of the apogee (its increase of longitude) being $11''.8$, the anomalistic year is completed when the Sun has described $360^\circ\ 0'\ 11''.8$.

Hence, its length

$$= \frac{360^0\ 0'\ 11''.8}{360} \times 365^d\ 6^h\ 9^m\ 11^s.5 = 365^d\ 6^h\ 13^m\ 58^s.8,$$

longer than the sidereal by $4^m\ 47^s.3$ and longer than the equinoctial by $25^m\ 7^s.2$.

The use of the anomalistic year consists, as we have seen in p. 477, in finding the exact place of the apogee. The horary motion which we computed at p. 519, is a portion of the *anomalistic* motion.

By means of the preceding results it is easy to convert one species of time into another, and to assign the number of degrees, minutes, &c. which the Sun and a star will respectively describe in a specified portion of sidereal time, or in an equivalent portion of mean solar time. For instance, the Sun describes an entire revolution of 360^0 in $24^h\ 3^m\ 56^s.5554$ of sidereal time. In one

mean solar day the motion of the sphere, or of a star, is $360° 59' 8''.33$, consequently, a star, in one mean solar hour, describes

$$\frac{360^0\ 59'\ 8''.33}{24} = 15^0\ 2'\ 27''.84708.$$

But hitherto no method has been given of converting either sidereal, or mean solar time, into apparent time, or of computing, from the instants of apparent time, (which instants, as we shall see, are marked by phenomena) the corresponding *mean solar* times and sidereal times.

In apparent solar time, the term *day* means the interval between two successive transits of the Sun over the meridian: which interval (see pp. 431, &c.) is a variable quantity *. There cannot, therefore, be any simple rule for converting apparent solar time into mean: since there cannot be a constant proportion between the two, as there is between sidereal and mean solar time.

The *correction* then to be applied to apparent time, in order to reduce it to mean time, is a variable correction: not to be expressed by a simple term, but by several variable terms that respectively expound the several causes that render inequable, the Sun's motion in right ascension.

This correction, or *equation*, by which apparent time is made equal to mean time, is technically called the *Equation of Time*: and our present concern is with the method of computing it.

For the purpose of elucidating such method, and of guiding us in it, let us feign mean solar time to be measured by a fictitious Sun, moving equably in the equator, with the real Sun's mean motion in right ascension, and consequently, (see p. 526,) at the rate of $59' 8''.33$, in twenty-four mean solar hours.

If this motion begin to be dated from the first point of Aries, the right ascension of the fictitious Sun, after an interval of time

* Not only variable according to the time of the year, but, in strictness, variable on the same days of civil reckoning at different places.

equal to t, will be equal to $59' \, 8''.33 \times t$. The right ascension of the real Sun depends upon, or may be computed from, his true longitude, and the true obliquity of the ecliptic, of which latter computation we have given instances in pp. 504, &c. In each case, the reckoning is made from the first point of Aries, and the equable regression of that point is taken account of, when $59' \, 8''.33$ is assigned as the mean increase of the Sun's right ascension in a mean solar day.

In the above case then (that of the equable retrogradation of the equinoctial point), the difference between mean solar time and apparent time, or the equation of time, is equal to the difference between the true right ascension of the real Sun, and the right ascension of the fictitious Sun, or, which is the same thing, between the true right ascension of the Sun, and his mean longitude.

But let us suppose, which indeed is the case, that the equable retrogradation of the equinoctial point is disturbed by a displacement of the pole of the equator (and consequently of the equator itself) such as is caused by *nutation:* then the longitude of the real Sun, and the right ascension of the fictitious Sun describing the equator will both be altered. The right ascension of the latter will no longer be

$$59' \, 8''.33 \times t, \text{ but } 59' \, 8''.33 \times t \pm \text{♈} \, \text{♈}' \times \text{cos. obliquity},$$

(see the figure of p. 357, in which ♈ ♈′ represents the effect of nutation) whilst the longitude of the Sun, no longer measured from ♈ but from ♈′ will be affected with the whole quantity ♈ ♈′. But, wherever the point ♈ be, the true longitude is always measured from it, and from such true longitude the right ascension must be computed. In this latter case, then, the equation of time is the difference of the Sun's right ascension, and of his mean longitude ($59' \, 8''.33 \times t$) $\pm$ ♈ ♈′. cos. obliquity. But this last term (♈ ♈′ cos. obliquity) is the nutation in right ascension of a star in the equator, or, technically, is the *equation of the equinoxes in right ascension;* if, therefore, we use this latter term, *The equation of time is the difference of the Sun's true right ascension, and of his mean longitude corrected by the equation of the equinoxes in right ascension.*

The *equation of the equinoxes in longitude* (the effect of nutation on the Sun's longitude) is (see p. 376,) $= 18''.034 . \sin. \Omega$: the equation of the equinoxes in right ascension, (the effect of nutation on the right ascension of the fictitious Sun, which is supposed to describe the equator) is

$18''.034 . \sin. \Omega . \sin. \text{obliquity} =$ (see p. 375,) $16''.544 . \sin. \Omega$.

Hence, if

S represent the Sun's true longitude,
A his true right ascension,
M his mean longitude,
E the equation of the centre,
R (see p. 501,) the reduction to the ecliptic,
P (see p. 511,) the effect of the several planetary perturbations,

$$S = M + E + P + 18''.034 . \sin. \Omega,$$

$$\text{and, } A = S \mp R = M + E + P \mp R + 18''.034 \sin. \Omega,$$

$$\text{but the Æ } (A') \text{ of the fictitious Sun} = M + 16''.544 \sin. \Omega;$$

$$\therefore A - A' \text{ (the equation of time)} = E + P \mp R + 1''.49 . \sin. \Omega,$$

and, expressed in time,

$$\text{the equation of time} = \frac{E + P \mp R}{15} + 0^s.0993 . \sin. \Omega.$$

The cosine of the obliquity ($\cos. 23^0\ 28'$) is, nearly, equal to $\frac{9173}{10000} = \frac{11}{12}$. Hence, since the equation of time is equal to the Sun's true right ascension, diminished by his mean longitude and the equation of equinoxes in right ascension, we have

$$\text{the equation of time} = A - M \mp 18''.034 . \sin. \Omega \times \frac{11}{12},$$

which, essentially, is the form under which Dr. Maskelyne expressed the equation of time (see *Phil. Trans.* 1764).

Since, the right ascension is derived from the true longitude, which itself depends, in part, on the effect of the planetary perturbations, we cannot, without the aid of Physical Astronomy,

compute the equation of time. For such a reason the Astronomers, who lived previously to Newton, were unable to compute it. They could indeed *nearly* assign its value, since the Earth is not considerably disturbed by the action of the planets.

The Solar Tables, of the present day, enable us to compute the effect of the planetary perturbations. They, in fact, assign the Sun's true longitude, when such perturbations are taken account of. They enable us, then, (although this is not the most convenient mode) to compute the equation of time.

Thus, on March 12, 1822,

Sun's mean longitude	11^s	19^0	$33'$	$43''.2$
longitude of perigee	9	9	50	54.9
mean anomaly	2	9	42	48.3
(see p. 468,); ∴ equation of centre (E)	0	1	48	18.2
sum of perturbations (P)	0	0	0	22.18
(see pp. 501, &c.) reduction (R).....	0	0	42	13.3
$E + P + R$	0	2	30	53.68

Hence, the equation of time (see p. 532,)

$$= \frac{2^\circ\ 30'\ 53''.68}{15} + .0993 \text{ sin. } \Omega\,;$$

$$\text{but, } \Omega = 10^s\ 23^0\ 54' \text{ and sin. } \Omega = -\ .5891\,;$$

$$\therefore \text{ the equation of time} = 10^m\ 3^s.57 - 0^s.058$$

$$= 10^m\ 3^s.5, \text{ nearly*}.$$

* The equation of time may be computed from an observed right ascension of the Sun, and from the Sun's mean longitude known from the Tables. For instance, by observations (reduced observations) at Greenwich, June 11, 1787,

	By Clock.	By Cat. (see pp. 371, &c.)	Diff.
Æ of Sun's centre	$5^h\ 17^m\ 9^s.6$		
of Procyon	7 27 15.58	$7^h\ 28^m\ 9^s.820$	$54^s.24$
of β Pollux	7 31 22.82	7 32 17.069	54.249

daily

In the above example, *E*, &c. were computed to a mean anomaly belonging to the *mean* noon of March 12, whereas, in strictness, the computations ought to have been for the *apparent* noon of that day. In other words, since the equation of time is nearly $10^m\ 3^s$, the Sun's true longitude ought to have been computed from the Solar Tables (which are constructed for mean time) for March 11, $23^h\ 49^m\ 57^s$ of mean solar time; since such is nearly the time of apparent noon, on March 12; and the equation of time, on the apparent noon of March 12, is the difference of the Sun's true right ascension at that time and of his mean longitude (corrected by the equation of equinoxes in right ascension) at the same time. The result of the computation, however, thus conducted, will differ, very slightly, from that which has been just obtained.

The equations of time are set down in the Nautical Almanack, and in the foreign Ephemerides, for every day of the year.

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daily rate of clock $0^s.84$; therefore, at the time of the transit of the Sun's centre, the error of the clock was

54.245 − 0.07	0^h	0^m	$54^s.17$
Sun's transit	5	17	9.6
therefore Sun's right ascension	5	18	3.77

Again, Sun's mean longitude 1787,	9^s	$11°$	$2'$	$20''$			
motion to June 11, ...	5	8	41	21.1			
	2	19	43	41.1	5^h	18^m	$54^s.74$
equation of equinoxes in right ascension					0	0	1.08
					5	18	55.82
					5	18	3.77
					0	0	52.05

The difference then of the true right ascension of the Sun, and of the Sun's mean longitude corrected by the equation of equinoxes in right ascension, on the *mean* noon of June 11, 1787, (for the Tables are constructed for mean time) was $52^s.05$, true or apparent time preceding mean. The mean longitude then at the time of observation, or on true noon, was less by the increase of the mean longitude during $52^s.05$, or by $0^s.142$: consequently, the equation of time was $52^s.05 - 0^s.142$, or $51^s.91$.

They enable us to convert apparent solar time into mean and sidereal time, and also, which is the reverse operation, sidereal time into apparent solar time. We will give some instances of these operations taken from M. Zach.

Example I.

Sidereal Time converted into Mean Solar Time, and true Time. Place of Observation, Greenwich.

Jan. 18, 1787, beginning of a solar eclipse by sid. clock	18^h	$4'$	$59''$
clock too slow	0	0	5
beginning of the eclipse by sidereal time	18	5	4
epoch of Sun's mean longitude for the beginning of 1787, and the meridian of Gothe . .	18^h	$40'$	$5''.895$
Sun's motion to January 18	1	10	57.996
Sun's motion in an interval of time representing the difference ($42'$ $55''$) of the longitudes of Gothe and Greenwich, ...	0	0	7.049
equation of equinoxes in right ascension	0	0	1.06
Sun's mean right ascension	19	51	12
Æ of the mid-heaven or sidereal time	18	5	4
approximate mean solar time..............	22	13	52
(see pp. 526, &c.) acceleration	0	3	38.52
mean time	22	10	13.48
equation of time	–	11	15.08
true or apparent time of the beginning of the eclipse.	21	58	58.4

Example II.

Mean Time converted into Sidereal.

Marseilles, 21^m 29^s east of Greenwich,

1787, *mean* time of Venus' transit over the meridian 0^h 17^m $25^s.5$.

By Vince's Tables, epoch of Sun's mean longitude for 1787, ...	9^s	11^0	$2'$	$20''$
motion to January 2, ...	0	0	59	8.33
for $17^m\ 25^s.5$	0	0	0	42.92
	9	12	2	11.25
Sun's motion in $21^m\ 29^s$	0	0	0	52.9
Sun's mean longitude, or Æ of mean Sun	9	12	1	18.35

and in time	18^h	48^m	$5^s.223$
equation of equinoxes	0	0	1.055
Sun's mean Æ from true equinox	18	48	6.278
culmination or transit of ♀	0	17	25.5
sidereal time, or, apparent Æ of ♀ ...	19	5	31.78

or, if we convert time into degrees,

Æ of ♀ .... $286^0\ 22'\ 59''.1$.

Example III.

True or Apparent Time converted into Sidereal.

Greenwich, June 11, 1787, Sun on meridian	0^h	0^m	0^s
equation of time	0	0	52.379
mean solar time of Sun's transit	23	59	7.621

Now, by Tab. I—III, Vince, vol. III, converting the degrees, &c. into time, at the rate of 15^0 for 1^h,

Sun's mean longitude on June 11, 1787, ..	5^h	18^m	$54^s.74$
equation of equinoxes in right ascension ..	0	0	1.08
	5	18	55.82
correction on account of $52^s.379$	0	0	0.14
distance of mean Sun from true equinox ..	5	18	55.68
distance of mean Sun from mid-heaven ...	0	0	52.379
Æ of mid-heaven or sidereal time	5	18	3.3

In computing the equation of time by the methods in the preceding page, we are obliged, in fact, to compute the Sun's true longitude: which is a laborious computation. In order to avoid or to lessen such labor, Tables and approximate methods have been devised (see Delambre's *Astronomy*, vol. II, pp. 207, &c. Vince's *Astronomy*, vol. III, pp. 20, &c.)

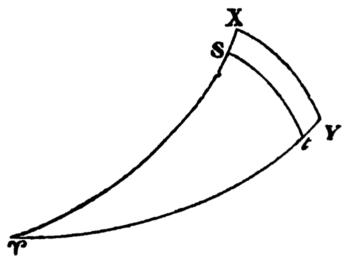
In the preceding reasonings, for the sake of simplicity, we have supposed the noon of mean time to be determined, by the aid of the noon of true or apparent time marked by the phenomenon of the real Sun on the meridian. But, if by means of the Sun's altitude observed out of the meridian, and a knowledge of his declination and of the latitude of the place, or by other means, we compute the hour angle measuring the time from apparent noon, we may, as easily as in the preceding case, compute the equation of time for such time, and thence deduce the corresponding mean solar time.

What has preceded contains the principle and the mode of computing the equation of time; all, therefore, that concerns the practical Astronomer. But if, for the purpose of new and farther illustration, we continue our speculations, we shall find that the equation of time, relatively to its causes, depends on two circumstances; *the obliquity of the ecliptic to the equator*, and *the unequal angular motion of the Sun in its real orbit.*

The Sun moves every day through a certain arc of the ecliptic: which, in other words, is his daily increase of longitude. If we suppose two meridians to pass through the extremities of this arc, they will cut off, in the equator, an arc which is the daily increase of the Sun's right ascension. This latter arc will not remain of the same value, even if the former, that of the ecliptic, be supposed constant. At the solstice it will be larger than at the equinox: the reason is purely a geometrical one: let $S\,\text{♈}$ be the ecliptic, and $\text{♈}\,y$ the equator, then by Naper's rule, if I be the obliquity, l the longitude, A the right ascension, D the declination, $1 \times \cos. I = \text{cotan.}\, \text{♈} S \times \tan.\, \text{♈} t = \dfrac{\tan. A}{\tan. l}$,

hence, $\tan. l \times \cos. I = \tan. A$, and, taking the differential,

$$dl \cdot \frac{\cos. I}{(\cos. l)^2} = \frac{dA}{(\cos. A)^2}, \text{ or, since } \cos. l = \cos. A \times \cos. D$$



$$dl . \cos. I = dA\,(\cos. D)^2, \text{ or } dA = dl . \cos. I . (\sec. D)^2.$$

Hence, I being the same, dA varies, if dl be given, as $(\sec. D)^2$; ∴ is least at the equinoxes and greatest at the solstices, and its value is easily estimated at the former, for since $D = 0$, $dA = dl . \cos. I$; at the latter, since

$$\sec. D = \frac{1}{\cos. D} = \frac{1}{\cos. I}, \quad dA = \frac{dl}{\cos. I};$$

$$\therefore dA \text{ (equinox)} : dA \text{ (solstice)} :: (\cos. I)^2 : 1$$
$$:: (\cos. 23^0\ 28')^2 : 1^2$$
$$:: 8414 : 10000.$$

Hence, even on the hypothesis of the Sun's equable motion in the ecliptic, the true right ascension will not increase equably; but since, by the very definition of the term, the mean longitude does, the equation of time, which is the difference of the true right ascension and the mean longitude (disregarding the equation of the equinoxes) would be a quantity, throughout the year, continually varying, and vanishing at the solstices.

The hypothesis, however, of the Sun's equable motion is contrary to fact; the Sun moves in an ellipse, and consequently, does not move uniformly, or equably in it. If a fictitious Sun, moving with the Sun's mean angular velocity, be supposed to leave, at the same time with the real Sun, the apogee, they will again come together at the perigee: but, in the interval, the fictitious Sun would constantly precede the real Sun: the latter therefore,

would be first brought on the meridian; true noon, therefore, would precede the noon of mean time, supposing, now, mean time to be measured by the imaginary Sun moving uniformly in the ecliptic.

If therefore, we hypothetically annul the first cause of the equation of time, by supposing the ecliptic to coincide with the equator, still from the second, (the elliptical motion of the Sun,) there would exist a difference between true and mean time; in other words, an equation of time, continually varying; vanishing, however, at the apogee and perigee.

But, both causes in nature exist; the Sun moves unequably, and not in the equator. From their combination then, the actual equation of time must depend. It cannot be nothing at the solstices, except the solstitial points coincide with those of the apogee and perigee, but, (see p. 486,) in the solar orbit, there is no such coincidence.

At what conjunctures then, will the equation of time be nothing? We have already, for the purposes of explanation, introduced two fictitious Suns, one moving equably in the ecliptic, the other in the equator; let the former be represented by S'', and the latter by S''', and the true Sun, that which moves unequably in the ecliptic, by S'; then, true time depends on S', and mean time on S'''; and consequently, when the meridian, passing through one, passes also through the other, then is mean time equal to the true, therefore no equation is requisite, or the equation of time is nothing. Let us suppose the two fictitious Suns S'', S''' to move from the autumnal equinox towards the perigee; S''', in this case, must constantly precede S'', till they arrive at the solstice, where the meridian that passes through one will pass through the other *. Hence, the real Sun S', which coincided

* We shall frequently use the expression of S' *rejoining* S''', or, *coinciding* with it. Nothing farther, however, will be meant by such expression, than that the meridian, which passes through the former in the ecliptic, passes through the latter in the equator; and when S' is said to precede S''', nothing more is meant, than that the point in the equator in which a meridian through S' cuts it, is beyond the place of S''', or, to the eastward of it.

with S'' at the apogee, being constantly behind it (see pp. 469, &c.) till their arrival at the perigee, must certainly be behind it, at and before the solstice, which is previous to the perigee (see p. 485.). Hence, before the winter solstice, the order of the Suns is

$$S' \; S'' \; S'''\text{*}.$$

At the solstice $S' \left\{ \begin{matrix} S'' \\ S''' \end{matrix} \right\}$; for S'' then ceases to be preceded by S'''. Immediately after the solstice, S'' takes the lead of S''': therefore, then, the order is

$$S' \; S''' \; S''.$$

But, at the perigee, S' must rejoin S'': it cannot effect that, except by previously passing S''': the moment of passing it is that in which true time is equal to mean time, in which, in other words, the equation of time is nothing.

The equation of time then is nothing, between the winter solstice and the time of the Sun's entering the perigee: and, for the year 1810, (when the longitude of the perigee was $9^s \; 9^\circ \; 39' \; 22''$) between Dec. 21, and Dec. 30. By the Nautical Almanack the exact time was Dec. 24, at midnight: since the equation for the noon of that day is $-$ 15^s, and, for the noon of the succeeding day, $+$ 15^s.

In the year 1250, when the perigee coincided with the winter solstice (see p. 486,) the equation of time was nothing on the shortest day.

Immediately after the passage of the perigee, S', the true Sun, moving with its greatest angular velocity (see p. 469,) precedes S''; therefore, since up to the vernal equinox S'' precedes S''', the order is

$$S''' \; S'' \; S';$$

and this order must continue up to the equinox; consequently, S''' and S' cannot come together: and therefore between Dec. 24, (for 1810,) and March 21, the equation of time cannot equal nothing.

* The symbol most to the right of the page denotes the preceding Sun.

After the vernal equinox, S''' precedes S'', and the order is

$$S'' \; S''' \; S',$$

S'', and S' are then, (see p. 472,) near the point of their greatest separation, but S'' and S''' begin to separate and reach the point of their greatest separation *, about 46° 14′ from the equinox that is, about the 8th of May. Now, this greatest separation, or, technically, greatest *equation*, is 2° 28′ 20″, or in time 9^m 52^s, whereas the greatest equation of the centre, being only 1° 55′ 33″, (pp. 473, &c.) the greatest corresponding separation in the equator cannot exceed 2° 6′ †, and that is already past. Hence, before S''

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* $1 \times \cos. I = \tan. A . \cot. l$, by Naper, or $\cos. I \times \tan. l = \tan. A$; $\therefore l > A$; $\therefore$ if Y be supposed the place of S''', so that, $\Upsilon Y = \Upsilon S$, Y is beyond t, and the *separation* is tY (since on that the difference solely depends.)

To find tY, is a common problem, (see Simpson's *Fluxions*, vol. II, p. 551. Vince's *Fluxions*, p. 27.) Since $tY = \Upsilon Y - \Upsilon t = l - A$;

$$\therefore \tan. tY = \frac{\tan. l - \tan. A}{1 + \tan. l . \tan. A} = \frac{\tan. A . (\sec. I - 1)}{1 + (\tan. A)^2 . \sec. I}.$$

Hence, since $d(tY) = d \tan. tY . (\cos. tY)^2$, which must $= 0$; if we take the differential of the quantity equal to it, make it $= 0$, and reduce it, there results

$$\tan. A = \surd \cos. I = \surd (\cos. 23° \, 27' \, 58'')$$

$$A = 43° \, 43' \, 50'', \text{ and } l \text{ (from equation, l. 2 of Note)} = 46° \, 14',$$

and $l - A$ (in its greatest value) $= 2° \, 28' \, 20''$.

† By p. 538, it appears that the arc of the equator, included between two meridians passing through the extremities of a given arc in the ecliptic, is greatest when the latter arc is at the solstice. The arc of the equator measures the *separation* of the Suns S'', S'''. Hence, putting in the formula of p. 230, $dl = 1° \, 55' \, 33''$, and $D = I$, which it is at the solstice, we have, very nearly,

$$dA = 1° \, 55' \, 33'' \times \sec. 23° \, 27' \, 58'' = 2° \, 5' \, 55''.$$

The two common problems then of the maximum equation of time, are not merely mathematical problems, exercises for the skill of the student, or Examples to a fluxionary rule, but of use in the discussion of the real problem of nature.

is at its greatest separation from S''', it is impossible that the order

$$S''\ S'''\ S'$$

should not have been changed. S' must have come nearer to S'' than S''' is: consequently, S''' must have passed S': but at the moment of passage, mean and true time are equal, that is, the equation of time is nothing: and this must happen between March 21, and the end of April. In the year 1810, it happened, according to the Nautical Almanack, on April 15, $11^h\ 12^m$.

This second point, at which the equation of time is nothing, being passed, the order of the Suns will become

$$S''\ S'\ S'''.$$

At the solstice, S'' must rejoin S''': but, previously to the solstice, it cannot effect that by passing S': since S'' does not rejoin S' till their arrival at the apogee, which point is more distant than the solstitial: the coincidence of S'' and S''' then can only take place, by S' previously passing S''': but, as before, the moment of passage, is the time when the equation of time is nothing: that circumstance therefore, must happen, before the summer solstice: therefore, between the middle of April and June 22: and, in 1810, according to the Nautical Almanack, it happened on June 15, 14^h.

In the year 1250, the equation of time was nothing on the longest day.

After this third *evanescence* of the equation of time, the order of the Suns will become

$$S''\ S'''\ S'.$$

At the solstice on June 22, S'' will rejoin S''': immediately afterwards, the order becomes

$$S'''\ S''\ S',$$

which will continue to the time of the Sun's entering the apogee: then, S'' rejoins S': and, immediately after, S'' moving with greater angular velocity than S' will precede it, and the order becomes.

$$S'''\ S'\ S''.$$

Now S' cannot rejoin S'' till their arrival at the perigee: but S' will rejoin S''' at the autumnal equinox, consequently, previously

to that time, S''' must pass S': but, as before, the moment of passage is, when the equation of time is nothing. It must happen then, between the time of the apogee and the autumnal equinox: between (for 1810) June 30, and September 24; and, by the Nautical Almanack, it happened August 31, 20^h.

It is plain, from the preceding explanation, that the days of the year in which the equation of time is nothing depend on the position, or the longitude of the perigee and apogee: and consequently, since those points are perpetually progressive, the equation of time will not be nothing on the same days of any specified year, as it was, of preceding years: nor, when not nothing, the same in quantity, on the corresponding days of different years.

The preceding statement (beginning at p. 537,) is to be regarded merely as a mode of explaining the subject of the equation of time. It is not essential, and might have been omitted; for, the two causes of inequality are considered and mathematically estimated, in the processes of finding the true longitude and true right ascension. But it has been inserted, since it serves to illustrate more fully, and, under a different point of view, a subject of considerable difficulty and importance.

With regard to results, very little is effected by the preceding statement. Four points are determined, at which, mean time is equal to apparent: in other words, four particular values (evanescent values) of the equation of time. But, according to the process in p. 533, we are enabled to assign its value for every day in the year: and accordingly, in constructing Tables of the equation of time, the above four particular values would be necessarily included amongst the 365 results.

If the question were, merely to determine when the equation was nothing, it would certainly be an operose method of resolution, to deduce all the values of the equation of time, and then, to select the *evanescent* ones. In such case, it would be better to have recourse to considerations like the foregoing (pp. 537, &c.). But, both these methods would be superseded,

if, which is not the case*, the equation of time could be expressed by a simple analytical formula.

The mere inspection of such formula, or some easy deduction from it, would enable us to assign the times when the equation of time vanished.

Instead of a formula, we must use a process consisting of several distinct and unconnected steps, for computing the equation of time. And, in point of fact, the process is quite as convenient as a formula could be; since the concern of the Astronomical Computist is not with special, as such, but with the general values of the equation of time.

If special values are sought after, it must be principally on the grounds of curiosity. The method of ascertaining four such values, independently of direct computation, has been already exhibited. And, on like grounds, a similar method might be used in the investigation of other special values: in determining, for instance, when the equation of time is of a mean value; or, when minute, the two causes of inequality counteracting each other; or, when large, the two causes co-operating. We will confine ourselves to two instances.

After the evanescence of the equation of time between the winter solstice and the perigee, the order, as we have seen, (p. 542,) is

$$S''' \; S' \; S'',$$

but S' is gaining fast on S'' in order to rejoin it at the perigee, and S'', after parting with S''' at the solstice, is preceding it, by still greater and greater intervals. Consequently, both causes of inequality conspire to make mean time differ from the true, and the equation of time goes on increasing till the Sun is about 40^0 distant from the vernal equinox, that is, past the point, at which the equation arising from the obliquity is a maximum, (see p. 541,) and before the point at which the equation from the Sun's ano-

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* Lagrange, however, although by no direct process, has succeeded in assigning a formula for the equation of time. See *Mem.* Berlin, 1772. So also has M. Schulze, *Mem.* Berlin, 1778. p. 249.

malous motion is a maximum. For the year 1810, the time would be about Feb. 10, and the maximum of the equation is $14^m\ 36^s$.

About the Summer solstice, on the contrary, between that and the apogee, the order is

$$S''' \quad S'' \quad S'.$$

S''' is indeed separating from S'', but S'' is approaching S' to reach it at the apogee: consequently, the two causes of inequality, in some degree, counteract each other, and the equation between the two periods at which it is successively nothing, (June 15, and August 31, for 1810,) never attains to the value of seven seconds.

In a similar way, we may form a tolerably just conjecture of the limits of the quantity of the equation of time, for other parts of the year.

The greatest quantity of the real equation of time can never reach the sum of the greatest equations arising from the separate causes. It must therefore be less than

$$2^0\ 28'\ 29'' + 2^0\ 6', \text{ or } 4^0\ 34'\ 29'',$$

or in time less than $18^m\ 15^s$ of mean solar time.

The equation of time computed for every day in the year, according to the method given in p. 533, or, by some equivalent method, is inserted in the Nautical Almanack; and, for the purpose of deducing mean solar, from apparent time. In order to regulate its application, the words *additive* and *subtractive* are interposed into the column that contains its several values. And, there will be no ambiguity belonging to that application, if we consider, that the equation is to be applied to a certain time marked by some phenomenon: which phenomenon is the real Sun on the meridian: determined to be so, either by a transit telescope, or by a quadrant, or declination circle that enables us to ascertain, when the Sun is at its greatest altitude. Apparent time, then, is what is instrumentally determined; and to such time, the equation, with its concomitant sign, must be applied, in order to deduce mean time, which, it is plain, is indicated by no phenomenon.

Thus, Dec. 31, 1810, the equation of time in the Nautical Almanack is stated to be $3^m\ 12^s.7$ *additive;* therefore, when the Sun was on the meridian, at its greatest height, on that day the mean solar time was $12^h\ 3^m\ 12^s.7$. Again, Nov. 13, 1810, the equation is stated at $15^m\ 33^s.2$ *subtractive;* therefore, on that day, the Sun was at its greatest height at $12^h - 15^m\ 33^s.2$, that is, $11^h\ 44^m\ 26^s.8$, mean solar time.

Independently of computation, very simple considerations will shew that this procedure is just. In the first instance, the true Sun precedes the mean; that is, is more to the east, or more to the left hand of a spectator facing the south: consequently, by the rotation of the Earth, from west to east, the meridian of the spectator must first pass through the hinder Sun, which, in this instance, is the mean Sun; 12^h therefore of mean time happens before the meridian has reached the true Sun, when it does reach it, then, the time is, in mean time, $12^h +$ the difference of right ascensions, or $12^h +$ the equation of time. In the second instance, the true Sun is behind the fictitious: therefore the meridian of the spectator first passes through the former: true noon therefore, or 12 hours apparent time, happens before the meridian has reached the fictitious mean Sun; before therefore the noon of mean solar time. The time consequently is not 12 hours, but 12 hours — some quantity, which quantity is the equation of time.

What has been given in the latter pages, has been for the purpose of illustration rather than for settling the grounds of, and arranging the method of computing, the equation of time. It may suit some students: others, perhaps, will be satisfied with the investigations that terminate at p. 537.

CHAP. XXIII.

THE PLANETARY THEORY.

On the general Phenomena of the Planets: their Phases, Points of Stations, Retrogradations, &c.

We have now passed, in our course of enquiry, through the theories of the fixed stars and of the Sun, and are arrived at the Planetary Theory. This latter theory has many points in common with the preceding ones. The planet Venus, by reason of the Earth's rotation, is transferred to the west, as Orion is and as the Sun is. By reason of the same rotation, she *rises* and *sets* as any fixed star is made to rise and set. But the points of the horizon at which Venus rises and sets, do not remain the same, which is a circumstance of distinction between that planet and the fixed stars: and indicative of a peculiar motion in Venus, whether such motion respects, as its centre, the Earth or the Sun.

The question, in truth, is not to be at once reduced to the above alternative. We may *conjecture*, besides the Sun and Earth, other points to be the centres of the planets' revolutions. But we shall here, as we have done before, avail ourselves of the results of previous investigations and restrict the range of our conjectures. Indeed, the restriction will be so close, that we purpose merely to enquire whether the phenomena of the planets (the phenomena of change of place and law of motion) can be explained on the hypothesis of the planets describing elliptical orbits round the Sun as a centre, and of their mutual perturbation.

We at once get rid of the suggested possibility of a simple revolution of the planets round the Earth, on this consideration:

namely, that, in such a case, the motion would take place, and seem to take place, in one and the same direction: whereas, as observation shews, the planets are sometimes stationary and sometimes retrograde.

These apparent quiescences and retrogradations, are some of the phenomena which it will be the business of this, and of the ensuing Chapters to explain, on the principle of the combination of the motions of the planets and of the Earth. In the first place, these phenomena will be explained in a popular way, on the principles of the Earth's rotation round its axis, and of the Earth's and planets' revolutions round the Sun. After this, the phenomena will be more scientifically explained, or the times and circumstances of their happening will be computed. But in order to effect this we must know the *elements*, as they are called, of the planetary orbits: such as their axes, the places of their nodes and of their aphelia, and their inclinations to the plane of the ecliptic. For this end we must have recourse to observations, and, according to modern practice, to observations of right ascensions and declinations. The *elements* being obtained, we may combine them according to Kepler's principles, and by means of his problem and other aids, compute the planet's longitude in his orbit. From such longitude, and a knowledge of the inclination of the orbit, and of the place of the node, we may compute the planet's *ecliptical* longitude and his latitude, and thence compute, by a Trigonometrical process, or by a Table of *reductions* (see p. 501,) the planet's right ascension and declination. The last step in this process, would be to compare these previously computed longitudes and latitudes, with longitudes and latitudes resulting immediately from observed right ascensions and declinations: or, which is in fact the same, the previously computed right ascensions and declinations, with the observed. Such comparisons, as in the Solar Theory, (see pp. 508, &c.) enable us to *correct* the elements of the orbit, from which the planet's longitudes and latitudes are to be computed.

The order then, briefly stated, is this: the explanation of the phenomena: extrication of the elements from observations: the subsequent correction of those elements by a comparison with

observations: and, in pursuance of the first of these objects, we will begin with the planet Venus.

This brilliant star when seen in the west, at the time of the setting of the Sun, is called the *Evening* Star *. It will be found, by observing it on successive nights, to vary its distance from the Sun: sometimes apparently moving away from the Sun, until it reaches a certain term of elongation, at other times, having passed such term, approaching the Sun. When the star begins, it continues, to approach: and, at certain epochs, it approaches so nearly to the Sun, as by reason of the Sun's effulgence, to be no longer distinguishable by unassisted vision. There are other epochs, rare, indeed, at which Venus passes over the Sun's disk, and is seen, during such transit, as a black spot on the disk. After either of these two sorts of epochs Venus ceases to be the evening star and will soon become the *morning* star †, and will be seen rising just before the Sun.

On successive mornings, *Venus* will rise still sooner: will continue to be separated from the Sun, till having reached an angular distance of about 45°, she will again approach, and finally rejoin the Sun. She then again becomes the evening star, and the same appearances, in the same order, are renewed.

These appearances prove, not decisively, that *Venus* describes either an oval, or a circle about the Sun, but that, at least, she oscillates about the Sun: they prove too, that her orbit can neither be round the Earth, as its centre, nor inclusive of the Earth; for, she is never seen in opposition; that is, in the production of a line drawn from the Sun through the Earth.

To the naked sight, or to unassisted vision, the disk of *Venus* appears circular and nearly of the same magnitude. But, the telescope and its micrometer ‡ prove both appearances to be delusive. Viewed through the former, *Venus*, when the evening

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* *Εσπερος*, Hesperus, Vesper. † *Φωσφορος*, Lucifer.

‡ An instrument for measuring small angles, and commonly attached to the telescope.

star, at her greatest separation from the Sun, assumes the form of a crescent, the convex illuminated part being towards the Sun, or towards the west. As she approaches the Sun, the crescent diminishes. Having passed the Sun, she appears as the morning star, and the crescent is turned the other way, or towards the east. Day after day, the crescent increases, till it is changed into a full orb, just at the time when *Venus* is about to rejoin the Sun.

In this last situation the disk of *Venus*, though most illuminated, is least in magnitude. It is greatest in magnitude, when the disk is least illuminated, and *Venus* is about to rejoin the Sun. These latter circumstances, relative to the magnitude of the disk, are determined by the micrometer.

This last-mentioned instrument enables us to determine the greatest and least apparent diameters of *Venus* to be about 60″, and 10″.

If we now enumerate the circumstances relative to *Venus*, they are as follow:

Venus, whatever be the Sun's place in the ecliptic, always attends on him, and is never separated by a greater angle of elongation, (technically so called) than 45°.

Venus is continually at different distances from the Earth: when at her greatest, that is, when her apparent diameter is the least, she shines with a full orb: when seen at her least distance, that is, when her apparent diameter is the greatest, her crescent is very small; and there are conjunctures, as we have noted, when *Venus* eclipses part of the Sun's disk, and passes over it like a dark spot.

Venus, when the evening star and separating from the Sun, moves from west to east; or according to the order of the signs, or, as the phrase may still be varied, *in consequentia*. Returning towards the Sun, from her greatest elongation, she moves towards the west, that is, *in antecedentia*, contrary to the order of the signs. And, in like manner, she moves, when the morning star, alternately, according and contrary to, the order of the signs.

These are the phenomena of observation, that are proposed for explanation, on the grounds of two hypotheses: the first, that *Venus* is an opaque spherical body illuminated by the Sun: the second, that *Venus* revolves round the Sun in an orbit which is interior to the Earth's orbit.

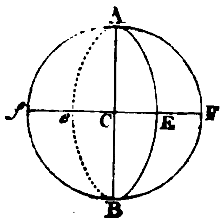
If *Venus* be a sphere, only half of it can be illuminated by the Sun. And the illuminated hemisphere, called, for distinction, the Hemisphere of *Illumination*, is thus to be determined. From the centre of the Sun, to that of *Venus*, conceive a right line to be drawn; perpendicular to this line, and passing through the centre of *Venus*, conceive also a plane to be drawn; then, such plane will divide the body of the planet into two hemispheres, the one luminous, the other dark.

But, a spectator, whatever be his distance from a sphere, can never see more than half of the same. The hemisphere which he sees, called the Hemisphere *of Vision*, is thus to be determined: conceive the eye of the spectator and the centre of the planet to be joined by a right line; a plane perpendicular to this line, passing through the centre of the planet, divides its body into two hemispheres; the one towards the spectator, is that of *vision*.

The two hemispheres, and their boundaries, the circles of illumination and of vision, do not necessarily coincide: indeed, they can coincide only when the Sun, which illuminates the planet, is between it and the spectator on the Earth's surface. In every other situation, part of the planet's illuminated hemisphere is turned away from the spectator; and, when the planet is between the Sun and spectator, wholly turned away: in other words, the planet's disk can either not be seen, or must appear as a dark circle or spot on the Sun's face.

When the spectator, Sun, and *Venus* (for of that planet we are now speaking) lie not in the same right line, the delineation of the illuminated disk, or phase, is reduced to a very simple proposition in orthographic projection. On the plane of projection which is always perpendicular to a line joining the eye of the spectator and the centre of the planet, it is required to delineate

the ellipse into which the circular boundary of light and darkness will be projected. The minor axis of the ellipse, will, as it is well known, bear that proportion to the major, which the radius bears to the cosine of the inclination of the planes. The inclination is equal to the angle formed by two lines, one drawn from the Sun to the centre of *Venus*, the other, from that same centre and directly from the spectator. Hence, if $AFBA$ represent the



disk, and we take $CF : CE ::$ rad. : cos. planet's inclination, then, describing, with the semi-axes AC, CE, the semi-ellipse AEB, we shall have the illuminated disk represented by $AFBEA$.

If $KVuL$ be the orbit of *Venus*, S the Sun, E the Earth; then, the angle of inclination of the planes of illumination, and vision at V, is the angle SVF, and at u, the angle SuF. In the latter, the angle is acute, in the former, obtuse; consequently, if CE in the above Figure be taken to represent the cosine of the acute angle, to the right of the line AB, Ce must be taken to the left of the same line, in order to represent the cosine of the obtuse angle SVF. At K, when the planet is in *superior* conjunction †, the angle SVF is equal to two right angles; consequently, the

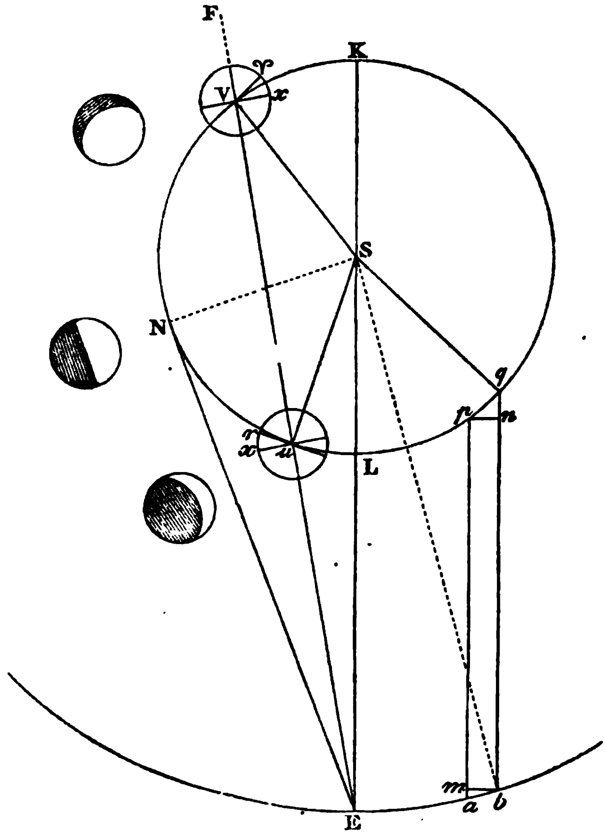
---

* ru is the half of the projection of the circle of illumination, xu of vision, and

$$\angle rux = \angle Fux - \angle Fur = 90^\circ - \angle Fur = 90^\circ - (\angle Sur - \angle SuF)$$
$$= 90^\circ - (90^\circ - SuF) \angle = \angle SuF.$$

† An inferior planet is in superior conjunction, when it lies in the direction of a line drawn from the Earth to the Sun, and produced beyond the Sun.

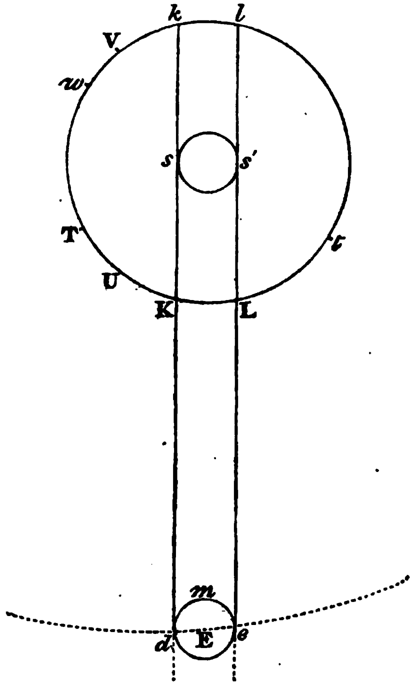
cosine (with a negative sign) becomes equal to radius, and the point



E, falls in *f* (Fig. p. 552,); or the whole orb is illuminated. At *L*, when the planet is in inferior conjunction* the angle, such as *SuF*, becomes nothing; therefore the cosine becomes equal to

* We have, for simplicity's sake, supposed the ecliptic and the plane of the orbit of Venus to be coincident. Such is not the case in nature. It will happen, and commonly, that the planet at the time of inferior conjunction will be above the Sun, in which case its bright crescent will be visible: and exactly at the time of conjunction, the line joining the horns of the crescent, will be parallel to the ecliptic.

radius, and the point E falls in F: or the whole orb is dark. From K to L, in the intermediate points, *Venus* exhibits all her varieties of phases; the full orb, near K; the half illuminated orb at N, where $SNE = 90^{0}$, and then the crescent diminishing, till its extinction at L*.



These phenomena that would happen if *Venus* an opaque spherical body be illuminated by the Sun, and revolve in an orbit round him, are strictly conformable to the phenomena that are observed, and have been described in the preceding pages.

Thus far then the hypothesis of *Venus's* revolution round the Sun is probable, and seems to involve no contradiction; it will be

* The phases which *Venus* at V, N, and u, exhibits to a spectator at E, are represented by the small circular Figures that are, respectively, to the left of the points V, N, and u (see p. 553.)

still farther confirmed, if we can shew, that it affords an adequate explanation of the other phenomena which the planet exhibits.

Suppose *emd* to be the Earth, and two tangents *dsk*, *es′l*, to the points *d* and *e*, to represent the respective horizons to a spectator at *d* and *e**. If the Earth's rotation be according to the order *emd*, when the horizon *dsk* of the spectator at *d* shall touch the Sun's disk, the Sun will *set* to that spectator; the moment after, by the rotation of the Earth, the point *k* will be transferred to some point between *k* and *V*, the line *dsk* will no longer touch the Sun's disk, or, the Sun will be below the horizon. But, *Venus*, if at *V*, will be above the line of the horizon, and above as an evening star, till the Earth, by its farther rotation, shall have so transferred the line *dsk*, that its extremity *k* shall be in some point between *V* and *U*. In the interval between this and the next night, *V* will have moved forward in its orbit to some point *w*; therefore, the line *dsk*, after leaving the Sun's disk, must revolve through a greater angle than it did the preceding evening, before it reaches *V* at *w*. The planet therefore, is now separated from the Sun by a greater angle of elongation: and the elongation on succeeding nights will still continue, till *V* reaches a point *T*, where a line drawn from *E* touches her orbit. Hence from superior conjunction at *k*, to the greatest elongation at *T*, *Venus* is continually separating or *elongating* from the Sun; and, if we refer her place to the fixed stars, will seem to move amongst them in a direction *kVwT*, that is, according to the order of the signs.

From *T* to *L* the inferior conjunction, the line *dsk*, after quitting the Sun's disk, will reach the planet after the description of angles still less and less, and the planet will be found approaching the Sun: but, referred to the fixed stars, will be found to change its place amongst them in a direction from *T* towards *L*, contrary to the direction of the former change of place, and

* In this explanation, of a popular nature, *Venus's* orbit and the Earth's equator *emd*, are supposed to be projected on the plane of the ecliptic, (represented by the plane of the paper,) and, the spectator is supposed to be placed in the equator.

contrary to the order of the signs. In other words, the planet is now *retrograde*.

Suppose now the planet to have passed the inferior conjunction at *L*. *Day breaks* to a spectator at *e*, when the line *es'l*, representing his horizon, touches the Sun's disk. But, before this has happened, the line *es'l* has passed the planet, or the planet is above the horizon, and has risen as the morning star: on succeeding mornings, the planet having moved forward in its orbit from *L* towards *t*, will rise before the Sun by greater and greater intervals; will continue, to appearance, separating from him, till its arrival at its greatest elongation *t*. From *L* to *t*, the planet will, as from *T* to *L*, still continue retrograde. From *t* to *l*, it will again approach the Sun, and move according to the order of the signs.

These phenomena, then, that would happen if *Venus* revolve either in a circular or elliptical orbit round the Sun, are in strict conformity with the phenomena that are observed, and which have been previously described.

In the preceding explanation of the phases and retrogradations of *Venus*, we have, for the sake of simplicity, supposed the Earth to be at rest at *E*. But, there is one phenomenon, that of the seeming quiescence of *Venus* during several successive days, which cannot be explained, except we depart from that supposition, and combine, according to the actual state of things, the motion of the Earth with that of *Venus*.

If *Venus* be at *L*, and the Earth at *e*, and both describe in the same time (24 hours for instance), two small arcs of their orbits, such arcs will be nearly parallel to each other. If, then, they were equal, during their description, *Venus* would be referred by a spectator on the Earth, to the same point in the heavens. But, *Venus* revolving round the Sun according to the laws of planetary motion (see p. 557, l. 16,) describes a greater arc than the Earth does in the same time. She must, therefore, at the end of the 24 hours, be referred by a spectator on the Earth, to a point in the heavens situated to the right of her former place. But, as *Venus* advances from *L* towards *t* in her orbit, the arcs of her

orbit (or tangents to them) will become more and more inclined to the arcs of the Earth's orbit. There will then be somewhere between L and t an arc pq (see Fig. p. 553,) such that, its obliquity compensating its greater length, two lines pa, qb, drawn to the contemporaneously described arc ab of the Earth's orbit, shall be parallel; when that circumstance happens, *Venus* must appear *stationary*.

We may determine the exact time of its happening by computing the angle bSq, which is, in the same time, the excess of the angular motion of *Venus* above that of the Earth *.

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* bSq may be thus computed: (see Fig. p. 553,).

Draw from p and b; pn, bm perpendicular to the parallel lines qb, pa, then $pn = bm$: call Sb, r, and Sq, r';

$$\text{then } pn = pq\,.\,\text{sin. } pqn = pq\,.\,\text{cos. } Sqb,$$
$$bm = ab\,.\,\text{cos. } mba = ab\,.\,\text{cos. } Sbq;$$

$$\therefore \frac{\text{cos. } Sqb}{\text{cos. } Sbq} = \frac{ab}{pq} = \frac{\text{vel. } \oplus}{\text{vel. } ♀} = \frac{\sqrt{r'}}{\sqrt{r}} \text{ (Newton, Sect. II. Prop. 4. Cor. 6;)}$$

$$\therefore \text{cos.}^2\, Sbq = \text{cos.}^2\, Sqb \times \frac{r}{r'}.$$

$$\text{But, sin.}^2\, Sbq = \text{sin.}^2\, Sqb \times \frac{r'^2}{r^2} \quad (\textit{Trigonometry}, \text{ p. } 16,)$$

$\therefore$ adding these two latter equations, and putting for $\text{cos.}^2\, Sqb$, $1 - \text{sin.}^2\, Sqb$,

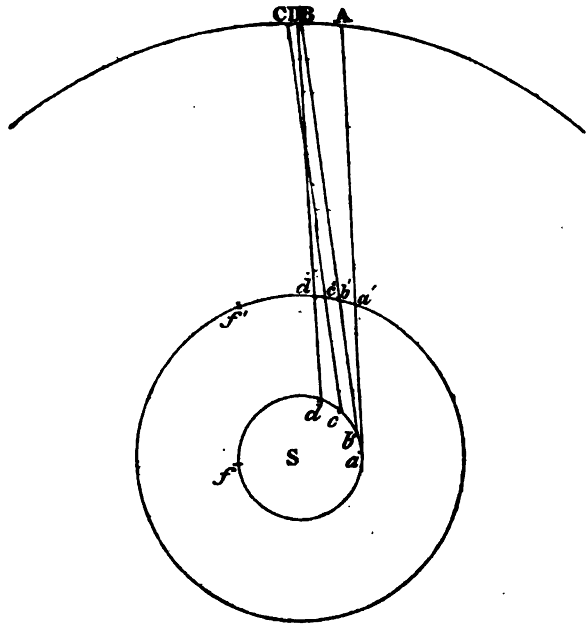
$$1 = \frac{r}{r'}\,(1 - \text{sin.}^2\, Sqb) + \frac{r'^2}{r^2}\, \text{sin.}^2\, Sqb,$$

$$\text{and sin. } Sqb = \sqrt{\left(\frac{r^3 - r^2 r'}{r^3 - r'^3}\right)} = \frac{r}{\sqrt{(r^2 + rr' + r'^2)}}.$$

$$\text{Hence, sin. } Sbq = \frac{r'}{\sqrt{(r^2 + rr' + r'^2)}}.$$

The two angles Sqb, Sbq, being thus determined, $bSq = 180° - (Sqb + Sbq)$ is known; and thence the time from conjunction at L. Thus, the mean daily motions of *Venus* and the Earth being $1° 36' 7''.8$, and $59' 8''.33$, the daily excess is $36' 59''.5$: therefore, if the angle bSq be $13°$, the time from conjunction will be $\frac{13°}{36' 59''.5}$, or about 21 days.

It is plain that *Venus* will be retrograde whilst moving through an arc such as NLt, whether the Earth be supposed to be at rest, or to be in motion. The case however, is different with a superior planet*, which can only be shewn to be retrograde by combining with its motion, the Earth's. Thus, let ab, bc, cd, be three equal arcs in the Earth's orbit, $a'b'$, $b'c'$, $c'd'$, three equal arcs in *Jupiter's* (for instance,) contemporaneously described, but less (see p. 557, l. 16,) let also A, B, C, D, be four points in the imaginary sphere of the fixed stars, to which a', b', c', d' are successively referred by a spectator at a, b, c, d. Now, if ABC be according to the order of the signs, the body in the orbit $a'\ b'\ c'\ d'$, is transferred in that direction or is *progressive;* whilst



the spectator moves from c to d, and the planet from c' to d', the latter, amongst the stars, is transferred from C to D towards B and A, that is, contrary to the order of the signs. During the

* A superior planet includes within its orbit, the Earth's; an inferior planet's orbit is included within that of the Earth's.

description then of the intermediate arcs cb, $c'b'$, the planet must have been stationary. The retrogradation will continue from c through opposition, where it will be the greatest, to a point f, situated similarly to c; that is, such that the angle made by two lines joining $f'f$, fS shall = the angle $c'cS$. From f through conjunction to c, the planet will move according to the order of the signs.

Here then is a material circumstance of distinction, in this part of their theory, between inferior and superior planets. In the explanation of the quiescences and retrogradations of the former, the Earth's motion is not an essential circumstance; it merely modifies their extent and duration. But, with superior planets, the Earth's motion is an indispensable circumstance. The very nature of the explanation depends on its combination with that of the planets.

In speaking of the stations and retrogradations of the planets, we have been obliged to use a language and phrases by no means descriptive of the observations by which those phenomena are ascertained. But, the student must be reminded upon this, as upon other occasions, to attend to the simple facts of observations. When a planet is stationary, the fact of observation is, that the right ascension continues the same: when retrograde, that the right ascension diminishes. The right ascension being determined by the hour, minute, &c. at which the observed body comes on the middle vertical wire of a transit telescope.

Jupiter, in treating of his retrogradations, has been assumed to be a superior planet. One proof of his being such, as well as that *Mars*, *Saturn*, and the *Georgium Sidus* are, is to be derived from their phases; which have not as yet been described.

Now, *Mars* exhibits no such variation of phases as *Venus* does; he is seen, indeed, sometimes a little *gibbous*, but never in the shape of a crescent, nor as a black spot on the Sun's disk. If we add to these circumstances, that he is seen at all angles of elongation from the Sun, we may presume that *Mars* revolves in an orbit round the Sun inclusive of the Earth's; that he is therefore a superior planet. He certainly cannot revolve round the

Earth, for then he would never be stationary, nor retrograde; nor can his orbit pass between the Sun and Earth.

Jupiter, Saturn, and the *Georgium Sidus* do not appear gibbous, but shine, almost constantly, with full orbs.

These phenomena can be accounted for, by supposing *Mars, Jupiter, Saturn,* and the *Georgium Sidus,* to be opaque spherical bodies illuminated by the Sun; and *Mars* to be the least distant: and, if not very distant (relatively to the Earth's distance), his illuminated disk may, in some situations, be so much averted from the spectator, as to give him the appearance of being a little gibbous; and, he will be most gibbous in quadratures: where, however, the breadth of the illuminated part will be to that of the whole disk as 175 to 200.

If we were to increase the distance of *Mars*, the above proportion would approach more nearly to one of equality. Hence the reason, why *Jupiter, Saturn,* and the *Georgium Sidus,* much more distant from the Sun than *Mars,* do not appear gibbous, even in quadratures.

From what has preceded, we may draw this conclusion; that, the adequate explanation of the phases, the stations, and the retrogradations of the planets, on the hypothesis of their revolution round the Sun, renders, at least, that hypothesis probable. But, since the explanation has been one, of obvious and general appearances, and not of phenomena precisely ascertained by accurate observations, the mere fact of a *revolution* has alone been rendered probable, without any determination of the nature of the curve of revolution. It may be either circular or elliptical. The system of Copernicus, therefore, is rather proved to be true, than Kepler's laws, or Newton's theory. Their truth, however, is intended to be shewn, and, that the planets revolve round the Sun in orbits very nearly elliptical: the deviations from the exact elliptical forms being such, as would result from the mutual disturbances of the planets computed according to the law of gravitation. For this end, phenomena, of a different kind from the preceding, must be selected and examined, and explanation, from being general, must become particular, and proceed by calcula-

tion. The *elements* of the orbits and the motions of the planets must be deduced from observations; arranged in Tables; again compounded according to theory; and, in this last state, as results, subjected to the test of the nicest observations.

The elements of the orbits of planets depend on certain distances, linear and angular, measured from the Sun. But, the observations, from which these elements are to be deduced, are made at the Earth. The first step then, in the succeeding investigation, must be towards the invention of a method, for transmuting observations made at the Earth, into observations that would be made by a spectator supposed to be placed in the Sun; in technical language, for converting *geocentric* into *heliocentric* angular distances.

This method is necessary for the extrication of the elements. For the examination of the system founded on those elements, the reverse method is required; in other words, we must be possessed of the means of converting *heliocentric* into *geocentric* angular distances.

CHAP. XXIV.

On the Method of reducing Observations, made at the Earth, to Observations that would, at the same time, be made by a Spectator situated at the Sun: or, on the Methods of extricating, from the Geocentric Observations of a Planet's Place, the Elements of the Orbit which it describes round the Sun.

In the theory of the fixed stars, the spectator is supposed to be placed in the centre of that sphere, which revolving, in twenty-four hours, round an axis passing through the poles of the Earth, produces the common phenomena of the risings, settings, and culminations of stars. In the solar system also, the spectator is supposed to be, very nearly, the centre of the solar motions. In both these cases, the observations are of right ascensions and declinations convertible, by rules already laid down, into longitudes and latitudes; in the case of the fixed stars, either geocentric, or heliocentric longitudes and latitudes; in the case of the Sun, its longitudes, seen from the Earth, differ from the longitudes of the Earth, seen from the Sun by the constant difference of 180 degrees.

The case is very different with the planets. These respect the Sun as the centre of their motions, which motions can only be observed at the Earth. It is necessary, then, if we would trace the orbit of a planet described round the Sun, and lay down the laws of its motion, that we should be able, from *geocentric* observations of a planet's place, and change of place, to infer what that place and change of place would be, were the spectator at the centre of the planet's motions.

The first steps, in this process, would be the same as in the sidereal and solar theories. The planet is to be observed on the

meridian, with the transit instrument and declination quadrant or circle, and, then, from such observed right ascension and declination, the planet's *geocentric* longitude and latitude are to be computed by the formulæ of Chapter VII, (see pp. 160, &c.).

We will give an instance in the computation of the geocentric latitude and longitude of Venus,

March 13,

♁ Æ $22^h\ 58^m$, declination $2^0\ 43'$ N : obliquity $23^\circ\ 27'\ 54''$,

Æ .... $344^\circ\ 30'$

Æ − 90 ... 254 30

$\frac{1}{2}$(Æ − 90).. 127 15 .... log. sin. 9.9009142

2

19.8008284

N. P. D..... $87^\circ\ 17'\ 0''$ .... log. sin. 9.9995117

I .... 23 27 54 .... log. sin. 9.6000890

− 2 log. r − 20.

2) 19.4014291

M .... 30 7 59 .. (log. sin. *M*) 9.7007145

$$\frac{\text{N.P.D.} + I}{2} \qquad 55^\circ\ 22'\ 27''$$

$$\frac{\text{N.P.D.} + I}{2} + M \quad 85\ \ 30\ \ 26 \ \ldots\ldots \text{log. sin.}\ 9.9986635$$

$$\frac{\text{N.P.D.} + I}{2} - M \quad 25\ \ 14\ \ 28 \ \ldots\ldots \text{log. sin.}\ 9.6298461$$

2) 19.6285096

(log. sin. .... $40^0\ 41'\ 38''$) 9.8142548

∴ comp. of lat. = 81 23 16

and latitude ... = 8 36 44

To find the longitude,

Δ	$81^\circ\ 23'\ 16''$	log. sin.	9.9950753
I	23 27 54	log. sin.	9.6000890
δ	87 17 0	(*d*)	19.5951643
2)	192 8 10		
$\frac{1}{2}$ sum	96 4 5	log. sin.	9.9975598
$\frac{1}{2}$ sum $-\delta$	8 47 5	log. sin.	9.1839025
		2 log. r	20
			39.1814623
		(*d*)	19.5951643
		2)	19.5862980
			9.7931490

Now 9.7931490 is the log. sin. of $38^0\ 23'\ 40''$, &c. and of

$$360^\circ + 38^0\ 23'\ 40'' = 398^0\ 23' \cdot 40''$$
$$\therefore 90 + L = 796\ \ 47\ \ 20$$
$$L = 706\ \ 47\ \ 20$$
$$= 360^0 + 346^\circ\ 47'\ 20'';$$

$\therefore$ rejecting 360^0,

the geocentric longitude of ♁, or $L = 11^s\ 16^0\ 47'\ 20''$.

By these means, then, that is, by meridional observations of the planet, and by computations, may its longitude and latitude be determined. Amongst the resulting values of the latitude, there must be some either nothing or very small. Now when the geocentric latitude is nothing, the heliocentric also is nothing, or the planet is in the plane of the Earth's orbit: or, technically, the planet is in its *node:* the node being the intersection of the orbit of a planet, with the plane of the ecliptic. We are able then, by examining the series of the values of the geocentric latitudes, (computed as above) to determine when a planet is in its node, and we also know the geocentric longitude corresponding to such a situation of the planet.

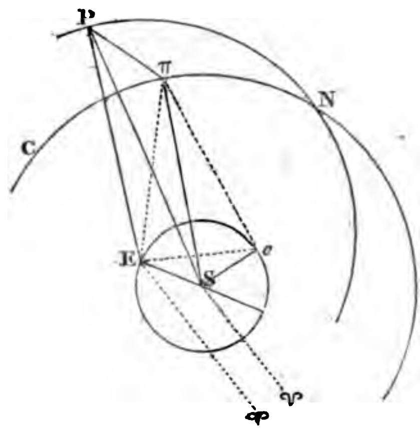
Some values of the latitude will, it has been said, be either nothing, or very small. The latter circumstance is likely to take place: for, it is very improbable that the planet should be, at the same time, on the meridian of the observer, and in the plane of the ecliptic: in the same way, as it is very unlikely to happen that the Sun should be, at once, in the solstice at noon, or in the equinoctial at noon. But the same artifice, or method of computation, which makes amends for the want of coincidence of the two events in the latter case, applies to the one now under consideration. Find, for instance, the longitude and latitude of the planet when just above the ecliptic (to its north) and, the next day, find the like quantities when the planet (supposing it to be descending towards the ecliptic) is just below, or to the south of, the ecliptic. *The Rule of Three*, or some equivalent rule of proportion, will give the longitude corresponding to a latitude that is nothing, or, in other words, will give the geocentric longitude of the descending node.

Before we proceed any farther we will just advert to a point which will soon be more fully discussed. Since we are able to compute the exact time of the planet's entering its node, we are able to determine the interval elapsed in its passage from the *descending* to the *ascending* node, and also the interval of time between two successive returns to the same node. The latter interval must be (supposing the places of the nodes, and the dimensions and positions of the orbit, not to have changed) the periodic time of the planet. The former interval, should it be exactly the half of the latter, would be a proof either that the orbit of the planet was circular, or, if elliptical, so placed as to have its axis major coincident with the line of the nodes.

We will now consider, on what conditions the reduction of geocentric longitudes and latitudes to heliocentric depends: or, what points, relative to the place of a planet, the position and dimensions of its orbit, are necessary to be settled previously to the accomplishment of such reduction.

Let NP be part of the orbit of a planet (superior according to the figure). $N\pi C$ part of the great circle of the ecliptic, E the Earth, S the Sun. Conceive $P\pi$ (part of a great circle) to be

drawn from P perpendicularly to the plane of the ecliptic. Now a spectator at E sees P distant from the ecliptic by an



angle $PE\pi$, which is, therefore, the *geocentric latitude* (G), and P, viewed from S, would appear to be distant from the ecliptic by the angle $PS\pi$, which is, therefore, the *heliocentric latitude* (H).

Suppose ♈ to be, what is called, the first point of Aries: then, since such a point is equally distant with the fixed stars, or so distant that the diameter of the Earth's orbit subtends at it an insensible angle, a line drawn from E to ♈ is to be held to be parallel to a line drawn from S to ♈. From this point ♈ longitudes are computed, therefore,

the geocentric longitude of P (L) is $\angle\, \pi E$ ♈,
the heliocentric longitude of P (P) is $\angle\, \pi S$ ♈,
the longitude of the Sun ($\odot$).... is $\angle\, SE$ ♈.

Hence,

$$L = \odot + \angle\, SE\pi = \odot + E,$$

E representing the angle $SE\pi$, which is technically called the angle of *Elongation*.

This is the denomination of one of the angles of the triangle $SE\pi$. The angle $ES\pi$ is called the *Angle of Commutation* (C),

the angle $S\pi E$, or rather, the angle SPE (the angle under which the planet sees the radius of the Earth's orbit) is called the *Annual Parallax*.

The examination of the parts of the triangle $SE\pi$, will shew us the conditions necessary for the deduction of heliocentric longitudes and latitudes from geocentric.

In the first place

$$\begin{aligned} \Upsilon S\pi (P) &= \angle SE\Upsilon + 180^0 - ES\pi \\ &= \odot + 180^\circ - C. \end{aligned}$$

Hence, we can determine P, the heliocentric longitude, if C the angle of commutation be previously determined.

SE is known from the solar theory,

$$SE\pi, \text{ or } E, = L - \odot,$$

is known since (see p. 564,) L the geocentric longitude can be computed, and the Sun's longitude is known from the solar theory: consequently, in order to determine the angle $ES\pi$ and all the other parts of the triangle, it is only necessary to know $S\pi$, which is denominated the *Curtate Distance*.

Now, $S\pi = SP \,.\, \cos. \angle PS\pi = r \,.\, \cos. H$,

consequently, in order to determine $S\pi$, we must know the values of r and H.

Let I (equal to the angle $PN\pi$) represent the inclination of the plane of the orbit to the plane of the ecliptic, then, by Naper's Rule for circular parts

$$1 \times \sin. N\pi = \cot. I \,.\, \tan. P\pi,$$
$$\text{or } \sin. N\pi \,.\, \tan. I = \tan. H.$$

In order then to determine H, we must previously know I, the inclination, and $N\pi$, the distance of the *reduced* place of the planet from the node of its orbit, which distance is evidently equal to the longitude of the planet minus the longitude of the node.

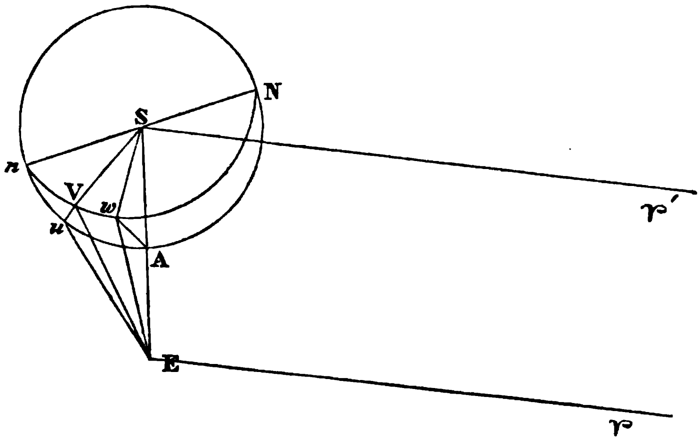
With regard to r (SP), its value may be determined, nearly, (on the supposition of a small eccentricity in the orbit) from

Kepler's law (see p. 455.). It is, however, the *mean distance* which is determined by such law. SP, therefore, is not exactly determined, except P move (which we have no reason to suppose) in a circle. If, therefore, we should be able to determine H exactly, still there would be some uncertainty in determining $S\pi = r \cdot \cos. H$, from the uncertainty respecting r's value, and, accordingly, there would be a corresponding uncertainty respecting the value of the heliocentric longitude determined from the angle $ES\pi$.

For the above reasons, since the heliocentric longitude (we are speaking of the original processes for determining the elements of a planet's orbit) cannot, generally, be exactly found, Astronomers have selected those particular positions of a planet in which its heliocentric longitude is known with certainty. Now such a position, if the planet be an inferior planet, such as Venus and Mercury are, is the superior, or inferior conjunction: in the former the planet's heliocentric longitude is equal to ($\odot$) the Sun's longitude: in the latter, to $180^0 + \odot$. In the case of a superior planet (one whose crbit embraces that of the Earth) its heliocentric longitude, in conjunction, is equal to $\odot$, and in opposition, equal to $180^0 + \odot$.

In such positions, then, the heliocentric longitude of a planet is known independently of any computation of such a triangle as $SE\pi$, and of a radius SP. It is necessary, indeed, to compute its geocentric longitude by the method of p. 564. Suppose Venus to be the planet, and near to her inferior conjunction, on March 8, 1822. Compute from the passage over the meridian (which will be near to noon) and the declination, the geocentric longitude: it will be found to be greater than the Sun's longitude, which, by the Solar Tables, or the Nautical Almanack, is $11^s\ 17^0\ 23'\ 39''$: on the 9th it will also be greater, on the 10th less: so that, at some time on March 9, (when Venus is on the meridian of some other observer) which is easily found by simple proportions, the geocentric longitude will have the same value which the Sun's longitude has at the same time: and at such a time, the geocentric longitude of the planet is the same as its heliocentric.

The diagram employed in p. 566, belongs to a superior planet: but what has been shewn applies equally to an inferior planet. The angle of *elongation* of the latter can never exceed a certain quantity: thus, if NV represent its orbit, the angle



SEu is the angle of elongation, which is greatest at that point at which a line drawn from E becomes a tangent to NAu.

This greatest elongation is called *Digression:* its value in the case of Venus is about 45° 42′: not always of the same value, because both the orbits of the Earth and Venus are eccentric, and inclined to each other.

The angle SVE, the *annual parallax,* may in the case of an inferior planet, be of any value between 0 and 180.

When, however, the planet is Mars, or Jupiter, or Saturn, the angle of elongation may be of any value between 0 and 180°: but the annual parallax can never exceed a certain limit: which limit in the case of Mars is .... 53°

of Jupiter 12

of Saturn 6

of the Georgium Sidus .. 3.

In the preceding disquisition we have endeavoured to bare to the view the real difficulties of the planetary theory, for the pur-

pose of pointing out the way of overcoming them. They are, in many cases, to be got rid of by being eluded: and, indeed, always so to be got rid of when that is the easier way. We here allude to what has been just said respecting the particular positions in which a planet is to be observed, which are those of its conjunctions and oppositions. In such positions, the difficulties of determining the heliocentric longitudes from the geocentric are eluded; or, all cause of uncertainty, respecting the exact values of the former, rescinded. The principle of the method is to be extended to other cases. In determining the inclination of the orbit, its eccentricity, the place of the aphelion, observations of the planet, when it occupies particular positions, are to be selected, or rather, particular positions of the planet and of its orbit: for instance, such would be the observations of a planet in conjunction, and, at the same time, near to the line of its apsides.

But, in these, as in most astronomical processes, there can be prescribed no general and absolute rules. The circumstances of the case must point out the method to be pursued. We must arrive at the end as we can. The simplest way is the best. It is frequently the real triumph of science to elude difficulties that are not easily grappled with.

If we revert to what has been said in pp. 567, &c. we shall easily discern the traces of the route we must pursue. The *nodes*, the *inclination* of the orbit, the *period* with the *mean distance* and *mean motion*, are, in the first place, to be determined approximately, and on the supposition of a circular orbit. In the next place, the *eccentricity* and place of the *aphelion*, are to be determined by a comparison of the mean, with the true longitudes, or, which is the same, by a comparison of the mean with the true motions: the true longitudes being (see p. 568,) what we can obtain, independently of the knowledge of the elements of the orbit, from observations of the planet in its conjunctions or oppositions: the mean longitudes being known from the period of the planet and its longitude at a given epoch.

This, it is plain, is the description of a process which can only give approximate results. But the approximate values of the

eccentricity, and of the place of the aphelion being obtained, the approximate value of the radius vector may be obtained, on which, as we shall soon shew, the determination of the place of the node depends. The place of this latter element may, therefore, by repeating the process for finding it, be more accurately found: or the approximate value of the radius vector may be applied to new or other observations for the same purpose. And it is after this manner, and not by the absolute results of any geometrical, or algebraical theorems, that the knowledge of the elements of a planet's orbit are gradually to be arrived at.

We shall proceed to give, under their separate heads, the methods of finding the elements of a planet's orbit.

Method of finding the Periodic Time, Mean Motion, and Mean Distance of a Planet.

From observations of the right ascension, and declination of the planet, compute (see p. 564,) its geocentric latitude and find when its latitude is equal to nothing. The planet is then in its node. Again, observe the planet and find when it next returns to the same node. The interval of the two computed times, is the periodic time of the planet; which may be nearly determined by one such process as has been just described, and exactly, by the mean of several; exactly, if the *retrogradation* of the nodes be not considerable.

The periodic time of Venus, found from the mean of several passages between its nodes, is, nearly, $224^{d}\ 16^{h}\ 41^{m}$.

The periods of Mars, Jupiter, and Saturn, may also be conveniently found by this method. But if we possess only a limited range of observations, the method loses some of its practical exactness, from our not being able to take the mean of several results. It is an excellent method for Venus, but nearly useless in the case of the Georgium Sidus.

This method, if the entrance of the planet into each node be observed, leads to something beyond the mere determination of the periodic time. It shews, whether or not the orbit be eccen-

tric, and to what extent at least it must be eccentric: and this will appear from the following detail, which Delambre has given us for finding the period of Mars.

(1.) July 23, 1807. ♂ in his *descending* node (☋) and his southern latitude increased till December 16. If we assume this latter time to be that of his greatest latitude, and the interval (145 days) between this greatest latitude, and his being in the node, to be $\frac{1}{4}$th of his period, the period will then be equal to 580 days.

(2.) May 21, 1808. ♂ in his *ascending* node (☊), and the interval elapsed in the passage between node and node (between ☋ and ☊) was 302 days. If that interval were half the period, the period would equal 604 days.

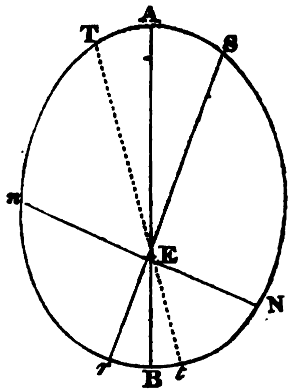
(3.) March 7, 1809. North latitude of Mars was 2° 49′, and on June 8th, was 0: at this latter time Mars had returned to his orbit, after a period of 687 days, which must be, very nearly, its true duration. The mean of several results, obtained as above, makes the period equal to

$$686^{d}\ 22^{h}\ 18^{m}\ 19^{s}.$$

Now, since the interval between node and node is not half the interval between two successive passages of the planet through the same node, it follows that the orbit is not circular, and, moreover, that the major axis is not coincident with the line of the nodes. Neither can the major axis be perpendicular to the line of the nodes: for, in that case, the planet when at the extremity of the axis, would have been at its greatest latitude, and the time from the node to the greatest latitude, would have been half the interval between node and node: whereas, (see above) the time from ☋ to the greatest latitude, was 145 days, but the time from ☋ to ☊ was 302 days ($= 2 \times 151$). This same result, however, which proves the major axis not to be perpendicular, shews also that it must be nearly so.

But we may draw farther inferences. The time from the descending to the ascending node, (from ☋ to ☊) being less than the other half of the period by the quantity 83 ($= 385 - 302$), we have (supposing Nn to represent the line of the nodes),

$$\frac{NAn - NBn}{NAn} = \frac{83}{385},$$



since the areas are proportional to the times. Now when Nn is perpendicular to AB, the difference between NAn and NBn is the greatest it can be. In such a position

$$\frac{AEN - NEB}{AEN} \text{ would equal } \frac{41}{193}, \text{ nearly,}$$

or, the time from B to N would be nearly 152 days,
and the time from N to A . . . . 193.

Now the period being, nearly, 687 days, in which the planet describes 360^0, the time of describing 90^0 would nearly equal 171 days, supposing the planet to depart from B, and to move with its mean motion: but (see l. 6,) the planet was really at N nineteen days previously: in nineteen days, however, the amount of the mean motion is equal to $360° \times \frac{19}{687}$, or nearly $10°$.

At the time, therefore, the real planet was at N, the fictitious planet or body would be, nearly, 10^0 behind. Now this difference, or angular distance is no other (see Chapter XVIII.) than the *equation of the centre*. Such equation, at the point N, is not exactly, although it is nearly so, at its greatest value. *The*

greatest equation of the centre, then, in Mars' orbit, cannot be less than 10°. In fact, it must be greater, not only from the cause just assigned, but because the difference of the times from B to N, and from N to A, would be greater than observation shews it to be, if Nn were (which it is not) perpendicular to AB the line of the apsides.

The same process for finding the period, and like inferences, relative to the degree of eccentricity, are applicable to Jupiter and Saturn. For instance, we have, according to M. Delambre,

in Oct. 13, 1794, (286 days) ♃ in ☋,
May 18, 1800, (138 days) ♃ in ☊;

therefore 5^{y} 218^{d}, or 2043 days is half a revolution.

Again,

1806, 239^{d} .... ♃ in ☊,
1794, 286................ ♃ in ☋,

11^{y} 318^{d}, or 4335 days is the period of Jupiter.

Hence, the difference between the two half revolutions, is about 249 days: the fourth of which is 62¼, in which time Jupiter describes about $5^{0}\ 4'\left(=360\times\frac{62.25}{4335}\right)$. The greatest equation, therefore, of the centre in Jupiter's orbit (see p. 575,) cannot be less than 5° 4'. The axis major of Jupiter's orbit is nearly perpendicular to the line of the nodes; which circumstance, as in the former case (see p. 575,) might be ascertained by an observation of Jupiter, at the time of his greatest latitude.

In the case of Saturn, the two half revolutions from node to node (from ☋ to ☊ and from ☊ to ☋) are nearly equal. The orbit of Saturn, therefore, is either nearly circular, or (which by other methods is proved to be the case) the line of its nodes is coincident with the axis major. We cannot in this case, from observations of the passages of the nodes, determine the quantity, than which the greatest equation cannot be less.

Since the periodic time is an important element, we will give other methods of determining it.

*Second Method of determining the Periodic Time**.

Observe the planet in opposition, then its place, with regard to longitude, is the same as if the observation were made at the Sun. Amongst succeeding oppositions, note that in which the planet is in the same part of the heavens, as at the time of the first opposition. The interval between the two similar oppositions is nearly the periodic time of the planet, or a multiple of the periodic time.

Since the planet, at the last of the two similar oppositions, will not be exactly in the place in which it was at the time of the first, the *error*, or *deviation*, must be corrected and accounted for, by means of a slight computation, similar, in principle, to several preceding computations, and the nature of which will be sufficiently explained by an Example.

Sept. 16, 1701, 2^h ♄'s long. in ☍ $353^\circ\ 21'\ 16''$ S. lat. $2^\circ\ 27'\ 45''$
(2) Sept. 10, 1730, $12^h\ 27^m$ ♄'s long. in ☍ $347\ \ 53\ \ 57$ S. lat. $2\ \ 19\ \ 6$
Interv. $29^y - 5^d\ 13^h\ 33^m$, diff. of long. $5^\circ\ 27'\ 19''$.

Hence, it is plain, we must find the time of describing this difference $5^\circ\ 27'\ 19''$: and the means of finding it may be drawn from other observations of the planet made in September 1731.

(3) Sept. 23, 1731, $15^h\ 51^m$ ♄'s long. in ☍ $0^\circ\ 30'\ 50''$ S. lat. $2^\circ\ 36'\ 55''$
Interval betw. (3) and (2) $1^y\ 13^d\ 3^h\ 24^m$, diff. of long. $= 12^\circ\ 36'\ 53''$

Hence,

$12^\circ\ 36'\ 53'' : 5^\circ\ 27'\ 19'' :: 1^y\ 13^d\ 3^h\ 24^m$: time required, which time $= 163^d\ 12^h\ 41^m$.

Hence, adding this time to the former interval between opposition and opposition, we have

$$\text{♄'s periodic time} = \left\{\begin{array}{rrrr} 29^y & 7^d & 0^h & 0^m \\ + & 163 & 12 & 41 \\ - & 5 & 13 & 33 \\ \hline 29 & 164 & 23 & 8 \end{array}\right. \quad \text{(7 Bissex.)}$$

* The periodic times of planets are important elements, and admit of being very exactly determined; and when determined, become the best means of determining the mean distances, which by parallax, or other methods, are very inaccurately found.

And consequently, *Saturn's* mean motion for one year, or mean annual motion $= 360^0 \times \dfrac{1^y}{29^y\ 164^d\ 23^h\ 8^m} = 12^\circ\ 13'\ 23''\ 50'''$.

If the major axis of *Saturn's* orbit be, like that of the Earth's, progressive, then the above determination of the periodic time will not be very exact. And indeed, it ought rather to be regarded as a first approximation, and as the means of obtaining the true value of the periodic time more exactly. Using it therefore as an approximation, we may, by comparing oppositions of the planet, distant from each other by so large an interval of time, that the inequalities of the several revolutions will be mutually balanced and compensated, determine the periodic time to much greater, and indeed, to very great exactness. Thus,

228 *A. C.* March 2, 1^h ♄'s long. in ☍ $98^\circ\ 23'\ 0''$ N. lat. $2^\circ\ 50''$
(2) Feb. 26, 1714, $8^h\ 15^m$ ♄'s long. in ☍ 97 56 46 N. lat. 2 3
* Interval $1943^y\ 105^d\ 7^h\ 15^m$, diff. of long. 26 14.

In order to find the time of describing $26'\ 14''$, as before, p. 575, &c.

(3) March 11, 1715, $16^h\ 55^m$ ♄'s long. in ☍ $111^\circ\ 3'\ 14''$ N. lat. $2^\circ\ 25'$
Interval between (2) and (3) $378^d\ 8^h\ 40^m$; diff. of long. $13^\circ\ 6'\ 28''$

$\therefore$ time of describing $26'\ 4'' = 378^d\ 8^h\ 40^m \times \dfrac{26'\ 4''}{13^\circ\ 6'\ 28''} = 13^d\ 14^h$.

Adding this to the former interval, we have $1943^y\ 118^d\ 21^h\ 15^m$ for the interval, during which, *Saturn* must have made a complete number of revolutions. Now, if the periodic time ($29^y\ 164^d\ 23^h\ 8^m$) previously determined, had been exactly determined, then, dividing the interval by the periodic time, the result would have been an integer, the exact number of revolutions. But, the period having been only nearly determined, the result of the division (the quotient) will be an integer and some small fraction: still the number of revolutions which can only be denoted by an integer, must be denoted by that same integer. And in the case

* 11 days are subtracted, in order to reduce it to the stile of the first observation, and 485 days added on account of the Bissextiles.

before us, it will be 66. The number of revolutions then being exactly 66, the exact time of one revolution

$$= \frac{1943^y\ 118^d\ 21^h\ 15^m}{66} = 29^y\ 162^d\ 4^h\ 27^m.$$

Hence, according to this more correct value of the periodic time, the mean annual motion is $12°\ 13'\ 35''\ 14'''$, and the mean daily $2'.0097$.

In the preceding method of determining the periodic time, *Saturn* was reduced to the same *longitude*. And longitude is measured from the first point of *Aries*, which point is continually moving westward $50''.1$ annually, and therefore, in $29^y\ 162^d\ 4^h\ 27^m$ moves through $24'\ 35''$. The period, then, of *Saturn*, which has been determined ($29^y\ 162^d\ 4^h\ 27^m$) belongs to his *tropical* revolution, and is shorter than that of his *sidereal*, by the time requisite to describe $24'\ 35''$, that is, about $12^d\ 7^h$. Hence, *Saturn's* period of sidereal revolution will be $29^y\ 174^d\ 11^h\ 27^m$.

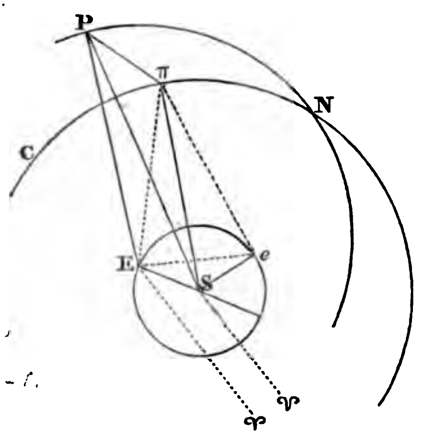
It is equally easy to determine, directly from observations, the period of the sidereal revolution. Since, instead of *reducing Saturn* to the same longitude, we should have so to reduce his place, that it should be at the same distance from a fixed star at the end, as it was at the beginning of the period.

But suppose a new planet to be discovered more distant than Saturn, must we be obliged to wait during a long term of years, to observe the successive returns of the planet to its node, in order to discover its mean period and distance, or, amongst the resources of Astronomical Science, can we find some means of supplying the defect of past observations, or of anticipating the results of observations to be hereafter made? We shall find an answer to this question by merely stating what has taken place with respect to the Georgium Sidus (or Uranus as the French call it). The planet was discovered in 1781, and in 1796 the Tables of its motions were inserted in the Nautical Almanack: indeed, so near the time of its discovery as the year 1782, the elements of its orbit, (as we find by the Memoirs of the Academy of Paris for that year) were computed by Lalande, and, amongst such elements, that of its period was stated to be 84 years.

This then is a sort of practical answer to the question just stated, and a proof that some method, other than has been described, was resorted to by Astronomers for discovering the period, and other elements of a planet, endowed with so very slow a motion.

The method of Lalande, one of trial and conjecture (of trial indeed, which after a few times was sure of succeeding) will easily be understood by adverting to what was said in pages 566, &c.

The angle of elongation (E) $= L - \odot$, L being the geocentric longitude, and $E\pi E$ the angle of parallax, (π) is the



difference of the heliocentric and geocentric longitude, and, therefore, is equal to $P - L$.

Now E is known from L and $\odot$ (see p. 566.), and since $\sin.\ \pi = \sin.\ E \cdot \dfrac{SE}{S\pi}$ we can find π, and thence $P = L + \pi$, if we can find $S\pi$, or, which is the same thing, if we *assume* a value (r) for $S\pi$ ($S\pi$ and SP are nearly equal) we shall have from the above equation of a corresponding value of π, and thence of P: suppose its value to be P'. Use the same process, with the same assumed radius $S\pi$, with a second and

third geocentric longitude, and let the two resulting heliocentric longitudes be P'' and P''', then we have

$$P'' - P', \; P''' - P'', \text{ and } P''' - P',$$

and from knowing the three times of observations (t', t'', t''') we know

$$t'' - t', \; t''' - t'', \text{ and } t''' - t'.$$

Take any one of these three differences, the last, for instance, then

$$P''' - P' : t''' - t' :: 360^0 : \text{period of the planet.}$$

But r is the assumed mean distance, accordingly, by Kepler's law (see p. 455,)

$$1^{\frac{3}{2}} : r^{\frac{3}{2}} :: 365.256384 : \text{planet's period.}$$

The agreement of this value with the former would be a proof that r had been rightly assumed. The disagreement, by its nature and degree, would point out to us the manner and extent of correcting the first assumption of r.

This is a description of the method which Lalande employed. He possessed three geocentric observations of the planet (♅) made in 1781, on April 25, July 31, and December 12, and he found the period (according to the method just described) by means of the first and third observation. The two values of the period (as it was probable they would) were found to disagree. Lalande, therefore, amended his first assumption: and assigned, partly by conjecture, and partly by the guidance of his first trial, a new value of the distance, and then examined it, as the former. By a repetition of like trials and examinations a radius vector was at length obtained, which agreed with all observations *.

* This method of M. Lalande's, is a kind of sample and exemplar of almost all astronomical processes. In these, at first, nothing is determined exactly. Approximate quantities are assumed and substituted, the results derived from them, examined and compared, and then other approximations, probably nearer to the truth, suggested. Astronomy leans for aid on Geometry; but the precision of Geometry does not extend beyond

the

We may state somewhat differently, but without any alteration of principle, the above process of approximation.

Should the first, or any observations of the planet shew the angle of elongation ($= L - \odot$) to be obtuse, the planet must be a superior one: in which case, 1 being the mean distance of the Earth, r must be > 1.

Assume $r = 1.5,\ 2,\ 2.5,\ 3,\ 3.5$, &c.

and form the corresponding values of π from

$$\sin.\ \pi = \frac{\sin.\ E}{r}:$$

thence, write down the corresponding values of

$$P = L + \pi.$$

Repeat these operations on succeeding observations, and then, by subtracting the heliocentric longitudes of one day, from those of the preceding day, deduce the heliocentric motions of the planet; suppose dP to represent this motion, and $d\odot$ the Sun's daily motion, then, since the angular velocity

$$= \frac{\text{area described in a given time}}{(\text{dist.})^2}$$

$$= \frac{\text{whole area}}{\text{period}} \times \frac{1}{(\text{dist.})^2},$$

and since the whole areas (if the orbits be circular) vary as the squares of the radii, and the periods vary as $(\text{radii})^{\frac{3}{2}}$, we have

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the limits of its theorems. In Astronomy scarcely one element is presented simple and unmixed with others. Its value when first disengaged, must partake of the uncertainty to which the other elements are subject; and can be supposed to be settled to a tolerable degree of correctness, only after multiplied observations, and many revisions. There are no simple theorems for determining at once the parallax of the Sun, the right ascension of a star, or the heliocentric latitude of a planet.

$$dP : d\odot :: \frac{r^2}{r^{\frac{3}{2}}} \times \frac{1}{r^2} : 1;$$

$$\therefore\ r = \left(\frac{d\odot}{dP}\right)^{\frac{2}{3}},$$

from which expression, since $d\odot$ is known from the Solar Tables, or the Nautical Almanack, r may be computed, and its several values corresponding to the several values of dP. Of the originally assumed values of r (see p. 580, l. 7,) that which, most nearly, approaches to one of these lastly deduced values of r, is the value nearest to the truth. Thus suppose one of the values from the expression

$$r = \left(\frac{d\odot}{dP}\right)^{\frac{2}{3}},$$

should be 19.3, then, since 19.5 is, of the originally assumed values, nearest to 19.3, we may conclude 19.5 to be nearly the true value, and whether the true value is between 19 and 19.5, or between 19.5 and 20, must be inferred from the two contiguous values of r, namely, from

$$r = \left(\frac{d\odot}{dP}\right)^{\frac{2}{3}}, \text{ and } r' = \left(\frac{d\odot}{dP}\right)^{\frac{2}{3}}.$$

The periodic time of a planet (P) being found, its mean daily motion (M) may be thence derived from this proportion,

$$P : 1 :: 360^\circ : M = \frac{360}{P},$$

P being expressed in days and parts of a day.

Thus, in the case of Venus, P being $225^d\ 16^h\ 41^m$, the mean motion is

$$= \frac{360}{225^d\ 16^h\ 41^m} = \frac{1^\circ}{.62415319} = 1^\circ.6027 = 1^\circ\ 36'\ 9''.7.$$

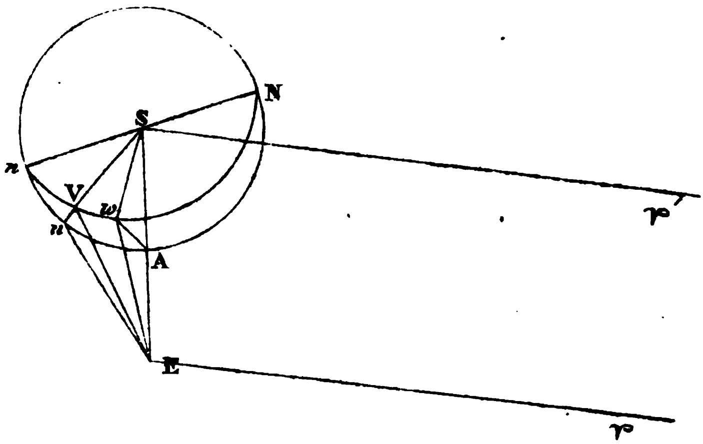
The mean distance (a) may be found by Kepler's law. Thus, 1 representing the Earth's mean distance from the Sun, and $365^d.256384$ being the value of the Earth's sidereal period,

$$(365.256384)^{\frac{2}{3}} : p^{\frac{2}{3}} :: 1 : a = \left(\frac{p}{365.256384}\right)^{\frac{2}{3}} *.$$

But although this is the best, it is not the only way of finding the distance of a planet. The distance of Venus may be found from her greatest elongation (technically called her *digression*). Thus, by examining a series of angles of *elongation* (E) formed from the expression

$$E = \pm (L - \odot),$$

it is found, that the greatest value of E is about $45^{0}\ 42'$, and



when E is the greatest, the angle SuE is a right angle, Eu being a tangent to nuA. In this case, then,

$$\dagger\ Su = SE . \sin.\ 45^{\circ}\ 42' = .7157, \text{ if } SE = 1.$$

These *digressions* of Venus would all be of the same value, if Venus and the Earth revolved in circular orbits. But, as we have

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* This is not exactly true: let μ = Sun's mass + the planet's mass, μ' = Sun's mass + Earth's mass;

$$\text{then } \left(\frac{365.256384}{p}\right)^{2} = \frac{\mu}{\mu'} \times \left(\frac{1}{a}\right)^{3},$$

is the exact equation from which a is to be deduced, (see *Physical Astronomy*, p. 30.)

† Vu should have been more inclined to SV, and then Su would be a line drawn from S to u.

seen (p. 449,) SE is a variable distance. Still the differences in the values of the *digressions* cannot be accounted for, by estimating the effects of the eccentricity of the Earth's orbit: the inference from which circumstance is, that Venus's orbit is also elliptical.

There are particular conjunctures from which, on the supposition of the orbit of Venus being elliptical, we could determine the value of its eccentricity. Suppose, for instance, we possessed, amongst our observations, two *digressions* (E and E'), one made when Venus was at the aphelion of her orbit, the other at the perihelion; in that case, if e were the eccentricity, R and R' the distances of the Earth from the Sun, we should have (r being the mean distance of Venus),

$$r + re = R \, . \, \sin. \, E,$$
$$r - re = R' \, . \, \sin. \, E',$$
$$\text{whence} \quad e = \frac{R \, . \, \sin. \, E - R' \sin. \, E'}{2r}$$
$$= \frac{R}{2r} (\sin. \, E - \sin. \, E'),$$

if we suppose $R = R'$.

We might also, (could we rely on the accuracy of the measurements) determine the relative values of the radii of the orbits of Venus and the Earth, from the apparent diameters of the former planet, at her greatest and least distances. Thus, should the least and greatest apparent diameters be, respectively, $10''$ and $60''$, we should have

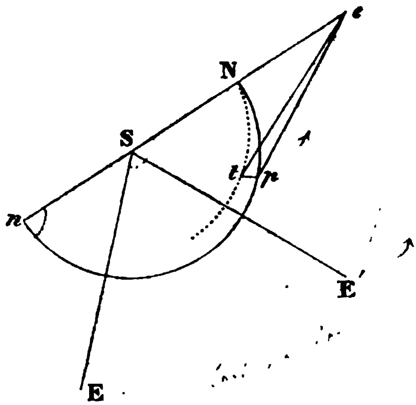
$$\frac{60}{10} = \frac{1 + r}{1 - r}, \text{ and } r = \frac{5}{7}.$$

Method of determining the Nodes of a Planet's Orbit.

The nodes of a planet's orbit, are those two points in it in which it is cut by the ecliptic. The node which the planet quits in ascending towards the north pole of the ecliptic, is called the *Ascending Node*, and its symbol is ☊. The reverse or ☋, is the

symbol of the *descending node*, or, of that node from which the planet moves towards the south pole of the ecliptic.

Let N, n represent the nodes; now by observations of the planet's right ascension and north polar distance, we can compute its geocentric latitude (see p. 563,) and thence determine



when the latitude is 0, or when the planet is in its node: let E, E' be the two positions, when the planet is respectively at n and N, then we have (see p. 582,)

$$^{*}SEn = \text{geocentric longitude of planet at } n - \odot,$$

$$SE'N = \odot' - \text{geocentric longitude of planet at } N,$$

and from the last method we know Sn, or SN; thence we can compute, in the triangles SEn, $SE'N$, the angles nSE, SnE, and NSE', SNE': and thence

$$\begin{aligned} \text{heliocentric long. of } n &= \text{geocentric long. of } n + \angle SnE, \\ \text{or} &= 180^\circ + \odot \ldots\ldots - \angle nSE, \\ \text{and helioc}^{c}\text{. long. of } N &= \text{geocentric long. of } N - \angle SNE', \\ \text{or} &= \odot' - 180^\circ \ldots\ldots + \angle NSE', \end{aligned}$$

$\odot$ and $\odot'$ representing the Sun's longitudes at the two times of observation.

The angle ESE' is proportional to the Earth's motion during the planet's passage from n to N.

---

* Conceive two lines drawn from E and E' to n and N, respectively.

Venus, of which the period is less than 225 days, may, in the space of a year, be observed three times in the ecliptic; the longitude of the node is, according to astronomical usage, to be estimated from the mean of a great number of observations at n and N.

In the above method, we have supposed the planet to be successively at n and N: but one observation is sufficient, as far as the principle of the method is concerned, to determine the longitude of the node. For example, in May 14, 1747, Mars was observed to be descending towards, and to be very near to, his descending node. By continuing the observations, and by a computation like that described in p. 575, Mars was found to be in his node on May 14, at $14^h\ 25^m\ 13^s$, whilst his geocentric longitude was computed to equal $7^s\ 6^0\ 13'\ 42''$.

Hence,

$$L = 7^s\ \ 6^0\ \ 13'\ \ 42''$$

$$\text{by Solar Tables } \odot = 1\ \ 23\ \ 46\ \ 47$$

$$\therefore \text{(see p. 584, l. 9,) } L - \odot \text{ or } E = 5\ \ 12\ \ 26\ \ 55$$

$$\text{but sin. } \pi\,(SnE) = \text{sin. } E \times \frac{SE}{Sn};$$

Sn being taken equal to 1.5446, and SE to 1.008;

$$\therefore \pi = 0^s\ \ 11^0\ \ 22'\ \ 55''$$

$$\text{but (see p. 584, l. 10,) } L = 7\ \ \ 6\ \ 13\ \ 42$$

$$\therefore \text{heliocentric longitude of } n, \text{ or } \pi + L = 7\ \ 17\ \ 36\ \ 37$$

which is the longitude of the descending node of Mars, at the time of observation.

The angle π (see l. 19,) depends on the value of $\dfrac{SE}{Sn}$.

The numerator SE is known from the solar theory: but the preceding method of pages 580, &c. determines solely the *mean distance* of Mars. If, therefore, from original osbervations, we were about to deduce the elements of that planet's orbit, we could only, in the first steps of the deduction, approximate to the longitude of the node: because we should, in such first steps, be

obliged to consider the orbit of Mars as circular, or, which is the same thing, we should be obliged to assume for Sn that value of the mean distance, which would result from the expression

$$Sn = \left(\frac{686.979619}{365.256384}\right)^{\frac{2}{3}} = 1.523694.$$

In this case then (see p. 585, l. 19,), we should have

$$\log. \sin. \pi = \log. \sin. 5^s\ 12^\circ\ 26'\ 55'' + .00471 - .1828965$$
$$= 9.3011888 = \log. \sin. 11^\circ\ 32'\ 28''.$$

Hence, the first approximate value of the longitude of the node would be greater than the one deduced by $9'\ 33''$: which is the error caused by supposing Mars' orbit to be circular, for the value of Sn in p. 585, was taken from the Tables of Mars.

When we determine, as above, the longitude of the node, from computing the time of the planet's entering the ecliptic, we do not require to be known the *inclination* of the planet's orbit. In a scientific arrangement, the determination of that element would be placed, after that of the node. But if we suppose the inclination to be known, or (which is the real astronomical usage) if, in performing the circuit of revision and correction, we wish, from an approximate value of the inclination, to correct by means of recorded observations, the elements of the orbit, we may compute the place of the node, by slightly modifying the above method. Thus, in the instance given, the observations of Lacaille were as follow:

May 14, 1749, $10^h\ 50^m\ 43^s$. geo. long. ♂ (L) $7^s\ 6^\circ\ 15'\ 20''$, lat. $25''.5$

	Sun's long.	1	23	38	10
	E	5	12	37	10
$\left(\text{from } \sin. \pi = \sin. E . \frac{1.008}{1.5446}\right)$	π	0	11	16	37
(heliocentric long. ♂); $\therefore \pi + L$		7	17	31	57.

But this is the heliocentric longitude of Mars, when his geocentric latitude was $25''.5$. If we could thence find the heliocentric latitude, and knew the inclination of the orbit to the ecliptic, we could thence deduce (see Figure of p. 582,) nu. With regard to the first point, the deduction of the *heliocentric*

from the *geocentric* latitude, since Vu is a tangent to the angles VSu, VEu to the respective radii Su, Eu*,

$$Su \,.\, \text{tan.}\ VSu = Eu \,.\, \text{tan.}\ VEu,$$

but $\dfrac{Su}{Eu} = \dfrac{\text{sin.}\ E}{\text{sin.}\ C}$ (C being ESu the angle of commutation)

$$\begin{array}{rl} \text{and since } E = & 5^s\ 12^\circ\ 37'\ 10'' \\ \pi = & 0\ \ 11\ \ 16\ \ 37 \\ \text{it is necessary that } C = & 0\ \ \ 6\ \ \ 6\ \ 13 \\ \hline & 6\ \ \ 0\ \ \ 0\ \ \ 0 \end{array}$$

$$\text{Hence,}\quad \text{tan.}\ VSu = \text{tan.}\ 25''.5 \times \frac{\text{sin.}\ 6^\circ\ 6'\ 13''}{\text{sin.}\ 17^\circ\ 22'\ 50''} =$$

tan. $9''.2$; $\therefore 9''$ is nearly the heliocentric latitude, which being very small, we may consider the right-angled triangle nVu as right-lined, and solve it accordingly: which we can do, if the angle Vnu (the *inclination*) be known. Let it be $1^\circ\ 51'$, then $nu = 4'\ 41''$, nearly, which being added to $7^s\ 17^\circ\ 31'\ 57''$, (the heliocentric longitude of ♂ decending towards and very near to, its node) there results for the heliocentric longitude of the node

$$7^s\ 17^\circ\ 36'\ 38'',$$

which, within one second, is the result of p. 585, l. 23.

In these methods, the determination of the place of the node is the more difficult the less is the inclination of the planet's orbit. For that reason it is difficult to determine the nodes of the orbits of Jupiter and the Georgium Sidus.

Method of determining the Inclination of the Orbit of a Planet to the Plane of the Ecliptic.

The longitude of the node being known by the preceding methods, compute the day on which the Sun's longitude will be the same, or nearly the same. The Earth will then be in the line of the nodes Nn, at some point e (fig. of p. 584,): observe, on that day, the planet's right ascension and north polar distance, and deduce (see pp. 563, &c.) the geocentric latitude (G);

* The lines SV, Vu should have been more bent to each other than they are in the Figure.

$$\text{then } tp = et \,.\, \tan. G = St \,.\, \frac{\sin. tSe}{\sin. Sep} \,.\, \tan. G$$

$$= \frac{\sin. Nt}{\sin. E} \,.\, \tan. G,$$

but, in the right-angled triangle Ntp, we have by Naper's Rules,

$$\sin. Nt = \cot. tNp \,.\, tp, \text{ or } \tan. I \,.\, \sin. Nt = tp,$$

I denoting the inclination,

$$\text{accordingly, } \tan. I \,.\, \sin. Nt = \frac{\sin. Nt}{\sin. E} \,.\, \tan. G,$$

$$\text{and } \tan. I = \frac{\tan. G}{\sin. E}.$$

The diagram that has been referred to belongs to an inferior planet: but, a like diagram, and the same process, will apply to a superior planet.

As an instance of the method, suppose we possessd the following observations, on Jan. 12, 1747, $6^h\ 6^m\ 33^s$:

long. ♄	6^s	26°	$12'$	$52''$,	lat. N. $2^\circ\ 29'\ 18''$
on the above day, ☉, or the Sun's long.	9	21	47	0	
∴ E	2	25	34	8	
Now, by Lalande's Table, ☋ or long. of node ...	9	21	31	0	

or the Earth was, then nearly, in a position such as e.

Hence, from the expression of l. 7,

log. tan. $2^\circ\ 29'\ 18''$	8.6380591
log. sin. 85 34 8	9.9986999
	8.6393592

and this result is the logarithmic tangent of $2^\circ\ 29'\ 44''.8$, which, accordingly, is the value of the inclination of Saturn's orbit from the above observation, and which must be very nearly its true value.

It is not its *exact* value, because the Sun's longitude being greater than the longitude of the node by $15'$, the Sun at the time

of observation, had passed the line of the nodes. About 6 hours previously, the Sun was in the line. In order, therefore, to correct the above result, we must correct, proportionally to such time, the geocentric latitude, and the geocentric longitude, and, consequently, (see p. 586, l. 26,) the angle E. The corrected place of the node is then to be deduced from the expression

$$\tan. I = \frac{\tan. G}{\sin. E},$$

G and E being now the corrected values.

But it is plain that this last result will differ very little from the former: for, the angle of elongation being 85° 34′ 8″, and the angle of parallax about 6°, the remaining angle of the triangle formed by the Earth, Sun, and Saturn, or the angle of *commutation*, will be 91° 34′: consequently, Saturn will be nearly at the same distance, both from the Sun and the Earth, and his heliocentric latitude will not differ much from his geocentric: but the latter is 2° 29′ 18″; therefore, since the inclination (which is measured by the greatest heliocentric latitude) is 2° 29′ 44″.8, the planet must be nearly at its greatest heliocentric latitude, and quantities, at or near to their *greatest* values, change very slowly.

The angle of elongation will vary with the geocentric longitude, and accordingly, in the present case, very little: but the inclination (see p. 588,) depends on the sine of the angle, which angle is between 85° and 86°, and consequently not far from that value at which the sine is a maximum. In this case then, as in the former, scarcely any alteration will take place in the new value of the sine of E. Hence, in the expression $\tan. I = \frac{\tan. G}{\sin. E}$, the resulting value of I will be nearly the same whether we use the original or the corrected values of G and E: or, which is the same thing, the inclination was very nearly determined by the first calculation.

---

* Log. sin. E 9.99870
log. ♄'s dist. .... .97949

9.01921 = log. sin. 6°.

The inclination may also be determined from observing the planet at a conjunction, when it has considerable latitude. Thus, suppose the planet to be Venus, at a point w of her orbit, (see fig. of p. 582,) such that A the reduced place in the ecliptic is in the same straight line with E and S: then, as before, we have

$$EA \,.\, \tan. AEw = SA \,.\, \tan. ASw.$$

$$\text{Let } SE = 1, \quad SA = \rho, \quad Sw = r, \quad ASw = H,$$

$$\text{then } (1 - \rho) \,.\, \tan. G = \rho \,.\, \tan. H.$$

But in the right-angled triangle Anw (right angled at A),

$$\sin. nA \,.\, \tan. I = \rho \tan. H;$$

$$\therefore (1 - \rho) \frac{\tan. G}{\sin. nA} = \tan. I.$$

Now nA is the longitude of the planet minus the longitude of the node. The latter quantity is supposed to be known by the preceding methods, and, the planet being in conjunction, its longitude is the same as the Sun's longitude: hence, if ☊ denote the longitude of the node n,

$$\tan. I = (1 - \rho) \frac{\tan. G}{\sin. (\odot - ☊)} *,$$

$$\text{but } \rho = r \,.\, \cos. I = \frac{r}{\sec. I} = \frac{r}{\sqrt{(1 + \tan.^2 I)}} = r \times \left(1 - \frac{1}{2} \tan.^2 I\right)$$

---

* The inclination of the orbit of Venus is about 3° 23′: suppose such an inferior conjunction to be observed, that the planet is 90° from its node: then $\odot - ☊ = 90°$, and

$$\tan. G = \frac{\sin. 3° \, 23'}{.276} = 214, \text{ nearly, and}$$

$$G = 12° \; 5'.$$

Again, suppose a like superior conjunction to be observed, then

$$\tan. G = \frac{\tan. I}{1 + \rho} = \frac{\tan. 3° \, 23'}{1.723} = .0343,$$

$$\text{and } G = 1° \, 58', \text{ nearly.}$$

Hence, as Delambre observes, it would be necessary, in order that Venus should be always seen in the zodiac, that the breadth of the zodiac should be, at the least, 24°.

if (as is almost always the case) I be very small, hence,

$$\tan. I = \left(1 - r + \frac{r}{2}\tan.^2 I\right)\frac{\tan. G}{\sin.(\odot - \Omega)},$$

from which I may be obtained by approximation, or the solution of a quadratic equation, or, in the expression of p. 590, if we make $\rho = r$, we may thence deduce an approximate value of I, which approximate value being substituted in $\rho = r\cos. I$, we may, from the same equation, obtain a new value of $\tan. I$.

We have now obtained the mean distance, the longitude of the node, and the inclination of the orbit of a planet: but, hitherto, nothing has been determined respecting the form of the orbit: indeed, in some of the previous determinations, we have been obliged to suppose the orbit circular, or to assume for the radius vector of the planet's orbit, its mean distance as it results from Kepler's law. We must now consider whether the steps that have been made good, will enable us to proceed farther, and to find out, what probably, and by analogy, exists, the *eccentricity* of the orbit; and then the place of the aphelion.

We have already seen, in a particular instance, from certain differences in the *digressions* of Venus, that her orbit is eccentric: but our present concern is, with some general method, of ascertaining and valuing the eccentricity and place of the aphelion of the orbit of any planet. It will not be difficult to find out the grounds of such method.

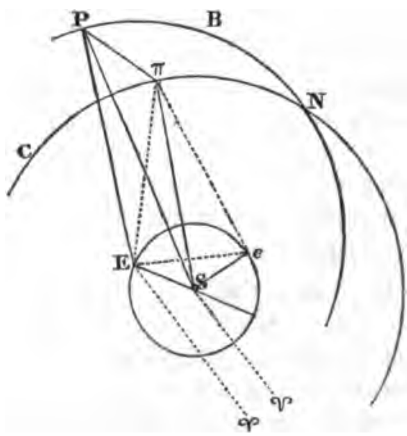
Suppose, for the sake of simplicity, the planet's orbit to lie in the plane of the ecliptic. Since, (see pp. 571, &c.) we know the mean motion, and, by observing the planet in conjunction, or opposition, the planet's true longitude (see p. 568,) we can, after any elapsed time, compute the planet's mean longitude. Let the elapsed time be the interval between two conjunctions: then, if the orbit were circular, the computed mean longitude would agree with the last observed longitude *; but a difference between them would be an indication of the orbit's eccentricity.

* Except, which is highly improbable to happen, the planet, at the times of the two conjunctions, should be in the aphelion, or perihelion of its orbit: for at those points the mean and true anomalies are the same.

This difference must depend both on the eccentricity, and the place of the aphelion. It must depend upon the former, because if, in a given position of the orbit, the eccentricity were increased, the difference between the computed and observed longitudes would also increase. It must depend on the place of the aphelion, because, if the planet be there at the time of the observed conjunction, the true and computed places of the planet will agree. The differences then of the computed and observed longitudes depend on the eccentricity, and the position of the axis major of the orbit, and it is a fit subject of mathematical investigation, to deduce the eccentricity and the place of the aphelion, from such differences.

We will now consider what effect on the preceding reasonings will be produced by restoring to the orbit its inclination.

Let N be the node of the orbit, then its longitude (see p. 583, &c.) is known. The longitude of the planet, when in conjunction, is



known, since it equals $180° + ☉$. Hence, deducting the longitude of the planet from the longitude of the node, there remains $N\pi$. Now since the elliptical motion takes place in the orbit NP, it is requisite to know NP, and like distances of the planet in its orbit from the node. But $N\pi$ being known, and the angle $PN\pi$; the distance NP may be determined, either by the solution of the triangle $PN\pi$ (right angled at π) or (see pp. 505, &c.)

by the formula of reduction: for, it is plain, the finding of NP, from $N\pi$ and the angle $PN\pi$, is analogous to the finding of the longitude, from the right ascension and obliquity. In the formula, therefore, of p. 506, l. 8, write NP instead of $\odot$, and $N\pi$ instead of Æ, and let t be the tangent of inclination, then

$$NP = N\pi + t^2 \cdot \frac{\sin. 2\,N\pi}{\sin. 1''} + t^4 \cdot \frac{\sin. 4\,N\pi}{\sin. 2''} + \&c.$$

If we set off, on the orbit of the planet, an arc $(A) = N\Upsilon$ the longitude of the node, we shall have $A + NP$, which is called the *longitude of the planet on its orbit*: and, accordingly, we shall have as many such longitudes, or as many such distances as NP, as there are observations of the planet in conjunction, or opposition.

Now three such observations are sufficient to determine the two elements of the eccentricity, and place of the aphelion: for, if we have three longitudes on the orbit (V, V', V'') we have, by taking the differences of the second and first, and of the third and second, two *differences* of longitudes, and, since the planet's period is known, we can compute two portions of its mean motion, corresponding to the two noted intervals of time, between the second and first observation, and between the third and second observation. The two differences of real longitudes compared, according to the elliptical theory, with the corresponding portions of mean motion, will give us two equations for determining the eccentricity and place of the aphelion.

Thus, suppose we have three observations of conjunctions or oppositions, then we know the three corresponding longitudes of the planet on the ecliptic, and, deducting from each the longitude of the node, we know three such arcs as $N\pi$, and by the formula of reduction, three such arcs on the orbit as NP, and, lastly, by adding to each the longitude (A) of the node, set off on the orbit, we know three longitudes on the orbit, such as $A + NP$: let these be, respectively, V, V', V'', and let e be the eccentricity (supposed to be very small), ϕ the longitude of the perihelion, the place of which, suppose to be at some point (B)

between N and P: let M, M', M'', be the mean anomalies reckoned from B: then we have (see Chapter XVIII.)

$$BSP = M + 2e.\sin. 2M, \text{ nearly,}$$
$$\text{or } V - \phi = M + 2e.\sin.(V - \phi), \text{ nearly,}$$
$$\text{similarly } V' - \phi = M' + 2e.\sin.(V' - \phi),$$
$$V'' - \phi = M'' + 2e.\sin.(V'' - \phi).$$

Hence, by subtraction

$$V' - V = M' - M + 2e.\{\sin.(V' - \phi) - \sin.(V - \phi)\},$$
$$V'' - V' = M'' - M' + 2e.\{\sin.(V'' - \phi) - \sin.(V' - \phi)\},$$

or

$$(1)\ (V' - V) - (M' - M)(= a) = 2e\{\sin.(V' - \phi) - \sin.(V - \phi)\},$$
$$(2)\ (V'' - V') - (M'' - M')(= b) = 2e\{\sin.(V'' - \phi) - \sin.(V' - \phi)\}.$$

Now V, V', V'' are known (see p. 568,) and $M' - M$, $M'' - M'$ are known from the period of the planet, and the times elapsed: thus, if t be the interval between the observations of V and V',

$$\text{planet's period} : 360^0 :: t : M' - M = \frac{t}{\text{period}} \times 360^\circ.$$

Hence, since a and b are known, we have two equations for determining e and ϕ.

Divide equation (1) by equation (2), then

$$\frac{a}{b} = \frac{\sin.(V' - \phi) - \sin.(V - \phi)}{\sin.(V'' - \phi) - \sin.(V' - \phi)},$$

the numerator of this fraction

$$= \sin.(V' - \phi).\left(1 - \frac{\sin.(V - \phi)}{\sin.(V' - \phi)}\right)$$

$$= \sin.(V' - \phi).\left(1 - \frac{\sin. V\cos.\phi - \cos. V.\sin.\phi}{\sin. V'.\cos.\phi - \cos. V'.\sin.\phi}\right)$$

$$= \sin.(V' - \phi).\left(1 - \frac{\sin. V - \cos. V.\tan.\phi}{\sin. V' - \cos. V'.\tan.\phi}\right)$$

$$= \sin.(V' - \phi).\left(\frac{\sin. V' - \sin. V - \tan.\phi(\cos. V' - \cos. V)}{\sin. V' - \cos. V'.\tan.\phi}\right),$$

similarly, the denominator of the above fraction (l. 21,)

$$= -\sin.(V'-\phi)\cdot\left(\frac{\sin. V' - \sin. V'' - \tan.\phi(\cos. V' - \cos. V'')}{\sin. V' - \cos. V'.\tan.\phi}\right).$$

Hence,

$$\frac{a}{b} = \frac{\sin. V' - \sin. V - \tan.\phi.(\cos. V' - \cos. V)}{\sin. V'' - \sin. V' - \tan.\phi(\cos. V'' - \cos. V')}$$

and, accordingly,

$$\tan.\phi = \frac{a.(\sin. V'' - \sin. V') - b.(\sin. V' - \sin. V)}{a.(\cos. V'' - \cos. V') - b.(\cos. V' - \cos. V)},$$

which is an equation for determining ϕ, the longitude of the perihelion.

In order to determine the eccentricity, we have, ϕ being determined by the preceding equation,

$$e = \frac{a.\sin. 1''}{2.[\sin.(V' - \phi) - \sin.(V - \phi)]}$$

$$= \frac{\frac{1}{4} a.\sin. 1''}{\sin.\frac{1}{2}(V' - V).\cos.\left(\frac{V' + V}{2} - \phi\right)}.$$

By these means ϕ and e* are approximately determined: and if we use their approximate values, we may extend the series for $V - \phi$, &c. (see *Physical Astronomy*, p. 32,) and obtain nearer values for $(V' - V) - (M' - M)$, &c. or for a and b, and thence, by means of the equations of l. 5, nearer values of ϕ and e.

The eccentricity (e), the longitude of the perihelion (ϕ), and the axis major ($2a$), being determined, we are able to compute the radius vector (r) from the expression

$$r = \frac{a.(1 - e^2)}{1 + e.\cos.(V - \phi)},$$

---

* The eccentricity and place of the aphelion are often *mathematically* determined by the solution of a problem, of which the conditions are, three given radii vectores, and three given longitudes: but it is plain, from the preceding matter, that the first condition, (that of the *given* radii vectores,) is not easily to be obtained. The knowledge of the period, leads only to the knowledge of the *mean* distance.

and, since the place of the node, and the inclination of the orbit are determined, we are able to compute (see figure of p. 592,) the curtate distance $S\pi$, on the supposition that SP, from which it is deduced, is the radius vector in an elliptical orbit. If, therefore, in any of the processes for determining the elements, the curtate distance $S\pi$ has been supposed to be derived from SP, considered as a mean distance, or constant radius (see p. 567,) we may now, with a truer value of $S\pi$, repeat the processes and correct their results.

The elements of a planet's orbit being now obtained, we will proceed to consider by what means those elements are to be employed in forming Tables of the planets' motions; and, then, by what methods, either recorded or future geocentric observations may be applied to the correction of existing Tables. These subjects will be briefly considered in the ensuing Chapters.

CHAP. XXV.

On the Formation of Tables of the Planets.—The Variations of the Elements of their Motions.—The Processes for deducing the Heliocentric Places of Planets from Tables.

In the planetary theory, as in the solar, the described orbits are supposed to be elliptical. The same process then, which, in the latter theory, gave us the Sun's true anomaly and radius vector from the mean anomaly, will give us (changing what ought to be changed) a planet's true anomaly, whether the planet be Venus, or Saturn.

This regards the elliptical place to be found by Kepler's problem. But the Earth being, according to the doctrine of universal gravitation, disturbed by the action of the Moon and the planets, does not describe an orbit exactly elliptical. By parity of reason, neither Venus nor Saturn can move in orbits exactly elliptical. Each disturbs the other. Their places, therefore, like the Sun's place, require a small correction, or rather several small corrections due to the several planets.

But as in no case these corrections for planetary perturbation are large, so in some they are too small to be worth taking account of. Mercury and Venus are in the above predicament. Their Tables are constructed solely by means of Kepler's problem, and are, therefore, much more easily constructed than the Tables of the other planets. The longitudes of Mercury and Venus are,

accordingly, to be had very readily from their Tables. For instance, suppose it were required to find Mercury's longitude in his orbit.

	Longitude.	Aphelion.
Epoch for 1793,	$2^s\ 28^0\ 5'\ 16''$	$8^s\ 14^0\ 14'\ 17''$
Mean motion to June 3,	9 0 13 34	0 0 0 24
. . . . for 5^h	0 0 51 9	
Mean longitude	11 29 9 59	8 14 14 41
Equation of centre	— 23 39 58.5	11 29 9 50
Longitude on orbit	11 5 30 0.5	3 14 55 9 the mean anomaly.

This is a process precisely similar to that by which in pp. 489, 490, the Sun's longitude was found: and, to a certain extent, all other processes for computing the longitude of a planet, be it Mars, or Jupiter, or Saturn, must resemble it, inasmuch as Kepler's problem is, in all, the main instrument in procuring a result.

The result by Kepler's problem solely, is the planet's elliptical place: which, in the case of the Earth, Mars, Jupiter, Saturn, and the Georgium Sidus, requires a correction. We will give an instance of Mars' longitude taken from his Tables.

Required the Heliocentric Longitude and Latitude of Mars, Nov. 13, 1800, 11^h 8^m 20^s.

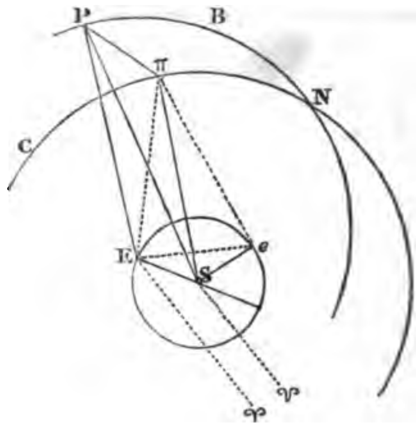
	Longitude.	*Aphelion.*	*Node.*
Epoch for 1800	7^s 22° $34'$ $21''.8$	5^s 2° $23'$ $17''$	1^s 18° $1'$ $1''$
Nov....	5 9 19 3.4	0 0 0 55.8	0 0 0 22.8
13^d....	0 6 48 46.5	0 0 0 2.4	0 0 0 1
11^h....	0 0 14 24.7		
8^m....	0 0 0 10.5		
20^s....	0 0 0 0.4		
Mean longitude	1 8 56 47.3	5 2 24 15.2	1 18 1 24.8
(*e*) Sum of equa.	0 10 13 26.9	1 8 56 47.3	1 19 10 14.2
Long. on orbit	1 19 10 14.2	8 6 32 32.1	0 1 8 49.4
Reduction....	0 0 0 − 2.2	the mean anom.	argument of lat.
Heliocen. long.	1 19 10 12		Heliocen. lat. $=0^\circ$ $2'$ $13''.4$ N.

In this process, *e*, the sum of the equations, contains, besides the equation of the centre ($= 10^\circ$ $13'$ $13''.5$), three small equations arising from the perturbations of *Venus*, the *Earth*, and *Jupiter*. The sum of these three equations is $13''.4$, which added to the equation of the centre make *e*.

The reduction − $2''.2$, applied to the longitude on the orbit, gives the heliocentric longitude, measured along the ecliptic, and from the mean equinox. If this result be corrected for the effect of nutation, (by applying the equation of the equinoxes) there will be obtained, the longitude measured from the *apparent* equinox.

In the fourth column, the *argument of latitude* is the difference of the longitude on the orbit (1^s 19° $10'$ $14''.2$), and of the longitude of the node (1^s 18° $1'$ $24''.8$). It is, in the annexed figure, *NP*: and it is properly called the argument of latitude,

because, the inclination of the orbit being given, the latitude depends upon it: for



$$1 \,.\, \text{sin. lat.} = \text{sin. } NP \,.\, \text{sin. } NP\pi^*.$$

There are no direct corrections, from the theory of *perturbation,* of the longitudes of Mercury and Venus, in the Tables of those planets. Still the Tables are not entirely constructed without the aid of such theory. If we revert to p. 598, l. 6, we shall see in the fourth column, under the head of *Aphelion,* 24″ to be added to the epoch of the aphelion, as a quantity due to the change of the aphelion's place, in the interval between January 1, 1793, and June 3, 1793.

Now such a change of place does not obtain in the elliptical theory, but arises from the disturbing forces of the system. Some, therefore, of the results of the theory of perturbation are made use of in constructing the Tables of Mercury and Venus.

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* If the inclination be taken equal to 1° 51′ 4″, we have

log. sin. 1° 51′ 4″	8.5092343
log. sin. 1 8 49.4	8.3014327
	6.8106670

which is the log. sin. of 2′ 13″⅓.

But the changes of the places of the aphelia are phenomena, or laws common to the orbits of all planets. We have another instance in the second Example. These changes are changes of *progression:* and their computation, on the principles of gravitation, was the second great proof of the truth of Newton's System, (see *Physical. Astronomy*, Chapters IX, XXII.)

In the second Example there is a small quantity to be added to the place of the node, and indicative of a change of its place in the interval between January 1, and November 13: (see the Chapters above cited).

The accounting for the *progressions* of the aphelia, and the *regressions* of the nodes (for such is the general statement of the laws of their motions), on the principle and law of gravitation, proves, to a certain extent, the truth of such law and principle. But, in determining the exact quantities (and the quantities are very minute) of such progressions and regressions, it is much better to use observations, than computations from theory. And observations are thus to be used: from those that are convenient for the purpose, find for a certain epoch the place of the node: repeat the process for another epoch: the difference of the two places is the change of the node's place in the interval between the two epochs: and the difference divided by the interval (if it be expressed in years and parts of years) will be the mean annual *regression* of the node. A like process will determine the *progression* of the aphelion.

We have now described and illustrated methods of deriving, from observations of right ascension and declination, the elements of a planet's orbit, and the variations and annual changes of those elements. The elliptical theory enables us, then, to form Tables of the planet: from which, at any epoch, its heliocentric longitude and latitude may be computed. The formula or Table of *reduction* to the ecliptic, gives the planet's longitude on the ecliptic. But in order to know at what time, and in what part of the heavens we ought to look for the planet, there is need of a method of deducing the geocentric longitude and latitude from the heliocentric. The geocentric longitude and latitude being known, the right ascension and declination of the planet may be

deduced: and, accordingly, if we use instruments placed in the meridian, we know at what time, and at what distance from the zenith, to look for the planet on the meridian. If the predicted, or computed, right ascension and declination should agree with the observed, a presumption would then arise of the Tables being right: and if, in many and various instances, the observed and computed places should be found to agree, a proof would be established of their being right.

But even now, as formerly, there are to be noted some small differences between the observed and Tabular places of the planets: differences, however, too great to be imputed solely to erroneous observation, and which must, therefore, arise, in part, from the errors of the Tables. In order to render the Tables more correct, the noted differences, just spoken of, must be used (as like differences, or errors were used in pages 511, &c.) in forming sets of equations, having indeterminate coefficients that represent the errors of the several elements of the computation. But this and the other matters, previously spoken of in this Chapter, will form the subject of the ensuing.

CHAP. XXVI.

On the Deduction of Geocentric Longitudes and Latitudes from Heliocentric.—Examples of the same: the Method of correcting the Tables of Planets.

In order to attain the objects, pointed out at the conclusion of the last Chapter, it is necessary to be possessed of a formula, or of rules for converting heliocentric longitudes and latitudes, furnished by the planetary Tables, into geocentric.

It is required to determine, from the Heliocentric, the Geocentric Longitude and Latitude of a Planet.

The heliocentric longitude of the planet, and the longitude of the Earth being known, (from the solar theory and Tables) that is, the angles formed by πS, ES, with S♈, E♈ being known, the angle $ES\pi$, *the angle of commutation*, is known.

Again, from the heliocentric latitude $\angle\, PS\pi$, and SP, given by the planetary theory, (see p. 595,) the *curtate* distance $S\pi$ may be computed, for

$$S\pi = SP \times \cos .\, PS\pi.$$

But, SE is also known by the solar theory (see p. 466,) therefore to determine $\angle\, SE\pi$, the difference of the heliocentric and geocentric longitudes, we have $\angle\, ES\pi$, SE and $S\pi$.

The angle $SE\pi$ may be thus determined:

Assume (see *Trig*. p. 28, &c.) an angle θ, such, that

$$\tan .\, \theta = r \times \frac{S\pi}{SE} = r \times \frac{SP\,.\cos .\, PS\pi}{SE}, \text{ then (see } Trig. \text{ p. 29, 30,)}$$

$$r \times \tan.\left(\frac{SE\pi - S\pi E}{2}\right) = \tan.\frac{ES\pi}{2}\tan.(\theta - 45^\circ)$$

from which formula $SE\pi - S\pi E$ may be computed, and $SE\pi + S\pi E$ being known, the separate angles $SE\pi$, $S\pi E$ may be determined.

The angle $SE\pi$, the angle of elongation, is the difference (see p. 566,) of the geocentric, and of the Sun's longitude. Hence,

geocentric long. planet = longitude of ☉ $\pm$ ∠ elongation.

The geocentric latitude may be thus determined,

$$\tan.PE\pi = \frac{P\pi}{E\pi} = \frac{S\pi}{E\pi}.\tan.PS\pi = \frac{\sin.\angle SE\pi}{\sin.\angle ES\pi}\tan.\angle PS\pi,$$

or,

$$\tan.\text{ geocentric lat.} = \frac{\sin.\ \angle\text{ elong}^{n}.}{\sin.\ \angle\text{ commut}^{n}.} \times \tan.\text{ heliocentric lat.}$$

Example.

The Heliocentric Longitude and Latitude of Jupiter *being, on July* 11, $5^h\ 48^m\ 39^s$, 1800, $6^s\ 29^\circ\ 9'\ 14''.3$, *and* $1^\circ\ 13'\ 42''$ *respectively, required the corresponding Geocentric Longitude and Latitude.*

Heliocentric long. ♃	6^s 29° 9′ 14″.3
(From Solar Tables) long. ☉	3 19 52 28 . 3
∠ $ES\pi$	3 9 16 46
∴ $\frac{1}{2}ES\pi$	1 19 38 23

θ computed from $\tan.\theta = r\ \dfrac{SP.\cos.\text{ heli}^c.\text{ lat.}}{SE}$ (p. 603, last line)

From Tables of the planet.	log. SP	.7355821
	log. cos. helio^c. lat.	9.9999001
	arith. comp. SE	9.9928989
	(log. tan. 79° 24′ 48″).........	10.7283811 (reject^g. 10)

$$\therefore \theta = 79^\circ\ 24'\ 48''$$

$\theta - 45^\circ = 34\ \ 24\ \ 48$ .... log. tan. 9.8357262

$\frac{1}{2} ES\pi\ = 49\ \ 38\ \ 23$ .... log. tan. 10.0706464

19.9063726

therefore, rejecting 10, 9.9063726 = log. tan. $\frac{SE\pi - S\pi E}{2}$;

$$\therefore \frac{SE\pi - S\pi E}{2} = 38^\circ\ 52'\ 16''.$$

$$\text{But } \frac{SE\pi + S\pi E}{2} = 49\ \ 38\ \ 23;$$

$$\therefore SE\pi = 88\ \ 30\ \ 39 = 2^s\ 28^\circ\ 30'\ 39''$$

But (p. 604, l. 17,) long. ☉ .... $= 3\ \ 19\ \ 52\ \ 28.3$

$\therefore$ (p. 604, l. 8,) geocen. longitude $= 6\ \ 18\ \ 23\ \ \ 7.3$

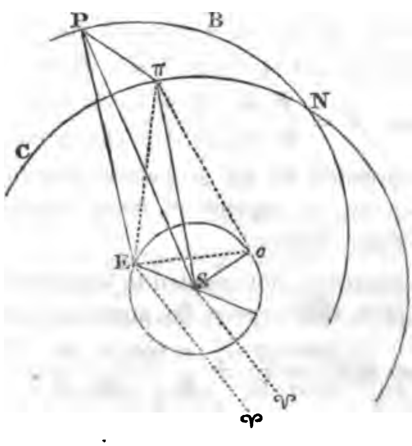
To find the Latitude (from the expression, p. 604, l. 12,)

log. sin. ∠ elon. ($SE\pi = 88^\circ\ 30'\ 39''$) 9.99985

ar. comp. sin. ∠ com. ($ES\pi = 99\ \ 16\ \ 46$) 0.00573

log. tan. heliocentric lat. (lat. $= 1\ \ 13\ \ 42$) 8.33126

$\therefore$ log. tan. geocentric lat. = 8.33684 (reject. 10)



$\therefore$ geocentric latitude $= 1^\circ\ 14'\ 39''$.

Or, the computation may be effected by the aid of the following formula,

L denotes the geocentric longitude,
P the heliocentric,
λ the heliocentric latitude,
E the angle of elongation,
π the angle of parallax,
r the radius vector SP,
R the radius vector SE,

$$\text{then, } \pi = P - L,$$
$$E = L - \odot,$$

$$\text{then, } \sin. E = \frac{r \,.\cos.\lambda}{R}, \quad \sin. \pi = \frac{r \,.\cos.\lambda}{R} \sin.(P - L)$$

$$= \frac{r \,.\cos.\lambda}{R} (\sin. P \cos. L - \cos. P \sin. L),$$

but also $\sin. E = \sin.(L - \odot) = \sin. L \cos. \odot - \cos. L \,.\sin. \odot$.

Equate these two values of $\sin. E$, and there results

$$r \cos.\lambda \sin. P \cos. L - r \cos.\lambda \cos. P \,.\sin. L$$
$$= R \sin. L \cos. \odot - R \cos. L \sin. \odot,$$

$$\text{and thence, } (R \cos. \odot + r \cos.\lambda \cos. P) \sin. L$$
$$= (R \sin. \odot + r \cos.\lambda \sin. P) \cos. L,$$

$$\text{and } \tan. L = \frac{R \sin. \odot + r \cos.\lambda \sin. P}{R \cos. \odot + r \cos.\lambda \cos. P},$$

which is an expression for the geocentric longitude in terms of quantities, given by, or capable of being computed from, the planetary and Solar Tables.

But this expression is not adapted to logarithmic computation. In order to adapt it, thus express the numerator and denominator,

$$\text{the numerator} = \left(\frac{\sin. \odot}{\cos. \odot} + \frac{r}{R} \frac{\cos.\lambda \,.\sin. P}{\cos. \odot}\right) R \,.\cos. \odot,$$

$$\text{the denominator} = \left(1 + \frac{r}{R} \cos.\lambda \frac{\cos. P}{\cos. \odot}\right) R \,.\cos. \odot.$$

Let $\frac{r}{R} \frac{\cos. \lambda . \sin. P}{\cos. \odot} = \tan. x = \frac{\sin. x}{\cos. x}$;

$$\therefore \frac{r}{R} \cos. \lambda \frac{\cos. P}{\cos. \odot} = \tan. x . \frac{\cos. P}{\sin. P} = \frac{\sin. x}{\cos. x} \frac{\cos. P}{\sin. P};$$

$$\therefore \tan. L = \frac{\sin. \odot \cos. x + \cos. \odot \sin. x}{\cos. x \sin. P + \sin. x \cos. P} . \frac{\sin. P}{\cos. \odot}$$

$$= \frac{\sin. (\odot + x) . \sin. P}{\sin. (P + x) \cos. \odot}.$$

We will apply this formula to the preceding instance, using the same numbers for r, R, &c.

First Operation. x computed.

log. r	.7355821
arith. comp. R	9.9928989
log. cos. λ	9.9999001
log. sin. P	9.6876697
log. sec. $\odot$	11.4685705
(rejecting 30)......	10.8846213

Second Operation. L computed.

$x =$	82°	34′	8″, nearly,		
$\odot =$	109	52	28.3	...... log. sec ..	11.4685705
	192	26	36.3	...... sin.	9.3333974
$P =$	209	9	14.3	...... sin.	9.6876697
$P + x =$	291	43	22.3	..... arith. comp.	10.0319914
				(rejecting 30)	10.5216290

Now 10.5216290 is the log. tangent of 18° 23′ 8″; and of 6^s 18° 23′ 8″, which latter quantity is evidently the true one in the present instance; therefore

$$L = 6^s \ 18^0 \ 23' \ 8'',$$

nearly the same result as before.

By these means, then, the geocentric longitudes and latitudes may be computed from the heliocentric, such as the planetary

Tables afford: the next step is to compare the computed geocentric longitudes and latitudes, with the observed, and from such comparison to derive the corrections of the Tables.

Let C be the computed longitude,

L the observed,

O the epoch of the Tables,

m the mean motion,

t the time elapsed since the epoch,

E the equation of the centre, corresponding to a mean anomaly A, then

$$C = O + mt + E;$$
$$\therefore dC = dO + dm.t + dE,$$

but, as in p. 511, E varies both from the variation of the eccentricity, and from the variation $(d\pi)$ of the longitude of the perihelion;

$$\therefore dE = \frac{dE}{de}de + \frac{dE}{d\pi}.d\pi;$$

$$\therefore dC = dO + t.dm + \frac{dE}{de}de + \frac{dE}{d\pi}d\pi.$$

Now dC the variation or error of the computed longitude, may be considered as the difference between the computed and the observed longitude: every comparison, therefore, of the two kinds of longitudes affords an equation like the one of l. 17, and four such equations will be sufficient for the elimination and determination of the errors of the eccentricity, epoch, &c.: but, instead of confining ourselves to a barely sufficient number of equations, it will be expedient to make use of a great number, and by their combination to obtain *mean* results, (see p. 511, &c.)

In the above method of correcting the elements of a planet's orbit, the orbit is supposed to be strictly elliptical: but it must deviate from such form, by the effect of *perturbation*. In order to estimate the parts of such effect, or, in other words, the partial effects of the several planets, it is necessary to assume a series of terms with indeterminate coefficients, and *arguments* depending on the angular distances of the disturbed and disturbing planets (see pp. 498, 519, &c.)

In the next Chapter we will turn our attention to the *synodical* revolutions of planets, and to the means of ascertaining, after what intervals of time, we may expect those rare phenomena of the *transits of Venus and Mercury, over the Sun's disk*: which indeed can only happen at peculiar conjunctions: such that the planet, when it has the same longitude as the Sun, shall be near to the node of its orbit: so near that its geocentric latitude shall either be less than the Sun's semi-diameter, or, in the extreme case, shall scarcely exceed it.

CHAP. XXVII.

On the Synodical Revolutions of Planets.—On the Method of computing the Returns of Planets to the same Point of their Orbit.—Tables of the Elements of the Orbits of the Planets.

In the preceding pages, the conjunctions and oppositions of planets have been spoken of, but hitherto no method has been given of computing the times between successive conjunctions, or successive oppositions.

In the method also of determining the mean motions of planets (see p. 375,) directions were given for observing the planet in the same, or nearly the same point of its orbit, but no process or formula given, of computing the time at which such event would take place.

Towards these points then our attention will be now directed: we shall find that they depend on the same principles, and require, in the business of computation, nearly the same formulæ.

The time between conjunction and conjunction, or between opposition and opposition, is denominated, a *Synodical* period. Suppose we assume, at a given instant, the *Sun, Mercury* and the *Earth* to be in the same right line: then, after any elapsed time (a day for instance,) *Mercury* will have described an angle m, and the Earth an angle M, round the Sun. Now, m is greater than M (p. 581,) therefore at the end of a day, the separation of *Mercury* from the Earth (measuring the separation by an angle formed by two lines drawn from *Mercury* and the Earth to the Sun) will be $m - M$: at the end of two days, (the mean daily motions continuing the same,) the angle of separation will be $2(m-M)$; at the end of three days, $3(m-M)$; at the end of s days, $s(m-M)$.

When the angle of separation then amounts to 360°, that is, when $s\,(m-M) = 360^0$, the *Sun*, *Mercury* and the *Earth* must be again in the same right line, and, in that case,

$$s = \frac{360^0}{m - M} \,\ldots\ldots\ldots\, (1).$$

In which expression s denotes the time of a synodical revolution, m and M being taken to denote the mean daily motions, but, as it is plain, m and M may denote any portions, however small, of the mean motions, and s will still be the corresponding time, however reckoned, whether by days, or hours, or seconds.

Let P and p denote the sidereal periods of the Earth and the planet; then, since $1^d : M^\circ :: P : 360^\circ$,

and $1 : m :: p : 360$,

$$M = \frac{360}{P} \text{ and } m = \frac{360^0}{p}; \therefore \text{ substituting}$$

$$s = \frac{360^\circ}{360^0\left(\frac{1}{p} - \frac{1}{P}\right)} = \frac{Pp}{P - p} \,\ldots\ldots\, (2),$$

and from either of these expressions, (1), (2), the synodical revolution of the planet may be computed.

We may differently express the synodic period; thus, if 1 be the Earth's mean distance, and r be the planet's mean distance, we have, by Kepler's law

$$P : p :: 1 : r^{\frac{3}{2}}; \quad \therefore \frac{P}{p} = r^{-\frac{3}{2}},$$

$$\text{and } s = \frac{P}{r^{-\frac{3}{2}} - 1} = \frac{365^d.256384}{r^{-\frac{3}{2}} - 1},$$

$$\text{or } s = \frac{365^d.256384}{1 - r^{-\frac{3}{2}}}.$$

The first expression belonging to inferior, the second to superior, planets: and from these or the former expressions of l. 4, 14, the synodical periods may be computed.

For instance, in the case of Mercury, $p = 87^d.969$;

$$\therefore s = \frac{365.256 \times 87.969}{277.287} = 115^d\ 21^h, \text{ nearly.}$$

In the case of the Moon, $m = 13^\circ.1763$, and M (the Earth's mean daily motion) $= 59'\ 8''.3$;

$$\therefore s = \frac{360^0}{m - M} = \frac{360}{12.1906} = 29^d\ 12^h, \text{ nearly,}$$

and the following Table may be formed by substituting in the expression of p. 611, l. 20, the respective values of r,

Planets.	Values of r.	Values of s.
☿	0.3871	$115^d.877$
♀	.7233324	583.920
♂	1.5236927	779.936
?	2.6	479.672
♃	5.202792	398.867
♄	9.5387705	378.090
♅	19.183305	369.656

It is upon this synodical revolution of the Moon, that its phases depend.

$$\text{Since } s = \frac{Pp}{P - p},\quad p = \frac{sP}{s + P};$$

therefore, from the Earth's period (P) known, and the synodic (s) observed, we can determine the periodic time (P) of the planet. This method will not be accurate, if only one synodic period be observed, since that will be affected with all the deviations of the planet's real from its mean motion. To obviate this, the return of the planet to a conjunction nearly in the same part of its orbit, at which a previous one was observed, must be noted; the interval of time divided by the number of synodical revolutions will give the time of a *mean* synodical period. For, in this case, there will take place, very nearly, a mutual compensation of the inequalities arising from the elliptical form of the planet's orbit.

By the above method, the sidereal periods of *Mercury* and *Venus* may be accurately determined.

One reason already assigned for the necessity of knowing those particular conjunctions at which the planet will be nearly in the same part of its orbit, is the mutual compensation that will probably take place of the inequalities (relatively to mean motion) arising from the planet's elliptical motion. Another reason is, that, on such conjunctions, depend observations of great importance in Astronomy; namely, the transits of *Venus* and *Mercury* over the Sun's disk. This will be manifest, if we consider that *Venus*, in order to be seen on the Sun's disk, must not only be in conjunction, but near the node of her orbit: at the next conjunction, after one synodical revolution, she cannot be near her node, and can only be again near, (supposing the motion of the nodes not to be considerable,) when she returns to the same part of her orbit as at the time of the first observation. The importance of knowing these particular conjunctions then is manifest; and we shall be possessed of the means of knowing them, by modifying the formulæ of p. 611, by which the times between successive conjunctions are computed.

The time (t) of a synodical revolution $= \dfrac{Pp}{P-p}$.

At the several times $\dfrac{2Pp}{P-p}$, $\dfrac{3Pp}{P-p}$, $\dfrac{4Pp}{P-p}$ and $\dfrac{nPp}{P-p}$, therefore, the planet is still in conjunction: it will, therefore, be for the first time in conjunction, and, besides, the Earth and planet will be in the same part of their orbits, when $\dfrac{nPp}{P-p} = P$, or when $n = \dfrac{P-p}{p}$. Now, n must be a whole number, but $\dfrac{P-p}{p}$ may not be a whole number; in such a case, therefore, after one revolution of the Earth, the planet cannot be in conjunction, or, if viewed, about that time, in conjunction, it cannot be in the same part of its orbit.

But, the conditions of the planet in conjunction, and in the same part of its orbit, although they cannot take place in 1 or 2 or 3 years ($P = 1$ year), yet they may take place in m years: and if such conditions take place, then must

$$\frac{nPp}{P-p} = mP,$$

$$\text{and } \frac{m}{n} = \frac{p}{P-p},$$

and the question now is purely a mathematical one, namely, that of determining two integer numbers m and n, such, that

$$\frac{m}{n} = \frac{p}{P-p}.$$

Thus, in the case of *Mercury*, whose tropical revolution is $87^d\ 23^h\ 14^m\ 32^s$ (= 87.968),

$$\frac{m}{n} = \frac{87.968}{365.256 - 87.968} = \frac{87.968}{277.288};$$

consequently, in 87968 periods of the Earth, in which will happen 277288 synodic revolutions, *Mercury* will be observed in conjunction, and in the same part of his orbit. But, this result is, on account of the length of the period, practically useless: we must find then the lowest terms of the fraction $\frac{87.968}{277.288}$, and if the lowest terms still give periods too large, we must investigate some integer numbers, which are very nearly in the ratio of 87968 to 277288; so that we may know the periods at which the conditions required will *nearly* take place.

$$\text{Now, } \frac{87968}{277288} = \frac{1}{\frac{277288}{87968}} = \frac{1}{3 + \frac{13384}{87968}},$$

$$= \cfrac{1}{3 + \cfrac{1}{6 + \cfrac{1}{1 + \cfrac{5726}{7664}}}}$$

and, by continuing the operation, there is at last obtained a remainder equal nothing, the greatest common measure being 8, and the fraction in its lowest terms $\frac{10996}{34661}$ *, which result, for obvious reasons, is of no practical use: we must therefore find two near integer numbers; and this we are enabled to do by the preceding operation, which, as we take more and more terms of the continued fraction, affords fractions alternately less and greater than the proposed $\left(\frac{87968}{277288}\right)$ but, continually, approximating, nearer and nearer, to its true value. Thus, the first approximation is $\frac{1}{3}$: or, in one year, in which happen 3 synodical periods, the planet will not be very distant from conjunction, nor from those parts of its orbit in which it was first observed. Again, the second approximation is $\cfrac{1}{3+\cfrac{1}{6}} = \frac{6}{19}$, or in 6 years, in which happen 19 synodical revolutions, the planet will be less distant than it was before, from conjunction, and from those parts of its orbit in which it was in the former instance. The third approximation is $\cfrac{1}{3+\cfrac{1}{6+1}} = \frac{7}{22}$, or, in 7 years, in which happen 22 synodical revolutions, the planet will be nearer to conjunction than it was at either of the two preceding points of time, and so on. This follows from the very nature of the process, by which the successive approximations are formed from the continued fraction (see Euler's *Algebra*, tom. II, p. 410, Ed. 1774); but it may be useful to exemplify its truth by means of the instance

* The operation in finding the *continued* fraction terminates, and gives a greatest common measure, because, since great accuracy is not requisite, we took $\frac{87968}{277288}$ to represent, which it does nearly, but not exactly, the ratio of the mean motions of *Mercury* and the Earth. If we had taken a fraction more exact to the true value, then the operation would not have happened to terminate.

before us. Thus, at the end of 1 year, since the diurnal tropical motion of *Mercury* is $4° \; 5' \; 32''.5 = 4°.092$, nearly, the angle described by that planet is

$$365.25 \times 4°.092 = 1494°.6, \text{ nearly,}$$

$= 4 \times 360° + 54°.6$, and consequently, *Mercury* at the end of 1 year, is elongated (reckoning from the Sun) from the line joining the Sun and Earth, and beyond that line, by an angle $= 54°.6$; again, at the end of 6 years, the angle described by the planet is equal to

$$(4 \times 360° + 54°.6) \times 6 = (\text{rejecting 24 circumferences})\ 327°.6;$$

or at the end of 6 years, *Mercury* is elongated from the line joining the Earth and Sun, by $327°.6$, or, not *up* to that line, by an angle $= 32°.4$.

At the end of 7 years, the angle described by *Mercury* is $(4 \times 360 + 54°.6) \times 7 =$ (rejecting 29 circumferences) $22°.2$: or *Mercury* is then (observing the analogy of the last expression, l. 12,) *beyond* the line joining the Earth and Sun, by that angle. At the end of 13 years, *Mercury*, (rejecting 54 circumferences) is separated from the line joining the Earth and Sun, and not *up* to that line, by an angle $= 10°.2$.

The series of fractions, formed as those in p. 614, were formed, is

$$\frac{1}{3}, \; \frac{6}{19}, \; \frac{7}{22}, \; \frac{13}{41}, \; \frac{33}{104}, \; \frac{46}{145}, \; \&c.$$

The denominators denote the number of synodical revolutions, corresponding to the number of years denoted by the numerators: the number of *periods* of the planet must evidently be

$$3 + 1, \quad 6 + 19, \quad 7 + 22, \quad 13 + 41, \quad \&c.$$

that is, 4, 25, 29, 54, &c.

and therefore the series of fractions, in which the denominators are the number of periods of *Mercury*, will be

$$\frac{1}{4}, \; \frac{6}{25}, \; \frac{7}{29}, \; \frac{13}{54}, \; \&c.$$

We may, on like grounds, and by like computations, determine the probable epochs, on which we ought to look out for the transits of Venus over the Sun's disk: which are phenomena of more practical importance than the transits of Mercury.

$$\text{Thus, if Venus's period } (p) = 224^d.7008240,$$
$$\text{the Earth's } (P) = 365.2563835,$$

the synodical period, or s, $= \dfrac{Pp}{P-p}$, $= 583^d.92$, nearly*, consequently in one synodical period, the Earth describes an angle equal to

$$360^0 \times \frac{583^d.92}{365.25}, \text{ or } 575^0.51, \text{ nearly,}$$

consequently, in n synodical periods, the Earth describes an angle equal to

$$575^0.51 \times n,$$

and when $575^0.51 \times n$, shall first become a multiple of 360^0, then there will first happen a conjunction of the Earth and Venus, in the same line from which they originally departed. If, therefore, Venus in this original position, was so near to the node of her orbit, that a *transit* took place, a transit will take place when

$$575^0.51 \times n = 360^0 \times m,$$

and we must now find, as before (see p. 614,) the integer values of n and m from the equation

$$\frac{m}{n} = \frac{57551}{36000}.$$

The series of quotients found as before in p. 614, are

1, 1, 1, 2, 28, 1, 81,

and the series of fractions

---

* Log. P	2.5625977
log. p	2.3516046
	4.9142023
log. $(P-p)$	2.1478477
(log. 583.92)	2.7663545

$$\frac{1}{1},\ \frac{2}{1},\ \frac{3}{2},\ \frac{8}{5},\ \frac{227}{142},\ \frac{235}{147},\ \&c.$$

from which series we are able to tell after what number of synodical periods Venus and the Earth will be nearly in the same parts of their orbits. Thus, taking the fourth fraction $\frac{8}{5}$, after 5 synodical periods, 8 circumferences will be nearly described, and on trial we find $575^{\circ}.51 \times 5 = 2877^{\circ}.55 = 360^{\circ} \times 8 - 2^{\circ}.45$, again, taking the next fraction, viz. $\frac{227}{142}$, we infer that, after 142 synodical periods, 227 circumferences will be nearly described; and more nearly described than the former 8 were in 5 synodical periods: or, which is the same thing, 142 synodical periods are nearly equal to 227 years: on trial we find

$$575^{\circ}.51 \times 142 = 81722^{\circ}.42 = 360^{\circ} \times 227 + 2^{\circ}.42.$$

Again,

$$575^{\circ}.51 \times 147 = 84599^{\circ}.97 = 360^{\circ} \times 235 - 0^{\circ}.03.$$

Hence, 235 years after a transit of Venus we may confidently expect another; and also after 235 + 8, or 243 years. In these computations, the alteration in the place of the node, that will happen in the interval of the transits, is not taken account of.

But, if we were guided merely by the preceding mathematical results, we should be in danger of missing some transits: for those results are founded on the probability of a transit's happening when Venus and the Earth are nearly in the same parts of their orbits, as they were at the time of a former transit. A transit, however, may happen when the planets are in parts of their orbits diametrically opposite, or, in other words, a transit may happen should there happen to be a conjunction when Venus is, or nearly, in the node of her orbit, opposite to that in which a transit has already happened. In order to find the probable periods at which the transits in the *opposite* node may happen, we must, instead of the equation of p. 617, write this

$$575^{\circ}.51 \times n = 180^{\circ} \times (2s - 1),$$

since, it is plain, a transit must happen, whenever, after n synodi-

cal periods, the angle described by the Earth shall be either 180°, or, a multiple of 180°. Form then a series of fractions, as before in p. 614, by dividing 57551 by 18000: which, since the successive quotients are

$$3, \quad 5, \quad 14, \quad 2, \quad 40,$$

will be

$$\frac{3}{1}, \quad \frac{16}{5}, \quad \frac{227}{71}, \quad \frac{470}{147}, \text{ \&c.}$$

and consequently, beginning with the third, in 71 synodical periods, 227 angles of 180° are described by the Earth: and on trial we find

$$71 \times 575^{\circ}.51 = 40861^{\circ}.21 = 180^{\circ} \times 227 + 1^{\circ}.21,$$

so that after 71 synodical periods the Earth has described a little more than 227 half circumferences, and, consequently, must be very nearly in the line drawn from the Sun, through the opposite node of Venus's orbit.

Since the Earth describes 227 times 180°, in 113 years and an half, it follows, if a transit happens at the beginning of 8 years, and not at the end, or, happening at the end of 8 does not (from the increase of Venus's latitude) happen at the end of 16 years, that the next period for expecting a transit will be 113 years, and that, agreeably to what has been before said, we ought to examine, or compute the latitudes of Venus at the periods 113 ∓ 8, that is, 105 and 121 years, since transits may happen at these periods.

M. Delambre has calculated the transits of Venus, over the Sun's disk, for 2000 years, some of which are subjoined.

Years.	Months.	Mean time of Conjunction.			Node.
1631	Dec. 6, ...	17^h	28^m	49^s	☊
1639	Dec. 4, ...	6	9	40	☊
1761	June 5, ...	17	44	34	☋
1769	July 3, ...	10	7	54	☋
1874	Dec. 8, ...	16	17	44	☊
1882	Dec. 6, ...	4	25	44	☊
2004	June 7, ...	21	0	4	☊

We now subjoin Tables of the elements of the orbits of planets, principally taken from Laplace, and reduced from the new French measures which he has adopted.

*Sidereal Periods of the Planets**.

Mercury	$87^d.969258$
Venus	224.700824
The Earth	365.256384
Mars	686.079619
Vesta	1335.205
Juno	1590.998
Ceres	1681.539
Pallas	1681.709
Jupiter	4332.596308
Saturn	10758.969840
The Georgian Planet	30688.712687

Movements in 100 *Julian Years of* $365^d.25$.

Mercury	415^c	2^s	14°	$4'$	$20''$
Venus	162	6	19	13	0
The Earth	100	0	0	45	45
Mars	53	2	1	42	10
Jupiter	8	5	6	17	33
Saturn	3	4	23	31	36
The Georgian Planet	1	2	9	51	20

* The tropical periods may be deduced from the sidereal, by deducting the times which the several planets require, respectively, for the description of an arc of longitude equal to the precession.

Mean Distances, or Semi-Axes of the Orbits.

Mercury	0.387098
Venus	0.723332
The Earth	1.000000 *
Mars	1.523694
Vesta	2.373000
Juno	2.667163
Ceres	2.767406
Pallas	2.767592
Jupiter	5.202791
Saturn	9.538770
The Georgian Planet	19.183305

* The Earth's distance is here assumed as a standard and = 1 : its distance from the Sun, in statute miles, is reckoned to be 93, 726, 900.

M. Bode of Berlin discovered the following curious law of the relative distances of the Planets:

Mercury	4	$= 4$
Venus	7	$= 4 + 3.2^0$
Earth	10	$= 4 + 3.2$
Mars	16	$= 4 + 3.2^2$
Ceres	28	$= 4 + 3.2^3$
Jupiter	52	$= 4 + 3.2^4$
Saturn	100	$= 4 + 3.2^5$
The Geórgian planet	196	$= 4 + 3.2^6$.

The distances of the next planets (should there be any) according to this law would be

$$388 = 4 + 3.2^7.$$
$$722 = 4 + 3.2^8$$
&c. =

We need scarcely mention that this law is empirical. It is not easy to see what led to the conjecturing of it.

Ratio of the Eccentricities (*a e*) *to the Semi-Axes at the beginning of* 1801: *with the Secular Variation of the Ratio,* (see p. 464). *The sign* — *indicates a diminution.*

	Ratio of the Eccentricity.	Secular Variation.
Mercury	0.205514	0.000003867
Venus	0.006853	0.000062711
The Earth	0.016853	0.000041632
Mars	0.093134	0.000090176
Juno	0.254944	not ascertained
Vesta	0.093220	not ascertained
Ceres	0.078349	not ascertained
Pallas	0.245384	not ascertained
Jupiter	0.048178	0.000159350
Saturn	0.056168	0.000312402
The Georgian Planet	0.046670	0.000025072

Mean Longitudes at the beginning of 1801; *reckoned from the Mean Equinox, at the Epoch of the Mean Noon of January* 1, 1801, *Greenwich.*

Mercury	166°	0′	48″.2
Venus	11	33	16.1
The Earth	100	39	10
Mars	64	22	57.5
Vesta	267	31	49
Juno	290	37	16
Ceres	264	51	34
Pallas	252	43	32
Jupiter	112	15	7
Saturn	135	21	32
The Georgian Planet	177	47	38

Mean Longitudes of the Perihelia, for the same Epoch as the above, with the Sidereal and Secular Variations.

	Long. Perihelion.	Sec. Var.
Mercury	74° 21′ 46″	9′ 43″.5
Venus	128 37 0.8	− 4 28
The Earth	99 30 5	19 39
Mars	332 24 24	26 22
Vesta	249 43 0	not ascertained.
Juno	53 18 41	not ascertained.
Ceres	146 39 39	not ascertained.
Pallas	121 14 1	not ascertained.
Jupiter	11 8 35	11 4
Saturn	89 8 58	32 17
The Georgian Planet	167 21 42	4

Inclinations of Orbits to the Ecliptic at the beginning of 1801, *with the Secular Variations of the Inclinations to the true Ecliptic.*

	Inclination.	Secular Variation.
Mercury	7° 0′ 1″	19″.8
Venus	3 23 32	− 4.5
The Earth	0 0 0	
Mars	1 51 3.6	− 1.5
Vesta	7 8 46	not ascertained.
Juno	13 3 28	not ascertained.
Ceres	10 37 34	not ascertained.
Pallas	34 37 7.6	not ascertained.
Jupiter	1 18 51	− 23
Saturn	2 29 34.8	− 15.5
The Georgian Planet	0 46 26	3.7

Longitudes of the Ascending Nodes on the Ecliptic, at the beginning of 1801, *with the Sidereal and Secular Motions.*

	Longitude of ☊.	Secular and Sidereal Variation.
Mercury	45° 57′ 31″	13′ 2″
Venus	74 52 38.6	− 31 10
The Earth	0 0 0	
Mars	48 14 38	− 38 48
Juno	103 0 6	not ascertained.
Vesta	171 6 37	not ascertained.
Ceres	80 55 2	not ascertained.
Pallas	172 32 35	not ascertained.
Jupiter	98 25 34	− 26 17
Saturn	111 55 46	− 37 54
The Georgian Planet	72 51 14	− 59 57

The use of the secular variation of the eccentricity has been already explained (see p. 464.) The secular variations of the longitudes of the perihelia and the nodes are *sidereal*: consequently, they cannot be immediately applied to find a longitude at an epoch, different from that of the Tables; but, in the first place, the precession of the equinoxes must be added, and then the result will be a variation relatively to the equinoxes, or tropics. Thus, the secular sidereal variation of the longitude of the perihelion of *Mercury's* orbit is stated to be 9′ 43″.5; therefore, if we assume the annual precession to be 50″.1, and consequently the secular to be 1° 23′ 30″, the secular variation, with *regard to the equinoxes*, is 1° 33′ 13″.5; and, accordingly, the longitude of the perihelion of *Mercury's* orbit, for the beginning of 1901, will be

$$74°\ 21'\ 46'' + 1°\ 33'\ 13''.5 = 75°\ 54'\ 59''.5.$$

For the beginning of 1821, it will be

$$74°\ 21'\ 46'' + 0°\ 18'\ 38''.7 = 74°\ 40'\ 24''.7.$$

Again, the *sidereal* secular variation of the perihelion of *Venus* is stated to be − 4′ 28″ (− indicating the motion of the perihelion

to be contrary to the order of the signs); therefore the variation with regard to the equinoxes, is

$$1^\circ\, 23'\, 30'' - 4'\, 28'' = 1^\circ\, 19'\, 2'';$$

and accordingly the longitude of the perihelion for the beginning of 1811, is

$$128^\circ\, 37'\, 0''.8 + 0^\circ\, 7'\, 54''.5 = 128^\circ\, 44'\, 55''.3;$$

and for the beginning of 1781,

$$128^\circ\, 37'\, 0''.8 - 0^\circ\, 15'\, 49'' = 128^\circ\, 21'\, 11''.8.$$

It is easy to see that, both for the nodes and perihelia, a column of the tropical secular variations might be immediately formed from the sidereal by the simple addition of $1^\circ\, 23'\, 30''$. The motions of the aphelia and nodes in Lalande's (vol. I. p. 117, &c.) and Mr. Vince's Tables, (vol. III. p. 17, &c.) are motions relative to the equinoxes.

CHAP. XXVIII.

On the Satellites of the Planets.—On Saturn's Ring.

THE planet *Jupiter* is always seen accompanied by four small stars, which are denominated *Satellites*, and sometimes, *Secondary* planets, *Jupiter* being called the primary.

The satellites of *Jupiter* were discovered in 1610, by Galileo: they are discernible by the aid of moderate telescopes, and are of some use in Practical Astronomy. *Saturn* also, and the *Georgian Planet*, are accompanied by satellites, not however, to be seen except through excellent telescopes, and of no practical use to the observer. The number of *Saturn's* satellites is seven, and of the *Georgian's*, six.

The satellites are to their primary planet, what the Moon is with respect to the Earth: they revolve round him, cast a shadow on his disk, and disappear on entering his shadow: phenomena perfectly analogous to solar and lunar eclipses, and which render it probable that the primary and their secondary planets are opaque bodies illuminated by the Sun.

That the satellites when they disappear, are eclipsed by passing into the shadow of their primary, is proved by this circumstance: that the same satellite disappears at different distances from the body of the primary, according to the relative positions of the primary, the Sun, and the Earth, but always towards those parts, and on that side of the disk, where the shadow of the primary caused by the Sun ought, by computation, to be. When the planet is near opposition the eclipses happen close to his disk.

There is an additional confirmation of this fact. The third and the fourth of *Jupiter's* satellites disappear and again appear on the same side of the disk; and the durations of the eclipses are found to correspond exactly to the computed times of passing through the shadow.

The motions of *Jupiter's* satellites are according to the order of the signs. The satellites are observed moving sometimes

towards the east, and at other times towards the west: but when they move in this latter direction they are never eclipsed; when the eclipses happen, the satellite is always moving eastward; when the transits over the disk happen, the satellite is always moving westward: the motion therefore towards the east, or, according to the order of the signs, must be the true motion.

By the same proof it is ascertained, that the satellites of *Saturn* perform their motions, round their primary, according to the order of the signs. But the satellites of the *Georgian Planet* may be thought to form an exception; at least, the direction of their motions is ambiguous; for, motions performed in orbits perpendicular to the ecliptic (and such, nearly, are the orbits of the satellites of the *Georgian*) cannot be said to be either direct or retrograde.

The mean motions and periodic times of the satellites are determined by means of their eclipses, and, most accurately, by those eclipses that happen near to opposition.

The middle point of time between the satellite entering and emerging from the shadow of the primary, is the time when the satellite is in the direction, or nearly so, of a line joining the centres of the Sun and the primary. If the latter continued stationary, then the interval between this and the succeeding central eclipse would be the periodic time of the satellite. But, the primary planet moving in its orbit, the interval between two successive eclipses is a *synodic period* (see p. 610.) This synodic period, however, being observed, and the period of the primary being known, the sidereal period of the satellite may be computed*. Instead of two successive eclipses, two, separated from each other by a large interval, and happening when the Earth, satellite, and primary, are in the same position (in the direction of the same right line, for instance,) are chosen, and then the interval of time divided by the number of sidereal periods, will give, to greater accuracy, the mean time of one revolution.

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* Since $\tau = \dfrac{Pp}{P-p}$, $p = \dfrac{P\tau}{P+\tau}$.

The mean motions of the satellites do not differ considerably from their true motions. Hence, the forms of their orbits, must be nearly circular. The orbit, however, of the third satellite of *Jupiter* has a small eccentricity: that of the fourth, a larger.

The distances of the satellites from their primary are ascertained by measuring those distances, by means of a *Micrometer*, at the times of the greatest elongations.

The distance of one satellite being determined, the distances of others, whose periodic times should be known, might be determined by means of Kepler's law, which states the squares of the periodic times to vary as the cubes of the mean distances.

In order to obtain such results, we suppose Kepler's law to be true. But we may adopt a contrary procedure, and, by ascertaining the periodic times and distances of all the satellites according to the preceding methods, determine the above-mentioned law of Kepler to be true. See *Principia Phil. Natur'*. lib. 3^{tius} p. 7, &c. Ed. *La Seur*, &c.

The eclipses of *Jupiter's* satellites are used in determining the longitudes of places, and, on account of this their practical usefulness, have been studied with the greatest attention. Thence has resulted the curious and important discovery of the *Successive Propagation of Light*, which is the basis of the theory of aberration (see pp. 254, &c.) The phenomenon that led to the discovery of the propagation of light was, that an eclipse of a satellite did not always happen according to the computed time, but later, in proportion as *Jupiter* was farther from the Earth. If, for instance, an eclipse happened, *Jupiter* being in opposition, exactly according to the computed time, then about six months afterwards, when the Earth was more distant from *Jupiter* by a space nearly equal to the diameter of its orbit, an eclipse would happen about 16 minutes later than the computed time. And by similar observations it appeared, that the *retardation* of the time of the eclipse was proportional to the increase of the Earth's distance from *Jupiter*. This fact, the connexion of the retarded eclipse with the Earth's increased distance from *Jupiter*, was first noted by Roemer, a Danish Astronomer, in 1674: who suggested as an hypothesis, and as an adequate cause of the retarda-

tion, the successive propagation of light*. Subsequent observations accord so well with this hypothesis, that it is impossible to doubt of its truth: and it receives an additional, although an indirect, confirmation from Bradley's Theory of Aberration which is founded thereon.

The following Table, exhibits the mean distances and sidereal revolutions of the satellites of *Jupiter, Saturn,* and the *Georgium Sidus.*

Mean Distances, (the radius of the planet being = 1.)		According to Laplace, Sidereal Revolutions.	According to Delambre.
Jupiter.		Day.	d h m s
1st. Satellite ..	5.81296	1.7691378	1 18 28 35.94537
2	9.24868	3.5511810	3 13 17 55.73010
3	14.75240	7.1545528	7 3 59 35.82511
4	25.94686	16.6887697	16 18 5 7.02098
Saturn.			d h m s
1st. Satellite ..	3.080	0.94271	0 22 37 32.9
2..........	3.952	1.37024	1 8 53 8.9
3..........	4.893	1.88780	1 21 18 26.2
4..........	6.268	2.73948	2 17 44 51.2
5..........	8.754	4.51749	4 12 25 11.1
6	20.295	15.94530	15 22 41 13.1
7..........	59.154	79.32960	79 7 53 42.8
Georgium Sidus.			d h m s
1st Satellite ...	13.120	5.8926	5 21 21 0
2..........	17.022	8.7068	8 17 1 19
3	19.845	10.9611	10 23 4
4..........	22.752	13.4559	11 11 5 1.5
5..........	45.507	38.0750	38 1 49
6..........	91.008	107.6944	107 16 40

* Light is propagated through a space equal to the diameter of the Earth's orbit in $16^m\ 26^s$.

On the Ring of Saturn.

Besides his seven satellites, *Saturn* is surrounded by a flat and thin ring of coherent matter. Dr. Herschel has discovered that the ring instead of being entire is divided into two parts, the two parts lying in the same plane.

The ring is luminous, by reason of the reflected light of the Sun; it is visible to us, therefore, when the faces illuminated by the Sun are turned towards us: invisible, when the opposite faces; invisible also, when the plane of the ring produced passes through the centre of the Earth; since then no light can be reflected to us; invisible also in a third case, when the plane of the ring produced passes through the centre of the Sun, since, in that case, it can receive no light from that luminary. The plane of the ring is inclined to that of the ecliptic in an angle of about $31^0\ 24'$, and revolves round an imaginary axis perpendicular to its plane in $10^h\ 29^m\ 16^s$: and, which is worthy of notice, this period is that in which a satellite, having for its orbit the mean circumference of the ring, would revolve according to Kepler's law *.

We have now gone through another great division of our subject. The Lunar Theory will next occupy our attention, which might, indeed, have taken its place before the Planetary.

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* The fact of the squares of the periodic times varying as the cubes of the mean distances, is frequently called, the *Third* law of Kepler.

CHAP. XXIX.

ON THE LUNAR THEORY.

On the Phases of the Moon.—Its Disk.—Its Librations, in Longitude, in Latitude, and Diurnal.

Of all celestial bodies, the Moon is the most important, by reason of its remarkable and obvious phenomena: the intricacy of the theory of its motions; and the usefulness of the practical results derived from such theory.

Some of the phenomena admit of an easy explanation, and require no great nicety of computation. Such are the phases of the Moon. Others, with regard to their general cause, admit also of an easy explanation; but, with regard to the exact time of their appearance and recurrence, require the most accurate knowledge of the lunar motions. Of this latter description, are the eclipses of the Moon.

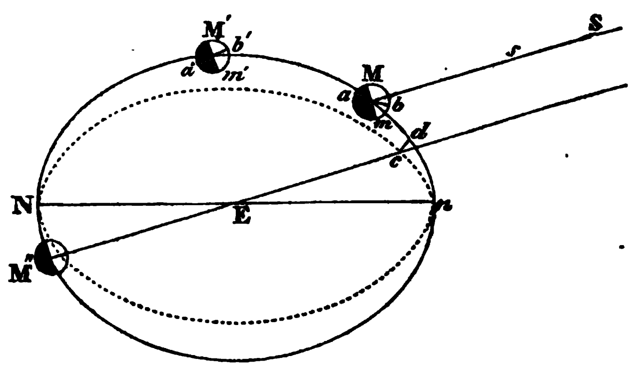
If therefore with a view to simplicity, we arrange the subjects of the ensuing Chapters, we ought first to place the phases of the Moon, next, the elements and form of the orbit, then, the lunar motions and their laws, and lastly, the lunar eclipses.

The explanation of the phases of *Mercury* and *Venus* was founded on the hypothesis, of their being opaque bodies illuminated by the Sun, and, of their revolution round the Sun. A similar explanation, on similar hypotheses, will apply to the Moon. We shall perceive the cause of its phases, if we suppose the Moon to shine by the reflected light of the Sun, and to revolve round the Earth: and, as in the case of the two inferior planets, the explanation does not require a knowledge of the exact curve in which the revolution is performed.

The Moon moves through 12 signs, or 360^0 degrees of longitude, in about 27 days. This fact is ascertained by observing, each day, on the meridian, its right ascension and declination, and thence deducing, by calculation, (see pp. 158, &c.) the corresponding latitude and longitude. Hence, in a period somewhat more than the preceding, the Moon is on the meridian at all hours of the day, and the angle, formed by two lines drawn from the Moon to the Earth and Sun respectively, passes through all degrees of magnitude. The *exterior* angle therefore, (see p. 553,) on the magnitude of which, the visible illuminated disk depends, passes also through all degrees of magnitude: and the Moon accordingly, like *Venus*, must exhibit all variety of phase; the crescent near conjunction; the *half Moon* in quadratures; and the entire orb illuminated, or the *full Moon* in opposition.

Venus revolves round the Sun, and the Moon round the Earth: but this difference of circumstance, in no wise affects the principle on which the phases depend: they are regulated by the inclination of the planes of the circles of illumination and vision: and their magnitude depends, as it was shewn in p. 553, on the versed sine of the exterior angle at the planet: that is, in Fig. p. 553, on the versed sine of the angle SuF.

The angle, analogous to SuF, in the annexed Figure, will be



contained between a line Ss drawn to the centre of the Moon at M, and a line drawn from E and produced through the same centre. This angle, by reason of the parallelism of the lines drawn

from E to the Sun, will equal the interior angle continued between cE and a line drawn from E to the centre of the Moon; which angle, in other words, is the angle of elongation.

Hence, in delineating the Moon's phases, we may use a simpler expression, and state *the visible enlightened part to vary as the versed sine of the Moon's elongation.*

If we suppose the Earth to be illuminated by the Sun, and to serve *as a Moon* to the Moon, the visible illuminated part of the Earth, will to a spectator at the Moon vary as the versed sine of the *Earth's elongation.* Let e be the latter angle, E the former: then by what has just preceded,

$$E + e = 180^0, \text{ nearly};$$

$$\therefore \cos. E = \cos. (180^0 - e) = -\cos. e,$$

$$\text{and } 1 - \cos. E = 1 + \cos. e, \quad 1 + \cos. E = 1 - \cos. e.$$

Hence, when the Moon's phase is ☽'s radius $\times (1 - \cos. E)$, the corresponding phase of the Earth

$$\{\oplus\text{'s radius} \times (1 - \cos. e)\}, \text{ is } \oplus\text{'s radius} \times (1 + \cos. E),$$

the larger, therefore, the Moon's phase is to us, the smaller, at the same time, is the Earth's phase to an inhabitant of the Moon. Thus, near conjunction when E is nearly 0, the Moon's phase is ☽'s radius $\times (1 - 1)$, nearly, whilst the Earth's phase is $\oplus$'s radius $\times$ 2, or the Earth is nearly at her *full*, to an inhabitant of the Moon, whilst the Moon is a *new* Moon to us. In such a situation the Earth's light is reflected towards the Moon, falls on its dark disk, and feebly illuminates it, producing the phenomenon called by the French *lumière cendrè.*

When the Moon is in opposition, $E = 180^0$, the Moon's phase is ☽'s radius $\times (1 + 1)$, or the Moon is at her *full*, and the corresponding phase of the Earth is expounded by, $\oplus$'s radius $\times (1 - 1)$, which being nothing, shews that the dark side of the Earth is then towards the Moon.

When $E = 90^0$, $\cos. E = 0$; $\therefore 1 + \cos. E$, and $1 - \cos. E$, are each $= 1$: consequently, in such a position,

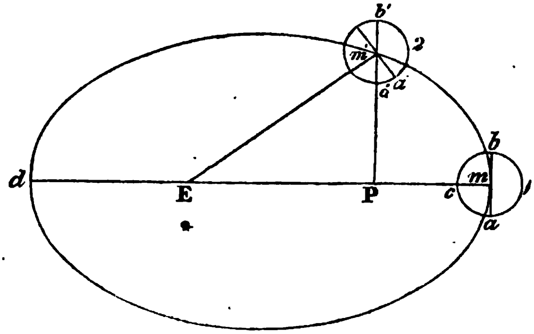
the Moon shews half of her illuminated disk to the Earth, while the Earth shews half of her illuminated disk to the Moon.

If $E = 60^0$, cos. $E = \frac{1}{2}$, therefore the Moon's phase is ☽ 's radius $\times \frac{3}{2}$, or the Moon is at her third quarter; the Earth's phase is ⊕'s radius $\times (1 - \frac{1}{2})$, or $\frac{2\,⊕\text{'s radius}}{4}$: or, the Earth, viewed from the Moon, is at her first quarter.

The period of the Moon's phases, or the interval of time which must elapse before the phases, having gone through all their variety, begin to recur, must depend upon the return of the Moon to a situation similar to that which it had, at the beginning of the period. If we date then the beginning of the period from the time of conjunction, (the time of new Moon,) the end of the period must be when the longitudes of the Moon and Sun are again the same. Now the longitude of the Sun is continually increasing;. when the Moon therefore has made, from its first position, the circuit of the heavens, it will be distant from the Sun, by the angular space through which, during the Moon's sidereal period, the Sun has moved. In order, then, to rejoin the Sun and to be again in conjunction, it must move through this space, and a little more; and when it does rejoin the Sun, a *synodic* revolution is completed. And the period therefore of the Moon's phases is a *synodic* period. From the inequality of the Moon's motion, this *synodic* period, or *lunation*, is not always of the same length.

If we conceive a plane passing through the centre of the Moon and perpendicular to a line drawn from the Earth to the Moon, then on such a plane the Moon's face will appear to be projected. This face, since the Moon has ever been an object of the attention of Astronomers, has been delineated, and a map made of its seeming Seas, Mountains, and Continents. But, one map of the same hemisphere has always served to represent the Moon's face: in other words, the same face of the Moon is always turned towards us. This is a curious circumstance, and the immediate inference from it is, that the Moon must revolve round its axis, with an angular velocity equal to that with which it revolves round the

Earth. For*, suppose in the position (1) a to be on the verge of the disk, then, if in the position (2) we still see the point a, in the verge, and in the same position, it must have been transferred, by rotation, through an arc $a'a$: since, in the case of



no rotation, $b'a'$, parallel to ba, would have been the position of ba. Now, a being seen on the verge of the Moon's disk, $\angle Em'a =$ a right angle $= \angle Em'a' + \angle a'm'a$. But since EPm' is a right angle, $\angle Em'P + \angle PEm'$ is one also: consequently,

$$\angle Em'a + \angle a'm'a = \angle Em'P\,(\angle Em'a) + \angle PEm;$$

$$\therefore \angle a'm'a = \angle PEm',$$

and the angle $a'm'a$ measures the rotation of the Moon round its axis that has taken place since it occupied the position (1), and the angle PEm', the angular motion of the Moon round E from the same position.

If the angle PEm', the measure of the Moon's true angular distance from one of the apsides of its orbit, increased uniformly, and the Moon's rotation round her axis were uniform, the above result would always take place; that is, the same face of the Moon ought always to be turned to the spectator: and such phenomenon

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* In the Figure, acb is supposed to represent the Moon's equator, and (which is not strictly true) to lie in the plane of the orbit: the axis of rotation, then, is perpendicular at m to that plane: perpendicular, for instance, to the plane of the paper, if the latter be imagined to represent that of the Moon's orbit.

ought constantly to be observed. But since, which is the case, the Moon's true motion differs from the mean, and the angle PEm' does not increase uniformly, the preceding result will not be precisely true, if we suppose, (which is a probable supposition,) the Moon's rotation round her axis to be uniform. If after any time, 3 days for instance, mEm' should measure the Moon's angular distance from the position (1), then, by reason of the Moon's elliptical motion, in 6 days twice the angle mEm' will certainly not measure the Moon's angular distance: but, on the supposition of the Moon's uniform rotation, twice the angle $a'm'a$ would measure the quantity of rotation in 6 days. Hence, if the Moon's angular velocity should be diminishing from the position at (1), at the end of 6 days the point a, previously seen on the verge of the Moon's western limb, would have disappeared, and some points towards the verge of the Moon's eastern limb would be brought into view; and such, by observation, appears to be the case, and the phenomenon is called the Moon's *Libration in Longitude*.

Since this libration in longitude arises from the unequal angular motion of the Moon in her orbit, it must depend on the difference of the true and mean anomalies, in other words, on the equation of the centre, or equation of the orbit; and would be proportional to that equation, and its maximum value would be represented by the greatest equation (6° 18′ 32″) in case the axis of the Moon's rotation were perpendicular to the plane of its orbit.

In the preceding reasonings, we have supposed the section bca, representing the Moon's equator, to be coincident with $mm'd$ the plane of the orbit: in other words, we have supposed the axis of rotation to be perpendicular to the same plane. Now, the axis is not perpendicular but inclined to the plane at an angle of 5° 8′ 49″; the preceding results therefore will be modified by this circumstance. For, take the extreme case, and suppose the axis of rotation to be parallel to the plane of the orbit, and in the position (1) to be represented by ce*: then it is plain, we should

* e, omitted in the Figure, ought to have been where cm produced cuts the circle eba.

at the position (1), see the pole *c*, and the hemisphere, projected upon a plane passing through *ba* perpendicular to the orbit; and, half a month after, at *d*, we should see the *opposite* pole *e*, and the *opposite* hemisphere, notwithstanding the equality between the Moon's revolution round the Earth, and her rotation round her axis. In intermediate inclinations then of the Moon's axis of rotation, part of this effect must take place, or must modify the preceding results. If in the position (1), the Moon's axis being inclined to the plane of her orbit, we perceive, for instance, the Moon's north pole and not her south, we shall in the opposite position at *d*, after the lapse of half a month, perceive the Moon's south, and not her north pole; and, this effect is precisely of the same nature, as that of the north pole being turned towards the Sun at the summer, and of the south pole at the winter solstice, (see p. 24.) The perpendicularity therefore of the axis of rotation to the plane of the orbit is a condition equally essential, with that of the equality of rotation and revolution, in order that the same face of the Moon should be always turned to the spectator.

This second cause, preventing the same face of the Moon from being always seen, is called, with some violation of the propriety of language, the *Libration in Latitude*. For, it is plain, from the preceding explanation, that there are properly and physically no librations, but librations only seemingly such.

There is a third libration, discovered by Galileo, and called the *Diurnal Libration*. If the two former librations did not exist, the same face of the Moon would be turned, not to a spectator on the surface, but, to an imaginary spectator placed in the centre of the Earth. Now, two lines drawn respectively from the centre and the surface of the Earth to the centre of the Moon, (the directions of two visual rays from the two spectators) form, at that centre, an angle of some magnitude; and, when the Moon is in the horizon, an angle equal to the Moon's horizontal parallax. Hence, when the Moon rises, parts of her surface, situated towards the boundary of her upper limb, are seen by a spectator, which would not be seen from the Earth's centre. As the Moon rises, these parts disappear: but as the Moon, having passed the

meridian, declines, other parts, situated near that boundary, which, whilst the Moon was rising, were the lower, are brought into view, and which would not be seen by a spectator placed in the centre of the Earth. The greatest effect of this diurnal libration will be perceived, by observing the Moon first at her rising, and then at her setting.

This last libration, like the two preceding, is purely optical.

The description of general and obvious phenomena requires only popular explanation, which is easily afforded. But the next steps, the accounting for, on principle and by calculation, minute phenomena, (if we may apply that term to effects detected only by the aid and comparison of numerous observations) are more difficult, whether those steps are to be made in the solar, planetary, or lunar theory: and we shall find them peculiarly so in the latter theory.

CHAP. XXX.

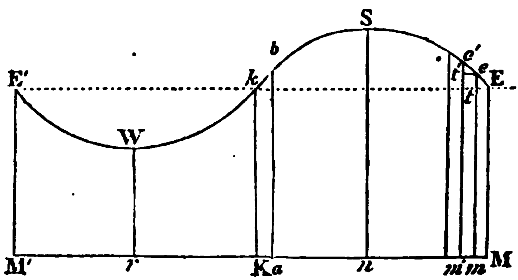
On the Methods of deducing, from Observations, the Moon's Parallax: the Moon's true Zenith Distance, &c.

According to modern Astronomical usage, the same kind of observations, namely, meridional observations, which are used in determining the places of the fixed stars, and the elements of the orbits of the Sun and the planets, serve also to determine the position and dimensions of the lunar orbit. But, by reason of the proximity of the Moon to the Earth, and the irregularity (if we may use such a term) of her motions, the *reduction* of the Moon's observed right ascensions and declination requires more scientific and longer computations.

The orbits of planets round the Sun, and of secondary planets round their primaries, would, if we abstract the mutual effects of planets, be elliptical. Now the elliptical is a regular motion. It is, therefore, the disturbing forces that render the motions of planets irregular; and, since the mutual influence of planets must be universally felt, there is no planet nor secondary, the motions of which are not, in some degree at least, irregular. The degree of irregularity depends on what may be called the peculiar circumstances of the planet, which are those of the vicinities and magnitudes of other planets. For instance, Jupiter and Saturn, (see *Physical Astronomy*, Chap. XIX.) bodies of great bulk, and, in a certain sense, not very distant from each other, mutually and powerfully disturb each other, or prevent what, according to our theories, would otherwise take place, namely, elliptical motion. In like manner the Earth's motion is rendered *irregular*, but not considerably so (see *Physical Astronomy*, Chap. XVIII.) by the actions of Venus and Jupiter, &c. The Moon is near to the Earth, but then its mass, relatively to the Sun's mass, is very

inconsiderable. It is, however, the Sun's mass which is almost the sole cause of the Moon's not describing an ellipse round the Earth, or, which, as we have explained it, prevents her motions from being *regular*, and which, therefore, makes them by reason of its largeness, very irregular.

The irregularities we are speaking of are real ones, and would be observable in the daily changes of right ascensions, and of north polar distances, even if the observer were placed in the centre of the Earth. Or, if from the Moon's right ascensions and north polar distances, her longitudes and latitudes were deduced, and then, on a line such as MM' representing the ecliptic, ordinates ME, me, &c. proportional to the latitudes were erected,



the curve $Ee e'$, &c. passing through their extremities would be a curve less *regular* than when (see p. 145,) under similar conditions, it represents the solar orbit. A consequence, or indication of such irregularity would be this, that from me, $m'e'$, &c. representing latitudes, or declinations, computed or observed for equal intervals Mm, mm', &c. an intermediate latitude or declination interpolated, for an intermediate interval, would be less exact in the lunar, than in the solar orbit.

It is plain, when observations are made by means of instruments placed in the meridian, that the north polar distances, and right ascensions of planets can only be known, at times intermediate of their meridional passages, by a species of interpolation. In the case of the Sun, its north polar distance at midnight, on March 1, is nearly the mean of his north polar distances on the noons of March 1 and 2 : and six hours past the noon of March 1,

is, nearly, his north polar distance on the noon of March 1, minus the decrease of north polar distance, proportional to six hours. This mode of computation, however, not exact even in the case of the Sun, is less exact when applied to the Moon.

In order to determine the inexactness of the computation, or of any other mode of interpolation, we must observe the heavenly body when it is out of the meridian. In the case of the Sun, for instance, observe its zenith distance, and note the distance in time from noon: then if the co-latitude (PZ) be known, we can from PZ, the horary angle ZPS, and ZS compute PS, and then compare PS, thus computed, with the interpolated value of PS.

But this brings us to the consideration of the second cause of irregularity: that which arises from the proximity of the observed body, and which proximity gives rise to the inequality of *parallax*. In the case of the Sun, its north polar distances, computed according to the above methods, and compared, are found, very nearly, to agree; which agreement is a proof of the smallness of the Sun's parallax. For parallax (see Chap. XII.) affects the zenith distance, and is the larger the greater the zenith distance. The north polar distances, therefore, found by adding to the co-latitude of the place the observed meridional zenith distances, would be incorrect, but would be less so than an intermediate zenith distance, observed out of the meridian. In the case, therefore, of a near heavenly body, it would be impossible that the north polar distances, found according to the above methods, should, on comparison, agree: and this we shall find to be the case with the Moon.

We shall give to this statement greater distinctness, by examining some of the recorded observations of the Sun and Moon.

In the second Volume of the Greenwich Observations, we find the following observations of the zenith distances of the upper and lower limbs of the Sun.

1783.	Barometer.	Thermometer.	Zenith Distance.	Corrected Zenith Distance.
May 4,	29.95	$48\frac{1}{2}$	☉ L. L. 35° 43′ 8″.9	35° 43′ 50″.76
			☉ U. L. 35 11 23 .9	35 12 4 .92
5,	29.84	$49\frac{1}{2}$	☉ L. L. 35 25 56 .9	35 26 38 . 34
			☉ U. L. 34 54 12 . 4	34 54 52 .95
6,	29.81	51	☉ L. L. 35 9 0 . 2	35 9 40 . 82
			☉ U. L. 34 37 17 . 3	34 37 57 . 16
7,	29.9	$47\frac{1}{2}$	☉ L. L. 34 52 19 . 4	34 53 0 . 46
			☉ U. L. 34 20 37 . 17	34 21 17 . 96

The last column contains the zenith distances, corrected or reduced according to the principles and formulæ of Chapter X. If we add together the respective corrected zenith distances of the lower and upper limbs, and take their half sums, the results will be the values of the zenith distances (Z) of the Sun's centre.

	Values of Z.	First Diffs. d'.	Secd. Diffs. d''.	Third Diffs. d'''.
May 4,	35° 27′ 57″.89	− 17′ 12″.25	+ 15″.60	+ 1″.27
5,	35 10 45 . 64	− 16 56 . 65	+ 16 . 87	
6,	34 53 48 . 99	− 16 39 . 78		
7,	34 37 9 . 21			

Here the several differences tend towards an equality, which is a proof (should the several values be represented by the ordinates me, $m'e'$, &c. of a curve Eee', &c.) of the *regularity* of that curve. The use of the Table of differences is to find an intermediate value of Z, and by means of what is called the *Differential Theorem*, (see Appendix to *Trigonometry*.) Thus, the intermediate value of Z corresponding to May 5, 8^h, would be, making

$a = 35^\circ\ 10'\ 45''.64,\ d' = -16'\ 56''.65,\ d'' = 16''.87,\ d''' = 1''.27;$

$x = \frac{8}{24} = \frac{1}{3},$

$$35^\circ\ 10'\ 45''.64 - 5'\ 48''.88 - 1''.87 + 0''.13 = 35^\circ\ 4'\ 55''.$$

This is not exactly the value of Z, since it has been obtained on the ground, that the interval between two successive meridional zenith distances, is exactly 24^h: which, (see Chapter XXII, on the Equation of Time) is not the case. In order to obtain an exact result, we must refer to the Volume of Observations above quoted, and examine the Sun's right ascensions at his transits on the 4th and 5th of May,

1784.	Sun's Right Ascension.	d'	d''.
May 4,	$2^h\ 45^m\ 53^s.9$	$+\ 3^m\ 51^s$	
5,	2 49 44.9	+ 3 51.4	+.4
6,	2 53 36.3	+ 3 51.9	+.5
7,	2 57 28.2		

Here the increase of the Sun's right ascension, between the transits on the 5th and 6th, is $3^m\ 51^s.4$: if, therefore, the eight hours should be eight hours of sidereal time, we should have

$$x = \frac{8}{24^h\ 3^m\ 51^s.4} = .33244,$$

from which value, as before, (see l. 2, &c.) we may deduce the value of Z, corresponding to eight hours of sidereal time, after the Sun's transit on May 5.

The values of Z are, in fact, meridional zenith distances. But, it is plain, an interpolated value cannot belong to the meridian of the place of observation; it may, however, be conceived to belong to the meridian of some other place, having a different longitude, but the same latitude. In point of fact, the result that has been obtained by the differential theorem is merely a mathematical result. We may, however, by slightly modifying the preceding

process, obtain a mathematical result, which, at the same time, shall represent a real quantity. Thus, if to the four values of Z, in the first column of the Table of p. 642, we add the co-latitude of the place, we shall obtain four north polar distances of the Sun, on the noons of the 4th, 5th, 6th, and 7th of May. An interpolated north polar distance is independent of the place of observation: and if we deduce it, as we deduced the value of Z, the deduced north polar distance, must be the same as the co-latitude (PZ) of the place added to that value of Z, because, in each computation, the differences d', d'', d''', are the same: since

$$(PZ + Z) - (PZ + Z') = Z - Z' = d', \&c.$$

If, therefore, in the above instance, the place of observation be Greenwich, the co-latitude of which is $38^\circ\ 31'\ 20''$, the Sun's north polar distance, on May 5 at eight hours of sidereal time, is equal to $38^\circ\ 31'\ 20'' + 35^\circ\ 4'\ 53''$, that is, to $73^\circ\ 36'\ 13''$.

But this determination supposes the observed zenith distance to be the same, as if the observer were near to the Earth's centre: in other words, it supposes the angle, subtended by the Earth's radius at the Moon, to be inconsiderable. We shall hereafter, in the Chapter on the Transit of Venus, see that the greatest angle which can be subtended by the Earth's radius, or, the Sun's horizontal parallax, does not exceed $9''$.

A shorter and easier method of proving the smallness of the Sun's parallax has been already described in pp. 326, &c.

If S represent the Sun, Z, the zenith, P the pole, the triangle ZPS can be solved if ZP, PS, and the angle ZPS be given or known. Thus, in the above instance,

$$ZP = 38^\circ\ 31'\ 20'',$$
$$PS = 73\ \ 36\ \ 13,$$

and in order to find the angle ZPS, we have

right ascension of mid-heaven	8^h	0^m	0^s
Sun's right ascension at noon	2	49	44.9
	5	10	15.1
acceleration (see p. 526,)			50.824
	5	9	24.276

ZS computed from these three values, and compared with ZS, found by observations made out of the meridian would shew, by the agreement of the two values, the smallness of the Sun's parallax.

But we shall find results of a different kind, if we examine and compare the Moon's places determined from zenith observations. In the Volume of the Greenwich Observations above referred to, we find

1784.	Barometer.	Thermometer.	Zenith Distance. Moon's Limb.	Right Ascension. Moon's First Limb.
Jan. 31,	30.35	32	L. L. $24^{\circ}\ 48'\ 13''.5$	$4^{h}\ 35^{m}\ 34^{s}$
Feb. 1,	30.08	$31\frac{1}{2}$	U. L. 23 12 13.3	5 51 35
2,	30.04	32	U. L. 23 33 43.2	6 27 28
3,	30.41	32	U. L. 25 19 0.9	7 22 0
4,	29.99	$33\frac{1}{2}$	U. L. 28 19 27	8 14 20

Correct on account of refraction, as in the former instance, the zenith distances of the upper and lower limbs, and add or subtract the Moon's semi-diameter: the results will be the zenith distances (z) of the Moon's centre, from which zenith distances we may, as before, form a Table of differences.

Values of z.	d'.	d''.	d'''.	d^{iv}.
$24^{\circ}\ 33'\ 37''$	$-1^{\circ}\ 6'\ 2''$	$+1^{\circ}\ 27'\ 28''$	$-3'\ 36''$	$-5'\ 1''$
23 27 35	+0 21 26	+1 23 52	−8 37	
23 49 1	+1 45 18	+1 15 15		
25 34 19	+3 0 33			
28 34 52				

Here the differences exhibit considerable irregularities, which arise from two causes: one real; the other, as it may be called,

optical, originating, mainly, from the Moon's proximity to the Earth, but varying, in degree, with the Moon's distance from the zenith. But from whatever causes the irregular values of Z arise, they are, as phenomena or results of observation, blended together, and it is necessary to institute an investigation, in order to distinguish the separate causes. Now, the first step in such investigation, is similar to the one made in p. 643, that is, we must find by interpolation, an intermediate value of the Moon's north polar distance, and from it and the horary angle ZPM, and the colatitude PZ, we must compute the Moon's zenith distance, which is to be compared with the Moon's *observed* zenith distance.

In order to find the value of x, or the interval proportional to eight hours of sidereal time on February 1, we must first deduct the Moon's right ascension on February 1, from her right ascension on January 31: that is, we must take the difference of $5^h\ 31^m\ 35^s$, and $4^h\ 35^m\ 34^s$, which is $56^m\ 1^s$. This $56^m\ 1^s$ is the angle which the meridian, after having passed through the Moon's centre, must describe, in addition to 24^h, before it can again reach the Moon's centre. Unity, therefore, denoting the interval between two successive transits,

$$1 : x :: 24^h\ 56^m\ 1^s : 8^h\ ; \ \therefore x = .3208.$$

Substitute this value for x, in the differential theorem, and the value of Z corresponding to 8^h (sidereal time) on February 1, is

$$23^\circ\ 27'\ 35'' + (21'\ 26'') \times .3208 + (1^\circ\ 23'\ 52'') \times .3208 \times -.3396$$
$$-8'\ 37'' \times .3208 \times .3396 \times .59304 + 5'\ 1'' \times .3208 \times .3396$$
$$\times .69304 \times .6697 = 23^\circ\ 24'\ 59''.033.$$

Hence the Moon's north polar distance is the above quantity added to $38^\circ\ 31'\ 20''$, or, is nearly equal to $61^\circ\ 56'\ 19''$. It is, however, the Moon's north polar distance, only on the supposition of the non existence of parallax. For if the Moon be so near to the Earth, that the radius of the latter subtends some measurable angle at the former: then (see the Chapter on Parallax) the observed zenith distances are not, in a certain sense, the true zenith distances: but every observed zenith distance will require, proportionally to its sine, a correction to *reduce* it to a true zenith distance.

If from observations contemporaneously made (see p. 325,) in different parts of the Earth, we knew the Moon's horizontal parallax, we could, by means of such a series as is given in p. 324, deduce such correction. But if, without quitting the place of observation, we wish to ascertain the existence and quantity of parallax, we must compute ZM (Z the zenith, M the Moon) from the co-latitude (PZ) an interpolated value of PM, and the horary angle ZPM. Now this horary angle, must, like PM, be obtained by interpolation.

In the case of a fixed star, and only in that case, the horary angle (the angle ZPs) is the difference of the right ascension of the mid-heaven (in other words, the sidereal time) and of the star's right ascension. In the case of the Sun, we must, as we have seen in p. 643, allow for the change of the Sun's right ascension, during his transit over the meridian, and the assigned instant of sidereal time. The computation for a like allowance, in the case of the Moon, is a little more operose. On the 1st of February (see the Table of p. 645,) the Moon's right ascension, at the instant of her transit, was $5^h\ 31^m\ 35^s$, and since her right ascension increases by unequal steps, we must find it at any time, intermediate of her meridional transits, by the differential theorem. If we form then a Table of differences, like the one of p. 645,

R. A. Moon's First Limb.	d'.	d''.	d'''.	d^{iv}.
$4^h\ 35^m\ 34^s$				
	$+ 56'\ 1''$			
5 31 35		$- 0'\ 8''$		
	$+ 55\ 53$		$- 1'\ 13''$	
6 27 28		$- 1\ 21$		$+ 22''$
	$+ 54\ 32$		$- 0\ 51$	
7 22 0		$- 2\ 12$		
	$+ 52\ 20$			
8 14 20				

we have $a = 5^h\ 51^m\ 35^s$, $d' = 55'\ 53''$, $d'' = -\ 1'\ 21''$, $d''' = -\ 51''$, $d^{iv} = 22''$ and (see p. 646,) $x = .32088$, and, accordingly,

$$\text{Æ of ☽'s 1st L.} = 5^h\ 49^m\ 35^s.48,$$

which is the right ascension of the Moon's preceding limb, eight hours after the Moon's transit of the meridian. But the sidereal time, at the time of the Moon's transit (in other words, the right ascension of the mid-heaven at that time, or the right ascension of the Moon's first limb) was $5^h\ 31^m\ 35^s$; eight hours, therefore, after the sidereal time, or right ascension of the mid-heaven, must be $13^h\ 31^m\ 35^s$, and accordingly, the horary angle must be $13^h\ 31^m\ 35^s - 5^h\ 49^m\ 35^s.48$, or $7^h\ 41^m\ 59^s.2$: from this must be subtracted the angle at the pole, subtended by the Moon's semi-diameter. Now the Moon's semi-diameter is $15'\ 4''$, and the polar distance (see p. 646,) of the Moon's centre is $61^\circ\ 56'\ 19''$; therefore the angle at the pole is

$$\frac{15'\ 4''}{\sin.\ 61^\circ\ 56'\ 19''} = 17'\ 4''.4 = 1^m\ 8^s.297;$$

consequently, the horary angle is $7^h\ 40^m\ 50^s.9$; we have then

$$\begin{array}{lcl} ZPM = 7^h\ 40^m\ 50^s.9 & = & 115^h\ 12^m\ 43^s.5 \\ ZP \ldots\ldots\ldots\ldots & = & 38\ \ 31\ \ 20 \\ PM \ldots\ldots\ldots\ldots & = & 61\ \ 56\ \ 19 \end{array}$$

whence, by the solution of a spherical triangle, according to the formula of *Trigonometry*, p. 171, Edit. 3, there results,

$$^* ZM = 82^\circ\ 13'\ 6'',\ \text{nearly}.$$

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* See *Trigonometry*, pp. 171, &c.

$$\begin{array}{rlr} \frac{c}{2} = 57^\circ\ 36'\ 21''.7 \ldots\ldots\ldots & 2\ \text{log. cos.} & 19.4599294 \\ a = 38\ \ 31\ \ 20 \ldots\ldots\ldots & \text{log. sin.} & 9.7943612 \\ b = 61\ \ 56\ \ 19 \ldots\ldots\ldots & \text{log. sin.} & 9.9456872 \\ \frac{a}{2}+\frac{b}{2} = 50\ \ 13\ \ 49.5 & & 19.1999778 \\ M = 23\ \ 27\ \ 33.5 \ldots\ldots\ldots & (\text{log. sin. } M) & 9.5999889 \\ \frac{a}{2}+\frac{b}{2}+M = 73\ \ 41\ \ 23 \ldots\ldots\ldots & \text{log. sin.} & 9.9821604 \\ \frac{a}{2}+\frac{b}{2}-M = 26\ \ 46\ \ 16 \ldots\ldots\ldots & & 9.6536248 \\ \therefore\ \frac{c}{2} = 41\ \ \ 6\ \ 32.8 \ldots\ldots\ldots & 2) & 19.6357852 \\ c = 82\ \ 13\ \ \ 5.6 \ldots\ldots\ldots & \left(\text{log. sin. } \frac{c}{2}\right) & 9.8178926 \end{array}$$

Suppose now the observed zenith distance to be 82° 49′ 10″, then the difference between the two, namely, 36′ 4″, would be an indication of parallax and *partly* its effect. It cannot represent the *whole* effect, because on the supposition of the existence of parallax, the meridional north polar distances, (obtained by adding the co-latitude to the observed meridional zenith distances), from which *PM* was obtained by interpolation, would be all wrong, and consequently *PM*, one of the given quantities in the triangle *ZPM* (see p. 648,) would be so also, and consequently, in the last place, the result of the solution, or the value of *ZM* would be incorrect. The difference 36′ 4″ then being only in part the effect, and not the measure of parallax, must be considered as a first approximation towards the true value of parallax. Under this point of view, if *P* (see pp. 323, &c.) should denote the horizontal parallax, we should have (see p. 323,)

$$\sin. P = \frac{\sin. p}{\sin. (D + p)}, \text{ or, nearly,}$$

$$P = \frac{p}{\sin. (D + p)} = \frac{36' \, 4''}{\sin. 82^{\circ} \, 49' \, 10''} = 36' \, 21''.$$

With this approximate value we may partly correct the observed zenith distances, and obtain more correct values of the north polar distances deduced from such zenith distances. Thus, since $P = 36' \, 21''$, and since the observed zenith distances on Feb. 1, (see p. 645,) was 23° 27′ 35″, we have (see p. 323,) the parallax of the meridional zenith distance

$$= 36' \, 21''. \sin. 23^{\circ} \, 27' \, 35'' = 868''.27 = 14' \, 28'', \text{ nearly.}$$

With this, as a correction, the series of zenith distances should be reduced (see p. 645,) and a new series of meridional polar distances, from which, as before, we may deduce by interpolation, or the differential formula, a more correct value of *PM* corresponding to 8^h. It is plain that this value of *PM* must be nearly the former value (61° 56′ 19″) minus the parallax on the meridian, that is, 61° 41′ 51″. Instead, therefore, of making $PM = 61^{\circ} \, 56' \, 19''$, make it, in the formula of solution of p. 648, 61° 41′ 51″, and the resulting value of *PM* is 82° 1′ 16″: subtract this from 82° 49′ 10″, the observed zenith distance, and

the difference, which is the second approximate value of the parallax, is 47′ 54″, and, therefore, as before

$$P = \frac{47' \; 54''}{\sin. 82^\circ \; 49' \; 10''} = 48' \; 16'',$$

and the parallax on the meridian $= 48' \; 16''. \sin. \; 23^\circ \; 27' \; 35''$ $= 19' \; 13''$, and, as before, deducting this from 61° 56′ 19″, the new value of PM is equal to 61° 37′ 54″, with which new value the side ZM is again to be deduced from the formula of p. 648.

The resulting value of ZM, is again to be deducted from the observed zenith distance, in order to obtain new values of p, and P, and after three more approximations, we shall deduce a value of P about 54′ 10″: which is nearly that of the Moon's horizontal parallax. This is the description of the process for ascertaining, at the same place of observation, the existence and quantity of the Moon's parallax. But if we knew by means of the method described in pp. 325, &c. and by the result of such observations as were made at the Cape of Good Hope and Berlin, the Moon's horizontal parallax, we could, in the first instance, find the parallaxes corresponding to the several zenith distances, (see p. 645,) correct such distances, and then deduce a series of north polar distances of the Moon, by adding the co-latitude of the place of observation to the zenith distances so corrected.

In what has preceded, we have pointed out and described two methods for determining the Moon's parallax, neither of which can be very conveniently practised. It was a rare occurrence that gave observations, contemporaneously made at places so far distant as the Cape of Good Hope and Berlin, and there are few Observatories provided, for observations out of the meridian, with instruments equally good as their mural quadrants and circles. The quantity and variation of the Moon'sparallax, now well known, has not been so known by one set of observations : but, like other astronomical elements, has been determined by the comparison of numerous observations, and with some small aid from theory.

The large quantity of the Moon's parallax, and its variations arising from the situation of the observer, and the change of

distance between the Moon and Earth, render it a subject of considerable astronomical importance. We shall, therefore, continue its discussion before we proceed to deduce the elements of the lunar orbit.

The Moon's horizontal parallax (P), is the angle which the Earth's radius subtends at the Moon. The Moon's apparent semi-diameter (D), is the angle which the Moon's radius subtends at the Earth. Hence,

$$P = \frac{\text{rad. } \oplus}{☽ \text{'s dist. from } \oplus},$$

$$D = \frac{☽ \text{'s rad.}}{☽ \text{'s dist. from } \oplus};$$

$$\therefore \frac{P}{D} = \frac{\oplus\text{'s rad.}}{☽ \text{'s rad.}},$$

the ratio, therefore, between the Moon's horizontal parallax and apparent semi-diameter, is a constant ratio, if the Moon and Earth be spheres; and, if the former be a sphere, is a constant ratio at the same place, whatever be the figure of the Earth.

If $P = 57' \, 4''.16844$, and $D = 15' \, 33''.8652$ *,

$$\frac{D}{P} = \frac{15' \, 33''.8652}{57' \, 4''.16844} = .27293,$$

or, by the method of continued fractions, is nearly $\frac{3}{11}$. Hence, from the observed apparent semi-diameter of the Moon, we may

* The ratio of the greatest and least apparent semi-diameters, is the same as the ratio of the perigean and apogean distances of the Moon,

$$\text{and } \frac{\text{the least apparent diameter}}{\text{the greatest apparent diameter}} = \frac{29' \, 30''}{33' \, 30''} = \frac{1-e}{1+e},$$

(if e be the eccentricity), whence $e = .0635$, whereas the eccentricity in the solar orbit only $= .0168$. The equation of the centre then, in the lunar orbit, must be about $7° \, 16'$. If, therefore, we set off from a circular motion, and call that the *regular* one, the Moon's motion, besides the causes already assigned (see p. 639,) will be still more irregular than the Sun's.

always deduce the corresponding horizontal parallax by multiplying the former by $\frac{11}{3}$: and *vice versâ*.

The horizontal parallax of the Moon is the angle subtended by the Earth's radius at the Moon. Hence, the Earth not being spherical, the horizontal parallax is not the same*, at the same instant of time, for all places on the Earth's surface. One proof that the Earth is not spherical, is by reversing this inference, namely, that the horizontal parallaxes computed for the same time are found not to be the same. Hence, in speaking of the horizontal parallax it is necessary to specify the place of observation. The Moon's parallax computed for Greenwich is different from the equatoreal parallax. Several corrections therefore, must be applied to an observed parallax, in order to compute, at the time of the observation, the Moon's distance from the centre of the Earth. For, that distance, it is plain, ought to result the same, whatever be the latitude of the place of observation.

The greatest and least horizontal parallaxes of the Moon, computed from observations at Paris, are, according to Lalande, (*Astron.* tom. II, p. 197,) $1° 1' 28''.9992$, and $53' 49''.728$, and the corresponding perigean and apogean distances respectively, 63.8419, 55.9164. The corresponding apparent diameters are $33' 31''$, and $29' 22''$.

The mean diameter, that which is the arithmetical mean between the greatest and least, is $31' 26''.5$; but, the diameter at the mean distance is smaller and equal to $31' 7''$.

Whatever be the quantity, which is the subject of their investigation, Astronomers are accustomed to seek for a constant and mean value of it, from which the true and apparent values are perpetually varying, or, about which they may be conceived to oscillate. In the subjects of time and motion, the search is after

* At the same distance the parallax varies as the radius vector of the spheroid. A Table, therefore, that gives the several values of the radii in a spheroid of a given oblateness, enables us to correct the equatoreal parallax. See Vince's *Astronomy*, vol. III. tab. XLV. p. 173.

mean time and *mean motion*, and by applying corrections or *equations* to deduce the true. The Moon's parallax not only varies in one revolution, from its perigean to its apogean, but the parallaxes which are the greatest and least in one revolution, remain not of the same value, during successive revolutions: they may not be the greatest and least, compared with other perigean and apogean parallaxes. But all may be conceived to oscillate about one fixed and mean parallax, which has been designated by the title of *Constant* Parallax, (*la Constante de la Parallaxe*).

We should obtain no standard of its measure, if we assumed it to be an arithmetical mean between its least and greatest values. For, the eccentricity of the lunar orbit varying, and consequently, the apogean and perigean distances, from the action of the Sun's disturbing force, the greatest parallax, if increased, would not be increased by exactly the quantity of the diminution of the least parallax; the mean of the parallaxes, therefore, would not always be the same constant quantity.

The constant parallax is assumed to be that angle, under which the Earth's radius would be seen by a spectator at the Moon, the Moon being at her mean distance and mean place: such, as would belong to her, when all causes of inequality are subtracted. But then, even by this definition, the *constant* parallax would be represented by the same quantity only at the same place; for, although the Moon's distance remains the same, the radius of the Earth, supposing it spheroidical, would vary with the change of latitude in the place of observation.

In order therefore, to rescind the occasion of ambiguity which might be attached to the phrase of *constant parallax*, Astronomers, in expressing its quantity, are accustomed to state the place for which it was computed. Thus, the equatoreal diameter being greater than the polar, the *constant parallax under the equator* (as it is termed) is greater than the *constant parallax under the pole:* the former, Lalande, by taking a mean of the results obtained by Mayer and Lacaille, states to be $57' \; 5''$, the latter $56' \; 53''.2$; the same author also states the constant parallaxes for Paris, and for the radius of a sphere, equal in volume to the Earth, to be respectively $56' \; 58''.3$, and $57' \; 1''$ (see *Astronomy*, tom. II. p. 315).

M. Laplace, however, proposes to deduce the several constant parallaxes from one alone: and to appropriate the term *constant*, to that parallax, belonging to a latitude, the square of the sine of which is $\frac{1}{3}$*. This parallax, by theory, he has determined to be 57′ 4″.16844, the corresponding apparent semi-diameter of the Moon being 31′ 7″.7304, (= 57′ 4″.16844 × .27293.)

This parallax being reckoned the mean parallax, the true parallax is to be deduced from it; if analytically expressed, to be so, by a series of terms: if arithmetically computed, by the application of certain *equations;* the terms and equations arising, partly, from mere elliptical inequality, and partly, from the perturbation of the Sun.

The terms due to the first source of inequality are easily computed: for, if we call P the horizontal parallax to the mean distance (a), then since we have any distance (ρ) in an ellipse expressed (see p. 459,) by this equation,

$$\rho = \frac{a.(1 - e^2)}{1 + e.\cos.\theta},$$

and since, the parallax × $\rho = P \times a$, we have the parallax = $P \times \frac{1 + e.\cos.\theta}{1 - e^2}$, and expanding as far as the terms containing e^3, &c. $= P\,(1 + e.\cos.\theta + e^2)$.

The terms due to the theory of perturbation are not easily computed. In the extent of mathematical science, there is no computation of equal importance and greater difficulty †.

The formula for the parallax, in which the constant quantity is 57′ 4″.16844, belongs to a latitude, the square of the sine of which is $\frac{1}{3}$. The corresponding formula for any other latitude is to be

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* Laplace chose this parallel, since the attraction of the Earth on the corresponding points of its surface, is very nearly, as at the distance of the Moon, equal to the mass of the Earth, divided by the square of its distance from the centre of gravity. Laplace, *Mec. Cel.* Liv. II, p. 118.

† The difficulty belongs equally to the formulæ for the latitude and longitude. See Lalande, tom. II, pp. 180. 193. 314.

deduced by multiplying the former by $\frac{r}{r'}$, or by applying a correction proportional to $r - r'$; r and r' being the radii corresponding to two latitudes, and computed on the supposition that the Earth is a spheroid with an eccentricity $= \frac{1}{300}$. [See Tables XLV, and XLVI; in the collection (1806) of French Tables, and the Introduction. See also Vince, vol. III, p. 50.]

The Moon's equatoreal horizontal parallax and apparent semidiameter, are inserted in the Nautical Almanack, and, for every 12 hours; the former is computed by the formula that has been mentioned (p. 654): the latter, by multiplying the parallax by .27293.

The Moon's distance may, as it has been already noted, be determined from her parallax; her greatest and least distances from her least and greatest parallaxes; and her mean distance from her mean parallax; and, taking for the value of the latter that determined by Laplace, we shall have

$$☽\text{'s distance} = \frac{57^0.2957795}{57'\ 4''.16844} \times \text{rad.} \oplus = \frac{57.2957795}{0.9511579} \times \text{rad.} \oplus$$

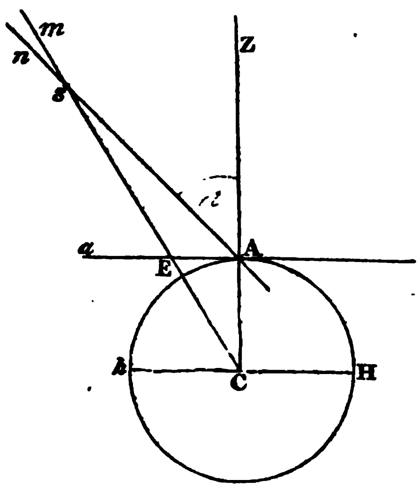
$= 60.23799 \times \text{rad.} \oplus$; therefore, if we assume the Earth's mean radius to be 3964 miles, the Moon's distance will be about 238783 miles.

The distances of the Sun and of the Moon from the Earth are inversely as their parallaxes. Hence, if the parallax of the former be considered equal to $8''.7$, the distances will be to each other, nearly, as 394 : 1.

Lacaille's method of determining the distance from the parallax applies successfully to the Moon, on account of her proximity to the Earth. It fails, with regard to the Sun, by reason of his distance. That distance is more than 24090 radii of the Earth: consequently, a radius of the Earth bears a very small proportion to it. The Sun's apparent diameter then seen from the surface of the Earth, is nearly the same, as if it were seen from the centre; and his diameter on the meridian cannot be sensibly larger than

his horizontal diameter. But, with the Moon, the case is different: since her distance is not much more than 60 radii of the Earth, her apparent diameter at its surface will be one 60th part greater than her diameter viewed from the centre: and as she rises from the horizon, and approaches the spectator, her apparent diameter will increase and be greatest on the meridian. It is easy to assign a formula for its augmentation.

Let *s* be the Moon, *p* the parallax represented by the angle



msn, *D* the ☽'s apparent distance from the zenith, Δ the ☽'s diameter viewed from the Earth's centre, *a* the augmentation of the diameter, then

☽'s real diameter $= \Delta \times Cs = (\Delta + a) \times As$;

$$\therefore \frac{\Delta + a}{\Delta} = \frac{\hat{C}s}{As} = \frac{\sin. CAs}{\sin. ACs} = \frac{\sin. D}{\sin. (D - p)}.$$

$$\text{Hence, } a = \frac{\Delta . \sin. D - \Delta . \sin. (D - p)}{\sin. (D - p)}$$

$$= \frac{2 \Delta \left\{\sin. \frac{p}{2} . \cos. \left(D - \frac{p}{2}\right)\right\}}{\sin. (D - p)}$$

(see *Trig.* p. 32.)

From this formula, in which $p = P . \sin. D$, (*P* the horizontal

parallax) a may be computed; but, in practice, more easily from a formula, into which, by the known theorems of Trigonometry, the preceding may be expanded. See Table XLIV, in Delambre's Tables; and the Introduction: also Vince, vol. III, p. 49.)

When the Moon is in the horizon, $p = P$, and $D = 90°$;

$$\therefore a = \frac{\Delta\,(1 - \cos. P)}{\cos. P} = \Delta\,.(\sec. P - 1).$$

Hence, the ☽'s horizontal diameter is greater than the diameter (Δ) seen from the centre, in the proportion of the secant of P to radius; that is, if we assume $P = 1°$, in the proportion of 1.0001523 : 1.

With the preceding value of the parallax (1°) the diameter (Δ) see p. 655, will $= 2^0 \times .27293 = 32'\,49''.9$, nearly, and accordingly the augmentation $= 32'\,49''.9 \times (\sec. 1° - 1)$

$$= 32'\,49''.9 \times .0001523$$

$$= 0''.3, \text{ nearly.}$$

It is plain, independently of any computation, that the Moon's horizontal diameter must appear larger than it would do, if seen from the centre: since the visual ray, in the latter case, is the hypothenuse, in the former, the side of a right-angled triangle. In order to find how much the Moon must be depressed, so that, if it could, it would be seen under the same angle, as when viewed from the Earth's centre, draw a line from the bisection of the radius joining the spectator and the Earth's centre, perpendicularly towards the Moon's orbit: the intersection with the orbit is the Moon's place, and the depression, below the horizon, is, as it is plain, half the Moon's horizontal parallax.

The Moon's parallax is necessary to be known for the purpose of determining, from its observed, its true zenith distance: from the true zenith distance, the Moon's north polar distance is found by adding to it the co-latitude. Lastly, from the north polar distance and right ascension, and the obliquity of the ecliptic, the Moon's longitude and latitude may be computed: and thence the elements of the orbit may be computed, or being computed, may be examined and corrected. This subject of the elements of the lunar orbit, will be briefly treated of in the ensuing Chapter.

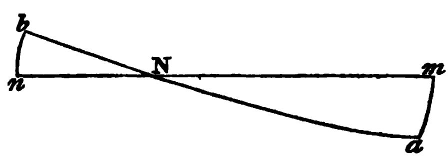
CHAP. XXXI.

On the Elements of the Lunar Orbit; Nodes; Inclination; Mean Distance; Eccentricity; Mean Motion; Apogee; Mean Longitude at a given Epoch.

The longitudes of the nodes are determined, as in the case of a planet. From the Moon's observed right ascensions and declinations, the corresponding latitudes and longitudes are computed: when the latitude is equal nothing, the Moon is in the ecliptic; in the intersection therefore of the ecliptic and its orbit: or, in other words, in its node: the longitude corresponding to such latitude (= 0) is the longitude of the node.

It will rarely happen (see p. 565,) that the latitude deduced from the meridional right ascensions, and polar distances, is exactly equal nothing: we must then, by proportion, compute the longitude corresponding to such latitude, if it may be called such. The object may be easily arrived at by the following method.

Let N be the place of the node, nNm a portion of the



ecliptic, am, bn (λ, λ') two latitudes, one to the south, the other to the north of the ecliptic: now by Naper's Rules

$$\tan. N = \frac{\tan. \lambda}{\sin. Nm} = \frac{\tan. \lambda'}{\sin. Nn};$$

$$\therefore \frac{\sin. Nm}{\sin. Nn} = \frac{\tan. \lambda}{\tan. \lambda'};$$

$$\therefore \frac{\sin. Nm - \sin. Nn}{\sin. Nm + \sin. Nn} = \frac{\tan. \lambda - \tan. \lambda'}{\tan. \lambda + \tan. \lambda'},$$

$$\text{or, } \frac{\tan. \dfrac{Nm - Nn}{2}}{\tan. \dfrac{Nm + Nn}{2}} = \frac{\sin. (\lambda - \lambda')}{\sin. (\lambda + \lambda')}.$$

$$\text{Hence, } \tan. \frac{Nm - Nn}{2} = \tan. \frac{nm}{2} \cdot \frac{\sin. (\lambda - \lambda')}{\sin. (\lambda + \lambda')},$$

from which expression, $Nm - Nn$ is known, since $Nm + Nn$, the difference of the longitudes on the two succeeding days of observation, is known: and, from the sum and difference of two quantities, we can determine the quantities themselves: in fact

$$Nm = \frac{Nm + Nn}{2} + \frac{Nm - Nn}{2},$$

$$Nn = \frac{Nm + Nn}{2} - \frac{Nm - Nn}{2}.$$

This method is capable of determining, besides the longitude of the node, the inclination of the orbit; for, since

$$\frac{\sin. Nn}{\sin. Nm} + 1 = \frac{\tan. \lambda'}{\tan. \lambda} + 1,$$

$$\frac{\tan. \lambda}{\sin. Nm} = \frac{\tan. \lambda + \tan. \lambda'}{\sin. Nm + \sin. Nn};$$

consequently,

$$\tan. N = \frac{\tan. \lambda}{\sin. Nm} = \frac{\tan. \lambda + \tan. \lambda'}{\sin. Nm + \sin. Nn}$$

$$= \frac{\sin. (\lambda + \lambda')}{\cos. \lambda \cos. \lambda' . 2 . \sin. \frac{mn}{2} . \cos. \left(\frac{Nm - Nn}{2}\right)}.$$

In which fraction, after the determination of the value of $Nm - Nn$, every thing is known.

In order to determine, whether the place of the node be fixed or not, or, if moveable, the direction and degree of its motion, repeat the above process for finding the longitude, and the difference between the two results will be, during the interval of the two observations, the motion of the node. Thus, if at the end of a month, we make a second computation of the place of the Moon's node, it will be found to have a longitude less than what it had at the beginning, by $1^\circ\ 28'$: at the end of two months, a longitude less by $2^\circ\ 55'$: and by like computations, or, rather by the comparison of very distant observations, the annual *regression* of the Moon's node, will be found to be $19^\circ\ 19'\ 43''$, and the period of the sidereal revolution of the node will be 6798 days *.

If we take the difference of two longitudes of the same node, we shall have, corresponding to the interval of time, the regression or motion of the node: if the interval be 100 years, the result will be *the secular motion* of the node. But, the mere difference of the two longitudes will not give the whole motion of the node, since the node may have regressed through several entire circuits of the heavens. For instance, in 100 years the mere difference of two longitudes is $4^s\ 14^\circ\ 11'\ 15''$: but, since the revolution of the Moon's nodes is performed in about $18^y\ 7^m$, in 100 years, besides this angle of $4^s\ 14^\circ\ 11'\ 15''$, five circumferences must have been described by the node: the proper exponent, therefore, of the secular motion of the node is

* There are certain phenomena which very plainly indicate the regression and its quickness. For instance, the star *Regulus* situated nearly in the ecliptic, (its latitude is about $27'\ 35''$,) was eclipsed by the Moon in 1757: the Moon therefore, must have been nearly in the ecliptic, and consequently, in its node. But, a few years after, the Moon, instead of eclipsing *Regulus*, passed at the distance of 5 degrees from the star. Again, if the Moon be observed at a certain time in conjunction with a star, and passing very near it, after the interval of a month, it will pass the star at a greater distance; after two months, at a still greater distance; and having reached a certain point, it will, in its conjunctions with the star, again approach it, and, at the end of about 19 years, pass it at the same distance, as at the beginning.

$$5 \times 360^\circ + 134^\circ\ 11'\ 15'' = 1934^\circ\ 11'\ 15'', (= 1934^\circ.1875.)$$

Hence, the tropical revolution of the node

$$= \frac{36000^\circ}{1934.1875} = 6798^d.54019 = 6798^d\ 12^h\ 57^m\ 52^s.416,$$

and since the equinoctial point in that time has regressed through 15′ 34″, the sidereal period is *less* than the former by nearly five days.

The annual regression of the node has been stated to be 19°.341875. This, as is plain from the mode of deducing it, is the *mean* regression. It will differ from the true annual regression, (that which belongs to any particular year, 1810, for instance,) by reason of several inequalities to which it is subject. And, as we shall hereafter see, the regression, besides its periodical inequalities, is affected with a secular inequality, by which its *mean motion* is, from century to century, retarded.

Inclination of the Moon's Orbit.

The inclination may be determined from the expression of p. 659, l. 17: or thus:

Amongst the latitudes computed from the Moon's right ascensions and declinations, the greatest, at the distance of 90° from the node, measures the inclination of the orbit. This, sometimes, is found nearly equal to 5°: at other times, greater than 5°. For instance, the greatest latitude of the new and full Moon, when at 90° from the node, is found equal to 5° nearly: but the greatest latitude when the Moon is in quadrature, and also 90° from the node, is found equal to 5° 18′. Hence the inclination of the Moon's orbit is variable: it is greatest in quadratures and least in syzygies.

Major Axis of the Moon's Orbit.

The Moon's distance is to be determined by her parallax. The method of Lacaille, described in Chap. XII, p. 325, (which is inapplicable, in the case of the Sun, on account of his great distance,) applied to the Moon, affords practical results of great exactness.

The degree of exactness is known from the probable error of observation, and the consequent error in the resulting distance: now, a variation of $1''$ in the parallax would cause a difference of about 67 miles in the determination of the distance*: therefore, as the Moon's parallax can certainly be determined within $4''$, the greatest error in the resulting distance cannot exceed 280 miles, out of about 240,000 miles.

Since, generally, the Moon's distance can be determined, her greatest and least may: and consequently, supposing her orbit to be elliptical, the major axis of the ellipse, which is the sum of the greatest and least distances, may be determined.

Eccentricity of the Moon's Orbit.

This is known from the greatest and least distances of the Moon, the apogean and perigean. Or, it may be determined from the greatest equation (see pp. 473, &c.) Its quantity, according to Lalande, (*Astronomy*, tom. II, p. 312,) is 0.055036: which gives for the greatest equation $6^{\circ}\ 18'\ 32''.076$, M. Laplace however, states the eccentricity for 1800 to be 0.0548553, which gives the greatest equation of the centre, $6^{\circ}\ 17'\ 54''.492$.

The Moon's Mean Motion.

By p. 611, the time (τ) of a synodic revolution equals $\dfrac{Pp}{P-p}$.

* Let $p =$ ☽'s parallax, then, see p. 651, ☽'s dist. $= \dfrac{\oplus\text{'s rad.}}{p}$. Let e be the error of parallax, then the corresponding errror in the Moon's distance

$$= \frac{\oplus\text{'s rad.}}{p} - \frac{\oplus\text{'s rad.}}{p+e} = \frac{\oplus\text{'s rad.}}{p}\left(1 - \frac{1}{1+\frac{e}{p}}\right)$$

$$= \frac{\oplus\text{'s rad.}}{p}\left(1 - 1 + \frac{e}{p}\right) = \frac{\oplus\text{'s rad.}}{p}\left(\frac{e}{p}\right), \text{ nearly,}$$

(rejecting the terms involving e^2, &c.) Hence, if $e = 1''$, and $p = 1^{\circ}$, and $\dfrac{\oplus\text{'s rad.}}{p}$, or the ☽'s dist. $= 240{,}000$ miles, the error $= \dfrac{1}{60.60} \times 240{,}000 = 67$ miles, nearly. In the case of *Mars*, an error of $1''$ includes in the distance an error of 40,000 miles.

Hence, if τ be computed from observation, since P the Earth's period is known, p, the Moon's, may be computed from the expression

$$p = \frac{P\tau}{P + \tau}.$$

If the Moon and Earth revolved equably in circular orbits, the above method would give accurately the Moon's period; but, since the Moon and Earth are subject to all the inequalities of a disturbed elliptical motion, the result obtained, by the above process, from one observed synodic revolution, would differ considerably from the mean period. In order, therefore, to obtain a mean period, we must observe and compute two conjunctions, or two oppositions, separated from each other by a long interval of time; and then, the interval divided by the number of synodic revolutions will give nearly the length of a mean synodic period, and very nearly indeed, if the Moon's apogee at the time of the second conjunction or opposition should be nearly in the same place in which it was, at the time of the first conjunction or opposition. From this *mean value* of the synodic period (τ), the mean period (p) may be computed from the above expression.

Now the phenomena of eclipses are very convenient for determining certain epochs of oppositions. And great certainty is obtained by their means. For, the recorded time of an eclipse by an antient Astronomer must be nearly the exact time of its happening; whereas, the assigned time of a conjunction or opposition happening long since, might, from the imperfection of instruments and methods, be erroneous, to a very considerable degree.

If we use two oppositions indicated by two eclipses, separated from each other by a short interval, we may deduce, but with no great exactness, (as has been already observed in this page,) the time of a synodic revolution. Thus, according to Cassini, a lunar eclipse happened in Sept. 9, 1718, $8^h\ 4^m$; another eclipse in Aug. 29, 1719, $8^h\ 32^m$. The interval between the two eclipses was $354^d\ 0^h\ 28^m$: and in the interval, 12 synodical revolutions had taken place; consequently, the mean length of one of these

twelve, is equal to $\frac{354^d\ 0^h\ 28^m}{12}$, equal to $29^d\ 12^h\ 2^m$.

This result cannot be exact: it is affected by the inequalities of the Moon's elliptical motion: for, independently of other causes, the place of the apogee of the Moon's orbit at the time of the second observation is distant from its place at the first by about 40^0.

In order to obtain a true mean result, we must employ eclipses very distant, in time, from each other. Such are, an eclipse recorded by Ptolemy to have been observed by the Chaldeans in the year 720 before Christ, March 19, $6^h\ 11^m$ (mean time at Paris, according to Lalande,) and an eclipse observed at Paris in 1771, Oct. 23, $4^h\ 28^m$. The interval between the eclipses, is 910044 days minus $1^h\ 43^m$, and expressed in seconds, 78627795420^s. In this interval 30817 synodic revolutions had happened; the mean length of one of these, then,

$$= \frac{78627795420^s}{30817} = 29^d\ 12^h\ 44^m\ 2^s.2.$$ Substituting this value in the expression, p. 663, l. 4, we may obtain the value of p.

The value of the synodic period, computed from different observations, is not always of the same magnitude. Its *mean* length therefore is subject to a variation, arising from a cause called the *Acceleration of the Moon's Mean Motion*, which will be hereafter explained.

According to M. Laplace, the mean length of a synodic revolution of the Moon for the present time, is

$$29^d\ 12^h\ 44^m\ 2^s.8032\ (= 29^d.530588).$$

The periodic revolution of the Moon computed from the expression of p. 663,

$$= \frac{365.242264 \times 29.530588}{365.25 + 29.530588} = 27^d.321582$$

$$= 27^d\ 7^h\ 43^m\ 4^s.6848.$$

This is the *tropical* revolution of the Moon, or the revolution with respect to the equinoxes, since the number which was substituted for P was 365.242264, which expresses the Earth's tropical revolution.

The diurnal tropical movement of the Moon

$$= \frac{360^\circ}{27.321582} = 13^\circ.17636 = 13^\circ\ 10'\ 34''.896.$$

The *sidereal* revolution of the Moon differs from the tropical, for the same reasons, (see p. 198,) as the sidereal year differs from the tropical: and the difference must be computed on similar principles: thus, the mean precession of the equinoxes being $50''.1$ in a year, or about $4''$ in a month, the sidereal revolution of the Moon will be longer than the tropical, by the time which the Moon, with a mean diurnal motion of $13^\circ.17636$, takes up in describing $4''$: which time is nearly 7^s. The exact length of a sidereal revolution is $27^d\ 7^h\ 43^m\ 11^s.510$, $(= 27^d.321661)$*.

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* We may easily deduce a formula of computation: thus, let p be the Moon's tropical revolution $(= 27^d.321582,)$ and x the sidereal period to be investigated; then, the arc of the precession described in the time $= \frac{50''.1 \times x}{365.25}$,

and the time of the Moon's describing it $= \frac{p}{365.25} \times \frac{50''.1}{360^\circ} \times x$.

Hence, $x = p + \frac{p}{365.25} \times \frac{50''.1}{360^\circ} \times x$, and thence

$$x = \frac{p}{1 - \frac{p}{365.25} \times \frac{50''.1}{360^\circ}},$$

= (expanding)

$$p \left\{ 1 + \frac{p}{365.25} \times \frac{50''.1}{360^\circ} + \left(\frac{p}{365.25}\right)^2 \times \left(\frac{50''.1}{360^\circ}\right)^2 + \&c. \right\}$$

in which, since $\frac{p}{365.25} \times \frac{50''.1}{360^\circ}$ is a very small quantity, two terms will be sufficient to give a value of x sufficiently near.

The same series may be used for determining the length of the sidereal

Since the equinoctial point (from which longitudes are measured) regresses, the Moon departing from a point, where its longitude is $= 0$, returns to a point at which its longitude is again $= 0$, before it has completed a revolution amongst the fixed stars. In like manner, the node of the Moon's orbit regressing, and faster than the equinoctial point, the Moon, quitting a node, will return to the same before completing a revolution amongst the fixed stars, and in a period less than the *tropical.*

This period may be thus found; the diurnal tropical movement of the Moon is $13^\circ\ 10'\ 34''.896$, and that of the node (see p. 661,) $= \dfrac{19^\circ.341875}{365.242264} = 3'\ 10''.6386$. Hence, the diurnal separation, which is the sum of the above quantities since the node regresses, $= 13^\circ\ 13'\ 45''.535$*: and consequently,

$$13^\circ\ 13'\ 45''.535 : 360^\circ :: 1^d : 27^d\ 5^h\ 5^m\ 35^s.6,$$

the revolution of the Moon with respect to its node.

This latter revolution may also be found by the aid of the formula given in the Note to p. 665.

By like processes, from the ascertained quantity of the apogee of the Moon's orbit, we may determine the *anomalistic* revolution

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sidereal from the tropical year, by substituting for p, $365^d.25$: in that case, the length of the sidereal year

$$= 365.25 \left(1 + \frac{50''.1}{360^\circ} + \&c.\right)$$

and a like series would serve to determine the length of an anomalistic year, substituting instead of $50''.1$, the quantity expressing the progression of the apogee.

* The Moon's motion with regard to its node may be found from eclipses; for, when these are of the same magnitude, the Moon is at the same distance from the node. Hipparchus, by comparing the eclipses observed from the time of the Chaldeans to his own, found that in 5458 lunations, the Moon had passed 5923 times through the node of its orbit: thence he deduced the daily motion of the Moon with regard to its node, to be $13^\circ\ 13'\ 45''\ 39'''\frac{2}{3}$. See Lalande, tom. II, p. 189.

of the Moon, M. Lalande (*Astronomie*, tom. II, p. 185,) states it to be $27^d\ 13^h\ 18^m\ 33^s.9499$, but M. Delambre, $27^d\ 13^h\ 18^m\ 37^s.44$ ($= 27^d.5546$.)

There is another revolution, of some consequence in the lunar theory, called the *Synodic Revolution of the Node:* this is completed when the Sun departing from the Moon's node first returns to the same. It is to be computed as the preceding periods have been. Thus, since the mean daily increase of the Sun's longitude is $59'\ 8''.33$, and the daily regression of the node is $3'\ 10''.638$, the sum of these quantities, which is the separation of the Sun from the node in a day, is $1^\circ\ 2'\ 18''.96$. Hence, $1^\circ\ 2'\ 18''.96 : 360^\circ :: 1^d : 346^d\ 18^h\ 28^m\ 16^s.032$ ($= 346^d.61963$ *.)

We will now exhibit, under one point of view, the different kinds of lunar periods and motions :

Synodic revolution	29^d	12^h	44^m	$2^s.8032$	$= 29^d.530588$
Tropical	27	7	43	4.6848	.. 27.321582
Sidereal...............	27	7	43	11.5101	.. 27.321661
Anomalistic	27	13	18	37.44	.... 27.5546
Revol^n. in respect of node	27	5	5	35.6	..... 27.212217
Tropical revolu^n. of node	6798^d	12^h	57^m	$52^s.416$	6798.54019
Sidereal	6793	10	6	29.952	..6793.42118

☽ 's mean tropical daily motion	13°	$10'$	$34''.896$
☽ 's sidereal daily motion	13	10	35.034
☽ 's daily motion in respect to the node	13	13	45.534

Place of the Apogee.

The Moon's diameter is least at the apogee, and greatest in the perigee : and since the diameter can be measured by means

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* This and the preceding periods are frequently found on like principles, but by different expressions, from the values of the *secular motions.* Thus, in 100 Julian years, each consisting of $365^d.25$, the *secular motion* of the Sun is $36000^\circ\ 45'\ 45''$ ($36000^\circ.7624998$) and the secular motion of the node (see p. 661,) $1934^\circ.1875$: and the sum of these is $37934^\circ.95$ nearly : thence $37934.95 : 360 :: 100 :$ period $= \dfrac{36000}{37934.95}$.

of a micrometer, or can be computed from the time it takes up in passing the vertical wires of a transit instrument, the times of the least and greatest diameter, or the times when the Moon is in her apogee and perigee, can be ascertained. Instead of endeavouring to ascertain when the Moon's diameter is the least, Lalande, *Astron.* tom. II, p. 162, says, that it is preferable to observe the diameters towards the Moon's mean distances when the diameter is about 31′ 30″. If two observations can be selected when the diameter was of the same quantity, then we may be sure that, at these two observations, the Moon was at equal distances from the apsides of its orbit. The middle time then between the two observations is that in which the Moon was in her apogee.

By finding the places of the apogee, according to the preceding plan, and comparing them, it appears that the apogee of the Moon's orbit is progressive *: completing a sidereal revolution in $3232^d\ 11^h\ 11^m\ 39^s.4$, and a tropical, in $3231^d\ 8^h\ 34^m\ 57^s.1$. Laplace states the sidereal revolution of the apogee to be $3232^d.579$, that is, $3232^d\ 13^h\ 53^m\ 45^s.6$. (See *Exposition du Systeme du Monde*, Edit. 2, p. 20.)

Mean Longitude of the Moon at an assigned Epoch.

By observations on the meridian, the right ascension and declination of the Moon are known; thence may be computed, the Moon's longitude. This resulting longitude is the true longitude, differing from the mean by the effect of all the inequalities, elliptical, as well as those that arise from the perturbations of the Sun and planets. The *mean* longitude therefore, is the difference of the true longitude and of the sum (mathematically speaking) of the *equations* due to the inequalities. In order, therefore, to determine the mean longitude, the lunar theory must be known to some degree of exactness. Any new inequality discovered will affect the previous determination of the mean motion: and accordingly, keeping pace with the continual improvements in the lunar theory, repeated alterations have been made in the quantity of the mean longitude. In the last Lunar French Tables,

* See *Physical Astronomy*, Chap. XIII.

the epoch of the mean longitude for Jan. 1, 1801, midnight at Paris, is $3^s\ 21^\circ\ 36'\ 30''.6$: which for Greenwich, Jan. 1, at noon, is $3^s\ 28^0\ 16'\ 56''.1$.

In order to determine the eccentricity of the Moon's orbit, considered as elliptical, and the deviations from the elliptical form caused by the actions of the Sun and planets, it is necessary to know the angular spaces described by the Moon, in her orbit. Such spaces are not immediately given by observation. We must make several steps to arrive at them. The first is the determination of the Moon's parallax: the second, the observation of the Moon's right ascension and zenith distance: the third, the correction of the zenith distance on account of parallax, in order to obtain the true declination. The fourth, the computation of the Moon's latitude and longitude: the fifth, the *reduction* of the Moon's longitude to a longitude on her orbit, to be effected by the same formula (see pp. 501, &c.) as that of the *reduction of the ecliptic to the equator*.

The comparison of the *reduced* longitudes, or the comparison of the arcs of the Moon's orbit, described in certain times, will shew us how much such arcs, with respect to their forms and laws of description, differ from elliptical arcs. This point will be considered in a subsequent Chapter. In the next we will advert to certain *secular* inequalities (arising, indeed, from the same source as the Moon's periodical inequalities) that affect those elements of the orbit, which we have just considered.

CHAP. XXXII.

On the Secular Equations that affect the Elements of the Lunar Orbit.

THE correction, which is called a *Secular Equation*, is strictly speaking periodical, but requiring a very large period, in order to pass through all its degrees of magnitude before it begins to recur. Its quantity, in general, is very small, and usually expounded by its aggregate in the space of 100 years.

The nodes, the apogee, the eccentricity, the inclination of the Moon's orbit, the Moon's mean motion, are all subject to secular inequalities. And the practical mode of detecting these inequalities is nearly the same in all.

If we subtract the longitude of the Moon's node now, from what it was 500 years ago, the difference is the regression of the node in that interval: the mean annual regression is the above difference divided by 500. If we apply a similar process to an observation of the Moon's node, made now, and to one made 1000 years ago, the result must be called, as before, the mean annual regression of the node; and this last result ought, if the regression were always equable, to agree with the former: if not, (as is the case in nature,) the difference indicates the existence of a *secular inequality*, requiring for its correction a *secular equation*.

By a similar method the motion of the perigee of the Moon's orbit does not appear to be a mean motion, but subject to a secular inequality.

But the most remarkable inequality is that which has been detected in the Moon's mean motion, and which is now known by the title of the *Acceleration of the Moon's mean Motion*. The

fact of such acceleration was first ascertained by Halley, from the comparison of observations: the cause of the acceleration has been assigned by Laplace *. Although the method of detecting the existence of these inequalities does not differ, in principle, from methods just described, yet, on account of its importance, we will endeavour to explain it more fully.

As we have before remarked, eclipses are a species of observations on which we may rely with great certainty; quite distinct from merely registered longitudes which must partake of all the imperfections of methods used at the times of their computation. Now, in the year 721 before Christ, with a specified day and hour, Ptolemy records a lunar eclipse to have happened. The Sun's longitude then being known, the Moon's, which must at the time of the eclipse differ from it by six signs, is known also. The Moon's longitude however, computed for the time of the eclipse and by means of the Lunar Tables, does not agree with the former †. In some part or other, then, the Tables are defective, or, without some modification, are not applicable to ages that are past.

The Moon's place computed from the eclipse is advanced beyond the place computed from the Tables by $1^{\circ}\ 26'\ 24''$; an error too great to be attributed to any inaccuracies in the coefficients of the equations belonging to the *periodic* inequalities, and which would seem rather to be the aggregate, during many years, of a small error in some reputed constant element, such, for instance, as the Moon's mean motion.

On the hypothesis then of an acceleration in the Moon's motion, or, in other words, if we suppose the Moon now to move more rapidly than it did 2000 years ago, the error of $1^{\circ}\ 26'\ 24''$ can be accounted for. With a mean motion too large, we should

* See Laplace, *Exposition du Syst. du Monde*, Edit. 2, pp. 20, 214, &c. also *Mec. Celeste*, pp. 175, &c. Lalande, tom. II, p. 185: Halley, *Phil. Trans.* Nos. 204, and 218, Newton, p. 481, Ed. 2. and Woodhouse's *Phys. Astron.* Chap. XXII.

† The *true* longitudes are not compared, but the *mean*.

throw the Moon too far back in its orbit. And, with the same motion, but for a point of time less remote than the preceding, we ought, if the hypothesis of the acceleration be true, to throw the Moon less far back in her orbit: for that would produce an error of the *same kind* as the one already stated, (p. 671). Now this is the case, and has been ascertained to be so, by means of an eclipse observed at Cairo by *Ibn Junis*, towards the close of the tenth century.

The acceleration of the Moon's motion therefore, discovered by Halley, may be assumed as established: or, in other words, in the former estimates of the quantity of the Moon's motion, a large secular inequality was included, which it is now necessary to deduct, in order that what remains may be truly a *mean motion*.

The variation in the mean motion of the Moon, will, it is plain, affect the durations of its synodic, tropical, and sidereal revolutions.

With this secular equation in the Moon's mean motion, the equations in the motions of the nodes and of the apogee are connected. The latter are subtractive, whilst the former is positive; and, according to Laplace, *Mec. Celeste*, tom. III, p. 236,) the secular motions of the perigee, of the nodes and mean motion, are to each other, as the numbers 3.00052, 0.735452, and 1.

The mean anomaly of the Moon, which is the difference of her mean longitude and the mean longitude of the apogee, must be subject to a secular equation, which is the difference of the secular equations affecting the longitudes of the Moon and of the apogee.

All quantities, in fact, dependent on the Moon's mean motion, the apogee and nodes, must be modified by their secular equations.

The Moon's distance from the Earth, the eccentricity and inclination of her orbit, are, according to M. Laplace, also affected with secular equations connected with that of the mean motion. But, the major axis is not. (See *Physical Astron.* Chap. XXIII.)

We will, in the next Chapter, explain briefly the origins, quantities, and variations of those inequalities, which during a month, a year, and the periodical revolution of the nodes, render the Moon's true place different from its elliptical, or, more generally speaking, from its mean place.

CHAP. XXXIII.

On the Inequalities affecting the Moon's Orbit.—The Evection.—Variation.—Annual Equation, &c.—The Inequalities of Latitude and Parallax.

By a comparison of the Moon's longitudes and of her distances deduced from her parallaxes, it appears that the lunar orbit is nearly an ellipse with the Earth in one of the foci. It appears also, that the Moon not only wanders from the ellipse which may be traced out as her mean orbit, and transgresses the laws of elliptical motion, but, that the ellipse itself is subject, in its dimensions, to continual variation: at one time, contracted within its mean state, at another, dilated beyond it.

In strictness of speech, neither the Earth's orbit nor the Moon's are to be called ellipses. If they are considered as such, it is purely on the grounds of convenience. It is *mathematically* commodious, or it may be viewed as an artifice of computation, first, to find the approximate place of each body in an assumed elliptical orbit, and then to compensate the error of the assumptions, and to find a truer place, by means of corrections, or, as they are astronomically called, *Equations*.

In a system of two bodies, when forces, denominated centripetal, only act, an accurate ellipse is described by the revolving round the attracting body; and, in such a system, the apsides, the eccentricities, the mean motions, &c., would remain perpetually unchanged. The introduction of a third, or of more bodies, and the consequent introduction of *disturbing forces*, destroys at once the beautiful simplicity of elliptical motion, and puts every element of the system into a state of continual mutation. Yet, the change and the departure from the laws of elliptical motion, are less in some cases than in others. The Earth's orbit ap-

proaches much more nearly to the form of an ellipse than the Moon's. The Sun's longitude, as we have seen in p. 496, computed by Kepler's Problem, did not differ from the true place by more than seven seconds: and that quantity, in those circumstances, represented the perturbations of the planets; and, the equations representing the perturbations were only four. But, in the case of the Moon, one inequality alone will require an equation nearly equal to two degrees, and the number of equations amounts to 28.

The quantity of perturbation, and the difficulty of computing it, depend less on the number than on the proximity of the disturbing bodies. In the case of the Sun, one equation suffices for the perturbation of *Venus*, and another for that of *Jupiter*. But, all the equations compensating the inequalities in the Moon's place, arise from different modifications of the Sun's disturbing force. It is not, however, solely the proximity, but the mass of the disturbing body, that gives rise *to equations*. The strictly *mathematical* solution of the problem of the three bodies (see Chap. XX.) is equally difficult, whatever be the mass of the disturbing body. The *practical* difficulty of merely approximating to the true place of the disturbed body, is very considerably lessened by supposing that mass to be small.

If we consider the subject merely in a mathematical point of view, the Moon's place, at any assigned time, results from the compound action of the Earth's centripetal force and the Sun's disturbing force; and the deviation from her place in the exact ellipse, arises entirely from the latter. We are at liberty to call the deviation, or error, one uncompounded effect: yet, since the quantity of the deviation cannot be computed from one single analytical expression, but must be so, by means of several terms, we may separate and resolve the effect into several, (analogous to the above-mentioned terms,) the causes of some of which we may distinctly perceive and trace in certain simple resolutions and obvious operations of the Sun's disturbing force.

Long before Newton's time and the rise of Physical Astronomy, this separation, or resolution of the error of the Moon's place from her elliptical place was, in fact, made. And, the

error was said to arise from three inequalities, distinguished by the titles of *Evection*, *Variation*, and *Annual Equation*.

These three inequalities were noted because they rose, under certain circumstances, to a conspicuous magnitude; and, were distinguished from each other, because they were found to have an obvious connexion with certain positions of the Sun and Moon and of the elements of their orbits. Although their real physical cause was not discovered, yet the laws of their variation were ascertained.

The other lunar inequalities have not, like the three preceding, been distinguished by titles. This is owing principally to their want of historical celebrity; they were not detected like the others, by reason of their minuteness and the imperfection of antient instruments and methods.

Some explanation has already been given, (Chaps. XIV, XV,) of the principles and modes of detecting and decompounding inequalities. The difference between an observed and computed place, indicates the operation of causes either not taken account of, or not properly estimated in the previous computation.

Take, for instance, the Moon: her mean place, computed from her mean motion, differs from her observed place; and the difference, if we suppose her to move in an elliptical orbit, is the equation of the centre, or, of the orbit, called the *First Lunar Inequality*.

Compute the Moon's place from a knowledge of her mean motion and of the equation of the centre, and then compare the computed, with the observed, place. In certain situations, a great difference will be noted between the places, ascending in its greatest value to nearly $1^{\circ}\ 18'\ 3''$. This difference is chiefly owing to the *Evection* discovered by Ptolemy, and named the *Second Lunar Inequality*.

In like manner, we may conceive the *Third Lunar Inequality* to be discovered. But, we will now proceed to consider more particularly the second inequality; the mode of ascertaining its

maximum; its general effect; the formula expressing the law of its variation; and its cause, reckoning as such, some particular modification of the Sun's disturbing force.

Evection. (See *Physical Astronomy*, pp. 236, &c.)

This inequality has a manifest dependence on the position of the apogee of the Moon's orbit. Let us suppose the Moon to quit the apogee, the line of the apsides to lie in syzygy, and that we wish to compute the Moon's place 7 days after her departure from syzygy, when, in fact, she will be nearly in quadratures. The Moon's place, computed by deducting the equation of the centre*, (then nearly at its greatest value and $= 6^0\ 37'\ 54''.492$,) from the mean anomaly (see Chap. XVIII.) will be found *before* the observed place by more than 80 minutes; in other words, the computed longitude of the Moon is so much greater than the observed longitude. But, if we suppose the apsides to lie in quadratures, the Moon's place, 7 days after quitting her apogee, computed, as before, by subducting the equation of the centre from the mean anomaly, will be found *behind* the observed place by more than 80 minutes; in other words, the computed longitude of the Moon is so much less than the observed.

It is an obvious inference, then, from these two instances, that some inequality, besides that of the elliptic anomaly, and having a marked connexion with the longitude of the lunar apogee, affects the Moon's motion.

What, from the two preceding instances, would be an obvious inference to an Astronomer acquainted solely with the elliptic theory of the Moon? In the first case, the computed place being *before* the observed, it would *seem* that the equation of the centre, to be subducted from the mean anomaly, had not been taken of sufficient magnitude; in the latter case, it would *seem* that the equation of the centre had been taken too large.

Let us take another case: suppose, instead of comparing the computed with the observed place, that it was intended to deduce

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* The anomlay is here supposed to be reckoned from apogee.

the quantity of the equation of the centre from an observation of the Moon in syzygy. In that case, the equation of the centre, reckoned as the difference of the true and mean longitudes, would result *too small* a quantity. And this circumstance has really happened. For, the antient Astronomers who determined the elements of the lunar orbit by means of eclipses, when the Moon is in syzygy, have assigned too small a quantity to the equation of the centre.

In the preceding instance, when the Moon is in syzygy and the apsides in quadrature, the determination of the equation of the centre would be too small by the maximum value of the *Evection* ($1^0\ 20'\ 29''.5$). But, in other positions of the apsides, the effect of the evection is to lessen, though not by its whole quantity, the equation of the centre.

Astronomers, having found that the augmentation and diminution of the equation of the centre arose from an inequality, soon ascertained the inequality to be periodical; in other words, that, after passing through all its degrees of magnitude, from 0 to its maximum value, it would recur. Now, of such recurring quantities the cosines and sines of angles are most convenient representations; for instance, $\pm K \cdot \sin. E$ is competent to represent the *Evection:* its maximum value is K, when $E = 90^0$: and it is nothing, when E is. If then, the value of K could be assigned and the form for E, the numerical quantity of the Evection could be always exhibited. After the comparison of numerous observations, and after many trials, it was found that

$$K = 1^0\ 20'\ 29''.5, \text{ and } E = 2(☽ - ☉) - A,$$

A representing the mean anomaly of the Moon, and ☽ − ☉ signifying the angular distance of the Sun and Moon, or, the difference of their mean longitudes viewed from the Earth.

In the *equation*

$$1^0\ 20'\ 29''.5 \cdot \sin.\ [2(☽ - ☉) - A],$$

$1^\circ\ 20'\ 29''.5$ is called the *coefficient*, and $2(☽ - ☉) - A$ the *argument*.

If we represent the equation of the centre by

$$(6^0\ 17'\ 54''.49) \sin.\ A,$$

in which, the coefficient $6^\circ\ 17'\ 54''.49$, is the greatest equation, and A (the mean anomaly) the argument, the Moon's longitude expressed by means of the two equations, that of the centre *, and the evection, would stand thus:

$$☽\text{'s longitude} =$$

$$☽\text{'s mean long.} - (6^\circ\ 17'\ 54''.49)\ \text{sin.}\ A$$
$$- (1^\circ\ 20'\ 29''.5)\ \text{sin.}\ [2(☽ - ☉) - A];$$

now in syzygies $☽ - ☉ = 0$; $\therefore \text{sin.}[2(☽ - ☉) - A] = -\text{sin.}\ A$; consequently, in this case, the former expression becomes

$$☽\text{'s longitude} =$$

$☽$'s mean long. $- (6^\circ\ 17'\ 54''.49)\ \text{sin.}\ A + (1^\circ\ 20'\ 29''.5)\ \text{sin.}\ A$, in which, the argument for the Evection assumes that form, which is the general one of the *equation of the centre;* and on this account, the former is sometimes said to *confound* itself with the latter, in syzygies. It also seems to lessen it, since the preceding expression may be put under this form,

$$☽\text{'s longitude} =$$

$☽$'s mean long. $- (6^\circ\ 17'\ 54''.49 - 1^\circ\ 20'\ 29''.5)\ \text{sin.}\ A$, in which, the coefficient of sin. A would be the difference of the two coefficients $6^\circ\ 17'\ 54''.49$, and $1^\circ\ 20'\ 29''.5$; and, accordingly, A being the argument of the *Equation of the Centre,* that equation would appear to be lessened.

The Evection itself, and, very nearly, its exact quantity, were discovered by Ptolemy in the first century after Christ, but the cause of it remained unknown till the time of Newton. That great Philosopher shewed that it arose from *one kind of alteration* which the Moon's centripetal force towards the Earth receives from the Sun's perturbation. Let us see how it may be explained:

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* If A be the mean anomaly, the equation of the centre cannot be represented by a single term such as a sin. A, but by a series of terms, such as $a\ \text{sin.}\ A + b\ \text{sin.}\ 2A + c\ \text{sin.}\ 3A + \&c.$ in which, however, the coefficients b, c, &c. decrease very fast.

When the line of the apsides is in syzygies, the Equation of the centre (p. 677,) is increased. The Equation of the centre depends on the eccentricity; (see pp. 662,) an increase therefore in the former would indicate an increase in the latter. Hence, if it can be shewn that the Moon's orbit must, when the line of the apsides is in syzygies, be made more eccentric by the action of the Sun's disturbing force, an adequate explanation will be afforded of the increase of the equation of the centre above its mean value; which increase is styled the *Evection*.

Again, when the line of the apsides is in quadratures, the Equation of the centre is lessened: the eccentricity therefore (see expression, p. 473,) is lessened: and now, in order to afford an explanation, it is necessary to shew that, in this position of the line of the apsides, the Sun's disturbing force necessarily renders the orbit less eccentric.

The Sun's disturbing force admits of two resolutions, one in the direction of the radius vector of the Moon's orbit: the other in the direction of a tangent to the orbit. The former sometimes augments, at other times, diminishes the gravity of the Moon towards the Earth, and always (see Newton, Sect XI, Prop. 66,) proportionally to the Moon's distance from the Earth. When the Moon is in syzygy, it diminishes; consequently, in the first case, when the line of the apsides is also in syzygy, the perigean gravity, which is the greatest, (since it varies inversely as the square of the distance) is diminished, and by the least quantity; the apogean gravity, the least, is also diminished, but by the greatest quantity: the disproportion therefore between the two gravities is augmented; the ratio between them becomes greater than that of the inverse square of the distance: the Moon, therefore, if moving towards perigee, is brought to the line of the apsides in a point between its former and mean place and the Earth: or, if moving towards apogee, reaches the line of the apsides in a point more remote from the Earth than its former and mean place. The orbit then becomes more eccentric; the equation of the centre is increased; and, the increase is the *Evection*.

Thus is the first case accounted for. In the second, the Sun's resolved force *increases* the gravity of the Moon towards

the Earth, and, as it has been said, proportionally to the distance. The perigean gravity, therefore, which is the greatest, is increased by the least quantity; the apogean, the least, is also increased, and by the greatest quantity. The disproportion, therefore, between these two gravities is lessened; the ratio between them is less than that of the inverse square of the distance. The Moon, therefore, if moving towards perigee, meets the line of the apsides, in a point more remote from the Earth than the mean place of the perigee: if moving towards the apogee, in a point between the Earth and the mean place of the apogee. The orbit, by these means, becomes less eccentric; the Equation of the centre is diminished, and, the diminution is the *Evection*.

We will now proceed to consider the third inequality called

The Variation. (See *Physical Astronomy*, pp. 217, &c.)

By comparing the Moon's place computed, from her mean motion, the equation of the centre, and the Evection, with her observed place, Tycho Brahe, in the sixteenth century, discovered that the two places did not always agree. They agreed only in opposition and conjunction, and varied most, when the Moon was half way between quadratures and syzygies, that is, in *Octants*. At those points the new inequality seemed to be at its maximum value (35′ 41″.6).

It appeared clearly from the observations, that this new inequality was connected with the angular distance of the Sun and Moon: and that its *argument* must involve, or, be some function of, that distance. At length, it was found, that the equation due to the inequality, was

$$(35' \; 41''.6) \,.\, \sin. 2\,(☽ - ⊙)$$

35′ 41″.6 being the *coefficient*, and 2 (☽ − ⊙) the *argument*.

According to the above form, the variation is 0 in syzygies and in quadratures, and at its maximum (35′ 41″.6) in octants.

If now, by means of this new equation, we farther correct the expression (p. 679,) for the Moon's place, we shall have

$$☽\text{'s longitude} =$$
$$☽\text{'s mean longitude} - (6^\circ\ 17'\ 54''.49)\ \sin.\ A$$
$$- (1\ \ 20\ \ 29.5)\,.\ \sin.\ [2\,(☽ - ⊙) - A]$$
$$+ (35'\ 41''.6)\,.\ \sin.\ 2\,(☽ - ⊙).$$

We will now proceed to Newton's explanation of the cause of this inequality.

One effect, from a resolved part of the Sun's disturbing force, we have already perceived in the Evection. The *Variation* is occasioned by the other resolved part, that which acts in the direction of a tangent to the Moon's orbit. This latter force will accelerate the Moon's velocity in every point of the quadrant which the Moon describes, in moving from quadrature to conjunction. The force will be greatest in octants and nothing in conjunction; and, when the Moon is past conjunction, the tangential force will change its direction, and retard the Moon's motion. The greatest acceleration, therefore, of the Moon's velocity must happen in syzygy: exactly at the termination or cessation of the accelerating force. At that point, therefore, the Moon's velocity must differ most from her mean, or, rather, from that velocity which she would have, if the effect of the accelerating tangential force were abstracted. When the Moon moves from that point, her place at the end of any portion of time, a day, for instance, will be beyond her mean place, or beyond the place of an imaginary Moon endowed with a motion from which the effect of *Variation* is abstracted. At the end of the second portion of time, the real Moon will have described a space less, by reason of the retarding force (see l. 15,) than the space described in the first, but, still, greater than the space described by the imaginary Moon; so that, at the end of the second portion of time, the two Moons will be distant from each other, by the effect of two separations; and, for succeeding portions of time, the real Moon will still continue describing greater angular spaces than the imaginary Moon, and the separation of the two Moons, which is the accumulation of the individual excesses, will continue, till the retarding force, by the continuance of its action, and the increase of its quantity, shall have reduced the Moon's velocity to its mean state: at that term which is the octant, the separation will cease to increase, and will

be at its greatest. And this greatest separation, 35′ 41″.6, is the maximum effect of the *Variation:* and the separation, previously described, in any point between conjunction and octants, is its common effect.

The preceding reasoning is precisely similar to that which was used in p. 469, on the subject of the greatest equation of the centre. At the apogee, the mean velocity differs most from the true, and then the two Suns are together; and, they are most separated, when the real Sun moves with its mean angular velocity.

We will now proceed to a fourth inequality called,

The Annual Equation. (See *Physical Astronomy*, pp. 237, &c.)

The two former inequalities, of which the periods are short, may be ascertained by observing the Moon during one revolution. But, in order to detect this fourth inequality, it is necessary to compare similar positions of the Moon, computed according to the theory of the three preceding inequalities, in different months of the year. If the computed place agreed with the observed place in January, it would not in March, and it would most differ in July. The inequality was soon found to have a connexion with the Earth's distance from the Sun, and its *equation* was at length found to be

$$11' \; 11''.97 \times \text{sin. } \odot\text{'s mean anomaly},$$

11′ 11″.97 being the coefficient, and ☉'s mean anomaly the argument.

According to the preceding form, the maximum (11′ 11″.97) of the annual equation happens when the Sun's mean anomaly is = 90°, or 270°. The equation is nothing, either when the Earth is in the aphelion or perihelion of its orbit.

If now, by means of this new equation, we farther correct the expression for the Moon's longitude, we shall have

$$\begin{aligned} &☽\text{'s longitude} = \\ ☽\text{'s mean longitude} &- (6^\circ \; 17' \; 54''.49) \text{ sin. } A \\ &- (1^\circ \; 20' \; 29''.5) \text{ sin. } [2(☽ - \odot) - A] \\ &+ \quad (35' \; 41''.6) \text{ sin. } 2(☽ - \odot) \\ &+ \quad (11' \; 11''.97) \text{ sin. } \odot\text{'s mean anomaly}, \end{aligned}$$

(see *Physical Astronomy*, p. 239.)

The annual equation is produced by the different positions of sun and earth to each other in the course of the motion of [illegible] and [illegible]

We will now proceed to an explanation of the cause of this inequality.

The *Variation* has been explained from the effect of that resolved part of the Sun's disturbing force which acts in the direction of the tangent; the *Evection*, from the effect of the resolved part in the direction of the radius vector, and which effect alters the ratio of the perigean and apogean gravities from that of the inverse square of the distance. The present inequality depends not, on any immediate effect, either of the one, or of the other resolved part; but on *an alteration in the mean effect* of the disturbing force in the direction of radius; and, which mean effect lessens the gravity of the Moon towards the Earth.

By the mean effect, that is meant to be understood, which is the result of the disturbing forces in the direction of the radius in one revolution. The disturbing force does not always diminish the Moon's gravity to the Earth; it does in opposition and conjunction, but it augments the gravity in quadratures (see Newton Sect. XI; Prop. 66). The augmentation however, is only half the diminution (Newton, Prop. 66, Cor. 7). In the course therefore of a synodic revolution, there results, what may be called a mean force tending to diminish the Moon's gravity to the Earth, the measure of the mean force being equal to (see Newton, Prop. 66.)

$$\frac{\odot\text{'s mass} \times \text{rad. } \rightmoon\text{'s orbit}}{\text{cube } \oplus\text{'s dist. from } \odot}.$$

By reason of this diminution, the Moon is enabled to preserve a greater distance from the Earth than it could do, by the influence of gravity alone. But, since the disturbing force acts in the direction of the radius, the equal description of areas is not altered (see Newton, Prop. 66). The area however varying as the product of the radius vector and the arc (the measure of the real velocity) and the former (see l. 26.) being increased, the real velocity must be diminished: so also must the angular, which varies inversely as the square of the distance.

These results are derived from that effect of the disturbing force of the Sun, which is a mean effect diminishing the Moon's gravity. If this mean effect of diminution be increased, similar

results will follow, but in an enlarged degree; the Moon's angular velocity will be still more diminished and her distance from the Earth increased: now the measure of the mean effect is

$$\frac{\odot\text{'s mass} \times \text{rad. ☽'s orbit}}{(\oplus\text{'s distance from }\odot)^3},$$

which will be increased, by diminishing the denominator: and is, therefore, in nature, increased when the Earth approaches the Sun. That circumstance happens in winter. In winter, therefore, the Moon's gravity to the Earth is more diminished, by the Sun's disturbing force, than in summer. Her angular velocity therefore is more diminished. A greater time is requisite to the description of a complete revolution round the Earth: in other words, a periodic month is longer in winter than in summer. Now, as the Earth approaches the Sun, its velocity increases. An acceleration therefore of the Earth's motion is attended, by reason of this new inequality, with a retardation of the Moon's, and reversely. On this account it is that, the *Annual Equation* is said to resemble the equation of the Sun's centre. For, supposing the Sun to be approaching his perigee, then his place (reckoning from apogee and neglecting the perturbations of the planets) is equal to the mean anomaly − the equation of the centre (E), E *decreasing* as the Sun approaches the perigee; if m be the Moon's place independently of the *annual equation* (e), then her place, corrected by that, is $m + e$, e increasing (since it varies as sin. ⊙'s mean anomaly,) and affected with a contrary sign.

When the annual equation is $\pm$ (11′ 11″.976) sin. ⊙'s mean anomaly, the corresponding Equation of the centre for the Sun is (1° 55′ 26″.3748) sin. ⊙'s mean anomaly.

We have now gone through the explanation of the three principal lunar inequalities, which were discovered before the time of Newton and the rise of Physical Astronomy. These inequalities were, by reason of their magnitude, *fished out*, (as a late writer has significantly expressed it) from the rest. The discovery of the rest, in number 28 *, is entirely due to Physical

* Strictly speaking there are more than 28. But Astronomers have *confined* themselves to this number, since other equations, that analytically present themselves, never rise to a numerical value worth considering.

Astronomy. Without the aid of this latter science, it would have been, perhaps, impossible, from mere observation and conjecture, to have assigned the forms of the arguments. These latter being ascertained, it is the proper business of observation to assign the numerical value of their coefficients.

The three equations that have been explained are, with regard to their magnitudes, eminent above the rest; but, it must not be forgotten, the other equations, on the footing of theory, are of equal importance, and in practice, considering the use that is now made of the Lunar Tables, of very essential importance.

The three equations, with all the others, are derived from theory by the same process. And, as we have seen, the causes of the former may, independently of any formal calculation, be discerned in certain modifications of the Sun's disturbing force. The causes of the other equations are not so easily discernible: yet, the sources of some of them may be pointed out in certain changes, which the conditions or circumstances belonging to the three principal equations must necessarily undergo.

For instance, suppose the Moon and the line of the nodes to be in syzygies; then, the Sun's disturbing force, represented by part of a line joining the Sun and Moon, lies entirely in the plane of the Moon's orbit; and two resolutions of it, one in the direction of the radius, the other of the tangent, are sufficient. But, the nodes are regressive; in a subsequent position of them, then, the line representing the Sun's disturbing force, will be inclined to the plane of the Moon's orbit: consequently, a threefold resolution of the force is requisite, the third being in a direction perpendicular to the plane of the Moon's orbit; consequently, if the line representing the absolute quantity of the disturbing force be supposed to be the same, the resolved parts in the directions of the radius and of the tangent must be less than they were before. The inequalities caused by them must therefore be less, and less, according to the position of the nodes. Hence, if the equation of the evection

$$1^{\circ}\ 20'\ 29''.5 \times \sin. 2\,[(☽ - ☉) - A]$$

were adapted to the first position of the nodes, it could not

suit the second, since the longitude of the nodes forms no part of the argument [2 (☽ — ☉) — *A*]. For this reason, therefore, a correction would be wanting for the Evection, that is a *new equation*, the argument of which should depend on the position of the nodes *. The same cause, the change in the Sun's disturbing force from its direction being more or less inclined to the Moon's orbit, must introduce new corrections, that is, *new equations*, belonging to the variation and annual equation.

Again, the annual equation arises from the change in that mean effect of the Sun's disturbing force by which the Moon's gravity is diminished. In adjusting therefore the value of the coefficient of the annual equation, the Moon's gravity must be supposed to be of a certain value: consequently, the Moon must be assumed to be at a certain distance from the Earth. When therefore the Moon is at a different distance, the *Equation*, if adjusted for the previous distance, cannot suit this: a small correction, therefore, or a *new Equation* will be necessary, the argument of which must involve or contain, in its expression, the Moon's distance, or her mean anomaly, or some term connected with these quantities †.

Again, the argument for the variation involves simply the angular distance of the Sun and Moon; and its coefficient must be supposed to be settled for certain values of the Moon's gravity and the Sun's disturbing force; and, consequently, when the Sun and Moon are at certain distances from the Earth. The changes therefore in those distances, which are continually happening, must render necessary two corrections, or two *new equations*: one for the approach of the Sun to the Earth, the other for the elongation of the Moon from the Earth. Generally, any equation

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* The equation in Lalande, p. 180, is

$$60''.4 \times \sin. 2 \text{ dist. } ☽\text{'s } ☊ \text{ from } ☉.$$

† The supplementary equation, according to Mayer, is

$$42'' \sin. (☽\text{'s mean anom.} - ☉\text{'s mean anom.})$$

which however is not the sole correcting equation due to this cause. See Lalande, *Astron.* tom. II, p. 178.

furnished with its numerical coefficient on the supposition of the Sun and Moon revolving round the Earth in circular orbits, will require new supplemental or subsidiary equations due to the real and elliptical forms of the orbits *.

Again, the inclination of the Moon's orbit is variable; therefore any equations adjusted to a mean state of inclination will require subsidiary equations, to correct the errors consequent on changes in that state.

From considerations like the preceding, the existence of the *smaller inequalities* is established: and, by an attentive consideration of the circumstances that occasion them, the forms of their arguments may be detected; with much less certainty however, than by the direct investigation of the disturbed place of the Moon.

It is one thing to prove the existence of an *inequality*, and another to establish the necessity of its corresponding *equation*. Whether it is expedient to introduce the latter, is a matter of mere numerical consideration. The correction of a correction, the subsidiary equation to a principal equation, is, in the lunar theory, very minute: and some equations, arising from the causes that have been enumerated, are so minute, as to be disregarded by the practical Astronomer.

We have at present considered only the inequalities that affect the Moon's longitude: but the Sun's disturbing force causes also *inequalities* in the Moon's *latitude* and in her *parallax*.

The inequalities of the latitude and of the parallax have nothing peculiar in them, nor distinct, (whether we regard their physical cause or the mode of ascertaining the laws of their variation,) from the inequalities of longitude. It is not necessary therefore to dwell on them, since the latter have been explained.

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* The evection, for instance, is variable from the variation of the distances of the Sun from the Moon and Earth: and for the purpose of correcting the evection, there are 4 subsidiary, or, as Lalande calls them, *accessary* equations, which in his Tables are the 5th, 6th, 7th, and 9th. See *Astron.* tom. II, p. 177.

We will only mention, that the principal inequality in latitude, and its law, were discovered by Tycho Brahé, and by the comparison of observations of the greatest latitudes of the Moon, at different epochs, and when that planet was differently situated, relatively to the nodes of its orbit. The equation is

$$(8' \, 47''.15) . \cos. 2 \odot\text{'s distance from } ☽\text{'s} ☊ .$$

(See Lalande, tom. II, p. 193. Mayer, *Theoria Lunæ*, p. 57. Laplace, *Mec. Cel.* Liv. VII, p. 283, &c. French Tables, Introduction.)

If the Moon's orbit coincided with the plane of the ecliptic, the Sun's disturbing force, resolved into the directions of a tangent to the Moon's orbit and of a radius vector, could only, by the first resolution, alter the law of elliptical angular motion, and, by the second, the length of the radius vector (such as it would be in an ellipse); in other words, it could only produce inequalities in longitude and in parallax, for the parallax varies inversely as the radius vector. But, the Moon's orbit being inclined to the ecliptic, the Sun's disturbing force (represented by a line drawn from the Moon towards the Sun) cannot be entirely resolved into the two former directions: a third resolved part will remain perpendicular to the plane of the Moon's orbit, which will cause the Moon to deviate from that plane; in other words, will cause inequalities in the Moon's latitude.

In order to correct these inequalities in the Moon's latitude, eleven equations are necessary, according to Lalande, (see *Astron.* tom. II. p. 193.) In the New French Tables an additional one is inserted.

The Lunar Tables we now possess, and which present us, under a commodious form, the results of the several preceding *Equations*, and from which in fact the Moon's place is computed in the Nautical Almanack, are of great extent and accuracy. It is almost unnecessary to observe, that they are the fruit of long and laborious research: of some conjectures, many revisions, and new helps from theory. The computers of the Nautical Almanack, have, within the space of forty years, used four different sets of Tables: 1. Mayer's Tables corrected by Mason :

2. Mason's Tables of 1780: 3. Mason's Tables, corrected by Lalande from Laplace's Equations of the Acceleration of the Moon's Motion, &c: 4. Burg's Tables edited by Delambre, and published by Mr. Vince in the third Volume of his *Astronomy*. The computers of the *Connoisance des Tems*, since 1817, have used Burckhardt's Tables.

The Moon's place, at any given time, is found by the addition of a great number of terms technically called *Equations*. An equation consists of its *coefficient* and its *argument*. The latter, although it may be found out by a species of orderly and regulated conjecture, is yet most surely obtained from theory, (see *Physical Astronomy*, Chap. XIV, p. 240.) The numerical value of its coefficient is best determined from observations. Now the Tables being once formed, a question arises concerning the means of examining and correcting them: in the first place then we must find their errors, and, in the second, from those errors find the corrections. As this is a subject of some complication, and as its development will afford an illustration of several of the preceding principles and processes, we will consider it fully in the ensuing Chapter.

CHAP. XXXIV.

On the Methods of finding the Errors and Corrections of the Lunar Tables.

The Moon's places, that is, its longitudes, latitudes, &c. are computed from the Lunar Tables, and then inserted in the Nautical Almanack. To examine then the accuracy of the longitudes and latitudes so inserted, is, in fact, to examine the truth of the Tables from which they were computed.

The means of examining the truths of the results in the Nautical Almanack, are, amongst other means, the observations made at Greenwich. Those observations are of north polar distances and right ascensions: but the immediate results of computations, made from the Lunar Tables, are lunar latitudes and longitudes: we must then, from the latter, derive the corresponding north polar distances and right ascensions, and compare them with the observed, or, we must institute a comparison between the latitudes and longitudes, computed from the observations, and the latitudes and longitudes computed from the Tables. We shall adopt the latter plan.

In the Greenwich Observations for 1812, p. 190, we find the following results obtained by means of the mural circle:

North Polar Distances.

1812.	Bar.	Therm. In.		N. P. D.
Nov. 18,	29.38	40	☽ L. L.	$75^{\circ}\ 34'\ 9''.7$
&c.	&c.			
	29.58	38	Arcturus	69 49 25.6

Transits over the Meridian.

* 3^h	57^m	$0^s.66$	☽ 2 L. 12^h 5^m $19^s.8$ mean time.
4	25	24.3	Aldebaran.
13	15	31.98	Spica Virginis.
14	7	18.43	Arcturus.

The above observations are, if we may use such an expression, in their rough state. In order to fit them for the computations of the Moon's longitude and latitude, they require several reductions.

(1.) In the first place the north polar distance must be corrected on account of the *index error* (see pp. 112, &c.)

(2.) According to the zenith distance of the lower limb, and the states of the barometer and thermometer, the north polar distance must be corrected for refraction, (see pp. 213, &c.)

(3.) The north polar distance, corrected as above, must be farther corrected, on account of parallax, (see pp. 311, &c.)

(4.) The north polar distance of the Moon's centre must be found by subtracting, from the distance of the lower limb, the Moon's semi-diameter.

(5.) If the computation be made for the time of the transit of the *Moon's second limb*, the above north polar distance, which is a *meridional* north polar distance, must be corrected for its change, during the Moon's passing over a space equal to its semi-diameter.

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* These transits were made with the mural circle: the old transit instrument being thought defective. They are called, in the Observations, *Corrected Transits*, being corrected on account of some small inequalities found to obtain in the intervals of the wires.

The mural circle not being a good transit instrument, it would be hardly fair, if the question were one of great accuracy, to examine the results of Lunar Tables by such an instrument. The observations, however, made with it, are sufficiently accurate for the purpose of illustration.

With regard to the reduction of the *transit* observations;

In the first place the observed transit is to be corrected on account of the error of the clock, (see pp. 104, &c.)

(6.) Secondly, the right ascension of the Moon's centre is to be found by subtracting, from the above right ascension of the second limb, the angle subtended at the pole of the equator by the Moon's semi-diameter.

Moon's North Polar Distance found.

☽ 's L. L. N. P. D.	75° 34′ 9″.7
Index Error *	+ 6.6
	75 34 16.3
Co-latitude	38 31 20
	37 2 56.3
Refraction	0 0 43.75
	37 3 40.05
Parallax	0 36 36.3
	36 27 3.75
☽ 's semi-diameter	0 16 40
	36 10 23.75
Co-latitude	38 31 20
N. P. D. ☽ 's centre	74 41 43.75

* The index error is derived by taking the mean of a great number of differences between the tabulated or computed north polar distances, and the instrumental distances, (see pp. 112, &c.). We will subjoin instances of results afforded by two stars; the process is precisely the same for any other.

Nov. 18,

Bar.	Therm. In.	Star.	Inst^l. N. P. D.
29.59	39	β Ursæ Minoris	15° 5′ 0″.5
29.58	38	Arcturus	69 49 25.6

Jan. 1,

Refraction.

(See pp. 245, &c.): also Tables of Refraction, in Vol. I. of Greenwich Observations, 1812,

Log. to 37° 3′ 40″	1.63327
Corr. barometer and thermometer.........	10.00774
(Log. 43.75)	1.64101

Jan. 1, 1812, N. P. D. β Ursæ Minoris			15°	4′	34″.25
Corrections.					
Refraction	(p. 243.)......	+ 25″.419			
Propor[l]. Annual Variation	(p. 407.)......	+ 12.98			
Aberration	(p. 286.)......	+ 4.24	....		33.869
Lunar Nutation	...........(Chap. XIV.)	− 8.33			
Solar Nutation		− 0.44			
			15	5	8.119
	Instrumental N. P. D.		15	5	0.5
	Index Error			+	7.62

Again,

Jan. 1, 1812, N. P. D. Arcturus			69°	50′	0″.11
Corrections.					
Refraction	+ 35″.68				
Aberration	+ 0.74				
Lunar Nutation	+ 7.64	....			− 27.78
Solar Nutation	+ 0.46				
Refraction	− 35.68				
Variation	− 16.74		69	49	32.33
Instrumental N. P. D.			69	49	25.6
Index Error				+	6.73
From β Ursæ Minoris				+	7.62
Mean....................................					7.17

This is the index error from two observations, one of each star: but the mean index error (6″.6) which has been used (see p. 693, l. 10,) in reducing the observations, was obtained from 149 observations, made, during 44 days, with 21 stars. Of such observations, 7 were made of β Ursæ Minoris, 10 of Arcturus. The mean of the 7 was 7″.16: of the 10, 6″.3.

Parallax

Horizontal equatoreal parallax $= 61' \, 7''.7$ * $= 3667''.7$,

Log. 3667.7	3.5643938
Correction	8841
	3.5635097
Log. sin. $36^{\circ} \, 52' \, 28''.4$	9.7781972
(Log. 2196.3)	3.3417069

In order to make the correction (5), we must find the time the Moon takes in describing its semi-diameter: now the angle at the pole subtended by the semi-diameter is (see p. 90,)

$$16' \, 40'' \times \text{sec. } 15^{\circ} \, 18' \, 26'' = 1036''.7 = 17' \, 16''.7,$$

but whilst the meridian, by reason of the Earth's rotation, is describing this angular space ($17' \, 16''.7$) the Moon moves to the eastward. We must find then the Moon's *retardation*. If we assume $13^{0} \, 30'$ for the mean angular retardation, we have

$$346^{0} \, 30' : 17' \, 16''.7 :: 24^{h} : 71^{s}.811 \dagger.$$

Therefore the Moon is $1^{m} \, 11^{s}.8$ in describing its semi-diameter: but it appears from the Nautical Almanack of 1812, (p. 126,) that the Moon's change of declination in 12^{h} was about $1^{0} \, 29'$, and consequently in $1^{m} \, 11^{s}.8$, about $8''.7$. Deducting, therefore, this quantity from the above meridional north polar distance, we have

$$\text{N. P. D. } \rangle\text{'s centre} = 74^{0} \, 41' \, 35''.05.$$

* There are two corrections in deducing the parallax from the horizontal equatoreal parallax: one, on account of the diminution of the radius of the Earth in an oblate spheroid: this in the latitude of Greenwich is effected by subtracting the logarithm .0008841 from the logarithm of the horizontal parallax. The second correction is on account of the angle, which a line drawn from the centre of the Earth to the place of observation, makes with the direction of the plumb-line at the same place. This correction is effected by subtracting $11' \, 11''.6$ from the zenith distance when its sine is to be multiplied into the parallax, in order to deduce the parallax of altitude.

† See a Table for this and like computations in Wollaston's *Fasciculus*, p. 79.

Moon's Right Ascension found.

First, to find the error of the clock, (see pp. 101, &c.) On Nov. 18, 1812, at the time of the Moon's transit.

Computed Right Ascension.		Observed R. A.	Clock too fast.
Aldebaran Æ, 1812, . .	$4^h\ 25^m\ 8^s.576$		
Aberr. prec^n. $4^s.30$ } Nutation -0.65 } . .	0 0 3.65		
See Chaps. XI, XII, &c.	4 25 12.226	$4^h\ 25^m\ 24^s.3$	$12^s.07$
Spica Virginis	$13^h\ 15^m\ 18^s.1$		
Aberr. prec^n. $1^s.71$ } Nutation . . . $-.56$ } . .	0 0 1.15		
	13 15 19.25	13 15 31.98	12.73
Arcturus	$14^h\ 7^m\ 5^s.28$		
Aberr. prec^n. $1^s.12$ } Nutation . . . $-.74$ } . .	0 0 0.38		
	14 7 5.66	14 7 18.43	12.77
Sum of times and errors		31 48 14.71	37.57
Mean time and error		10 36 4.9	12.52

gain of clock* in $10^h = 0^s.7$, nearly.

Hence, at $3^h\ 57^m\ 0^s.66$ the time of the Moon's passage, the clock was $12^s.04$ too fast, and, accordingly,

Æ ☽'s 2 L. . . . . $3^h\ 56^m\ 48^s.63 = 59^\circ\ 12^m\ 9''.45$

(see p. 695,) angle subtended by ☽'s radius 0 17 16.7

Æ ☽'s centre 58 54 52.75

Hence, the elements and process for computing the longitude and latitude of the Moon, at the time of the transit of its second limb over the meridian of Greenwich, are as follow (see pp. 158, &c.)

* There is no rate of the clock given in the Greenwich Observations, the clock having been taken down and adjusted to sidereal time, on the 18th.

Latitude.

☽'s Æ 58° 54′ 52″.75 .... sin. $\frac{1}{2}$ (90 — Æ).... 9.4280638
2

18.8561276

N. P. D...... 74° 41′ 35″.05 .... sin. 9.9843137
I........ 23 27 35.1 .... sin. 9.5999970

S......... 98 9 10.15 2) 18.4404383

$\frac{1}{2}$ *S*...... 49 4 35.07 9.2202196
9 33 28.1 ∴ $M = 9° 33' 28''.1$

$\frac{1}{2}S + M$, nearly, 58 38 3 .... sin. 9.9313873
$\frac{1}{2}S - M$, nearly, 39 31 7 .... sin. 9.8036816

2) 19.7350689

(sin. 47° 29′ 10″) 9.8675345

∴ the distance from the north pole of the ecliptic is 94° 58′ 20″
and the latitude (south) 4 58 20

Longitude.

Δ = 94° 58′ 20″ .... sin. 9.9983626
I = 23 27 35.1 .... sin. 9.5999970

δ = 74 41 35.05 .... 19.5983596

2) 193 7 30.15

(nearly) $\frac{1}{2}$ sum .... 96 33 45. .... sin. 9.9971450
$\frac{1}{2}$ sum — δ..21 52 10 .... sin. 9.5711180

(20 added) 39.5682630
19.5983596

2) 19.9699034

(sin. 75° 0′ 14″) 9.9849517

∴ 90° + longitude = 150° 0′ 28″,
and longitude = 60 0 28.

Such are the values of the latitude and longitude of the Moon, computed from immediate observations. In order to compare

such values, with the values of the latitude and longitude inserted in the Nautical Almanack, we must reduce the latter, which are computed for Greenwich at the apparent times of its noon and midnight, to the observed time of the transit of the Moon's limb. In the record of the observations (see p. 692,) the mean time of such transit is expressed. As we wish, however, to explain every part of the present investigation, we will now deduce the mean and apparent times of the transit.

On the 18th the Sun's transit was not observed at Greenwich: we will, therefore, compute it after the manner of pp. 527, &c.

Sun's mean longitude, 1812, ...	9^s	9°	$59'$	$50''.9$
Motion to Nov. 18, ...	10	17	22	42.2
Mean longitude Nov. 18, ...	19	27	22	33.1

In time (rejecting 24^h)	15^h	49^m	$30^s.2$
Equation of equinoxes..............			$-$ .64
	15	49	29.56
Right ascension Moon's second limb....	3	56	48.63
Apparent time of transit	12	7	19.07
Acceleration	0	1	59.15
Mean time of transit	12	5	19.9

Value of the Moon's Latitude and Longitude, at $12^h\ 5^m\ 19^s.9$ *computed from the Nautical Almanack.* See the *Nautical Almanack* for Nov. 18, 1812, &c.

Moon's Latitude.	First Diff. d'.	Second Diff. d''.	Third Diff. d'''.
18th Noon $4^\circ\ 59'\ 58''$			
	$-\ 1'\ 18''$		
Midnight 4 58 40		$-\ 5'\ 13''$	
	$-\ 6\ 31$		$+\ 16''$
19th Noon 4 52 9		$-\ 4\ 57$	
	$-\ 11\ 28$		$+\ 20$
Midnight 4 40 41		$-\ 4\ 37$	
	$-\ 16\ 5$		
20th Noon 4 24 36			

Now the intervals between these latitudes are 12 hours of apparent time and, therefore, in applying the differential theorem, we must find the value of x in such time. If, therefore, we assume the latitude of the Moon, on the midnight of Nov. 18th as the first term, we have

$$x = \frac{5^{m}\ 19^{s}.9 + 14^{m}\ 27^{s}.1}{12} = .027476;$$

$\therefore$ since $d' = -6'\ 31''$, $\quad xd' = -10''.74$,

and since $d'' = -4'\ 57''$, $\quad x.\dfrac{x-1}{2}\,d'' = +3.97$,

$$d''' = +20'', \quad x.\frac{x-1}{2}.\frac{x-2}{3}\,d''' = +0.17;$$

$\therefore$ latitude $= 4^{\circ}\ 58'\ 40'' - 6''.6 \ldots\ldots = 4^{\circ}\ 58'\ 33''.4$,

nearly, but the latitude computed from immediate observations was, see p. 697, $\}$ 4 58 20

the error of the Tables 0 0 13.4

Longitude.			
Moon's Longitude.	d'.	d''.	d'''.
18th Noon $1^{s}\ 22^{\circ}\ 12'\ 25''$			
	$+7^{\circ}\ 35'\ 25''$		
Midnight 1 29 47 50		$-2'\ 54''$	
	$+7\ 32\ 31$		$-1'\ 17''$
19th Noon 2 7 20 21		$-4\ 11$	
	$+7\ 28\ 20$		$-1\ 5$
Midnight 2 14 48 41		$-5\ 16$	
	$+7\ 23\ 4$		
20th Noon 2 22 11 45			

Here, the first term being $1^{s}\ 29^{\circ}\ 47'\ 50''$

$$d' = 0\ \ 7\ \ 32\ \ 31 = 27151'',$$
$$d'' \quad -\ 4\ \ 11 = -251,$$
$$d''' \quad -\ 1\ \ 5 = -\ 65,$$

and $x = .027476$,

therefore we have, by substitution, from the differential theorem

$$\left.\begin{array}{r} ☽\text{'s long., on 18th at } 12^h\ 19^m\ 47^s, = 1^s\ 29^0\ 47'\ 50'' \\ +\ 12\ 26 \\ +\ 0\ 3.35 \\ -\ 0\ 0.57 \end{array}\right\} = 2^s\ 0^0\ 0'\ 18''.78$$

but (see p. 697,) the longitude computed from immediate observations was. . . . . $2^s\ 0^0\ 0'\ 28''$

∴ error of Lunar Tables . . . . $-\ 9''.22$

We subjoin two other instances, in which the zenith distances of the Moon were observed by the brass mural quadrant, and the transits by the old transit instrument, (see pp. 33, 65, of Greenwich Observations.)

1811.	Transits reduced.	Rate of Clock.	Stars, &c.
Sept. 27,	$19^h\ 37^m\ 41^s.50$	+ 0.46	γ Aquilæ
	19 41 58.78	+ 0.48	α Aquilæ
	19 46 26.80		β Aquilæ
	20 11 47.08		☽ 1 L. $7^h\ 48^m\ 37^s.4$ mean time
	21 56 39.96		α Aquarii.
Sept. 28,	19 37 42.26	+ 0.76	γ Aquilæ
	19 41 59.56	+ 0.78	α Aquilæ
	19 46 27.70	+ 0.90	β Aquilæ
	20 35 25.36		α Cygni
	21 12 55.32		☽ 1 L. $8^h\ 45^m\ 38^s.8$ mean time

Sept.	Bar.	Therm. In.	Refraction.		*Zenith Distances.* Ext^r. Division.
27,	29.22	51	2′ 23″.3	☽ 's L. L.	68° 44′ 18″.2
28	29.36	53	2 4.7	☽ 's L. L.	65 53 23.8

Moon's North Polar Distance found.

27th, instrumental zenith dist. ☽ 's L. L...	68°	44′	18″.2
Error of Collim.			−5
	68	44	13.2
Refraction	0	2	23.2
	68	46	36.4
Parallax	0	55	26.2
	67	51	10.2
Moon's semi-diameter	0	16	15.7
	67	34	54.5
Co-latitude	38	31	20
North polar distance of Moon's centre on the meridian	106	6	14.5
Change of north polar distance			+ 6.23
North polar distance of Moon's centre when 1 L. is on the meridian ...	106	6	20.73

The values of the parallax and change of north polar distance, used in lines 5 and 9, are thus computed:

1st *Parallax.* Equatoreal horizontal parallax 59′ 40″,

Log. 3580	3.5538830
(See p. 50, Vince, vol. III.)	8841
	3.5529989
Log. sin (68° 46′ 36″.4 − 11′ 11″.6)	9.9689466
(Log. 3326.18)	3.5219455

2nd. *Change of the Moon's North Polar Distance during the time of the describing its Semi-diameter.*

Time of describing Moon's radius (p. 695,) ..	1ᵐ 10ˢ,5
Change of declⁿ. S. (*Naut. Almᵏ.*) in 12 hours	− 1° 4′
in 12ᵐ	− 1′ 4″
in 1ᵐ 10ˢ.5 ...	− 6.23
Or, decrease of north polar distance...........	− 6.23.

Again on 28th, zenith distance L. L.	65°	53′	23″.8
Collim.			− 5
	65	53	18.8
Refraction	0	2	4.7
	65	55	23.5
Parallax, (see below l. 11,)	0	54	58
	65	0	25.5
Moon's semi-diameter	0	16	27
Zenith dist. of Moon's centre on meridian	64	43	58.5
Change of north polar distance (l. 17,) . .			+ 9.2
	64	44	7.7
Co-latitude	38	31	20.
North polar distance of the Moon.	103	15	27.7

Parallax.

Horizontal equatoreal parallax 60′ 25″ = 3625″

Log. 3625	3.5593080
	8841
	3.5584239
Log. sin. 65° 44′ 11″.9	9.9598359
(Log. 3298)	3.5182598

Change in North Polar Distance.

Time of describing Moon's radius	1ᵐ 10ˢ.9
By Nautical Almanack, change in 12ʰ	− 1° 34′
in 12ᵐ	− 1′ 34″
In 1ᵐ 10ˢ.9	9.2

Moon's Right Ascension found.

First, error of clock found on the 27th,

	R. A. *from Theory and Tables.*			R. A. by Clock (p. 700,)	Clock too fast.
Aquilæ γ	19ʰ	37ᵐ	18ˢ.60	. . . . . . 41ˢ.5	22ˢ.9
Aquilæ α	19	41	35.88	. . . . . . 58.78	22.9
Aquilæ β	19	46	4.02	. . . . . . 26.8	22.78
α Aquarii	21	56	7.2	. . . . . . 29.96	22.76
4)	81	1	5.7		4) 91.34
therefore at	20	15	16.4	mean error of clock	22.83

Moon's transit by clock, p. 700, ...	20ʰ	11ᵐ	47ˢ.08
True Æ Moon's 1 L. on the 27th.	20	11	24.25

Next, gain of clock in 24^h from three stars of the Eagle (see p. 700,) = $\frac{1}{3}(.76+.78+.9)=.82$; ∴ in 25^h. . . . 0^h 0^m $0^s.83$, nearly,

Clock too fast on 27th.	0	0	22.83
too fast on 28th	0	0	23.66
Moon's transit by clock	21	12	55.32
True Æ Moon's 1 L. on the 28th	21	12	31.66
Hence, expressed in space, on 27th, right ascension Moon's 1 L. . .	302°	51′	3″.75
Angle subtended by Moon's radius (975″.58 × co-sec. 106° 6′)	0	16	55.4
Right ascension of Moon's centre	303	7	59.15
On 28th, Right ascension Moon's 1 L.	318	7	54.9
Angle of Moon's radius (987″. × co-sec. 103° 15′)	0	16	54.03
	318	24	48.9, nearly.

Computation of the Moon's latitude and longitude, (see pp. 159, &c.)

Latitude. Sept. 27th.

Moon's Æ	303°	7′	59″.15		
	90				
2)	213	7	59.15		
	106	33	59.57	. . . . sin.	9.9815873
					2
					19.9631746
North polar distance	106°	6′	20″.73	. . . . sin.	9.9826106
I	23	27	42.5	. . . . sin.	9.6000333
2)	129	34	3.23	2)	19.5458185
$\frac{1}{2}S$	64	47	1.61		9.7729092
	36	21	21.8		M = 36° 21′ 21″.8
$\frac{1}{2}S+M$	101	8	23.4	. . . . sin.	9.9917392
$\frac{1}{2}S-M$	28	25	39.8	. . . .	9.6776523
				2)	19.6693915
				(43° 6′ 45″.8).	9.8346957

Hence, complement of the latitude = 86° 13′ 31″.6
and latitude = 3 46 28.4.

Longitude. Sept. 27th.

$$\begin{array}{rlr}
\Delta = & 86^{\circ}\ 13'\ 31''.6 \ldots \text{sin.} & 9.9990567 \\
I = & 23\ \ 27\ \ 42\ .\ 5 \ldots \text{sin.} & \underline{9.6000333} \\
\delta = & \underline{106\ \ \ 6\ \ 20.73} & 19.5990900 \\
2) & \underline{215\ \ 47\ \ 34.84} & \\
\tfrac{1}{2}\text{ sum} \ldots\ldots\ldots & 107\ \ 53\ \ 47.42 \ldots \text{sin.} & 9.9784604 \\
\tfrac{1}{2}\text{ sum} - \delta \ldots\ldots & 1\ \ 47\ \ 26.69 \ldots \text{sin.} & \underline{8.4948395} \\
 & (20 \text{ added}) & 38.4732999 \\
 & & \underline{19.5990900} \\
 & 2) & \underline{18.8742099} \\
 & & 9.4371049
\end{array}$$

which is the log. sine of 15° 52′ 41″.4, and of 375° 52′ 41″.4.

Hence, taking the last value, (which the value of the Moon's right ascension points out as the right one),

$$\begin{array}{rl}
90^{0} + \text{longitude} = & 0^{s}\ 751^{0}\ 45'\ 22''.8 \\
\text{and longitude} = & 0\ \ 661\ \ 45\ \ 22\ .8 \\
(\text{rejecting } 360^{0}) = & 0\ \ 301\ \ 45\ \ 22\ .8 \\
= & 10\ \ \ \ 1\ \ 45\ \ 22\ .8.
\end{array}$$

Latitude. Sept. 28th.

$$\begin{array}{lll}
\text{Moon's Æ} \ldots\ldots\ldots & 318^{\circ}\ 24'\ 48''.9 & \\
 & \underline{90} & \\
 & 2)\ \underline{228\ \ 24\ \ 48\ .9} & \\
 & 114\ \ 12\ \ 24\ .4 \ldots\ldots \text{sin.} & 9.9600290 \\
 & & \underline{2} \\
 & & 19.9200580 \\
\text{North polar distance } 103^{\circ}\ 15'\ 27''.7 \ldots\ldots\ldots & \text{sin.} & 9.9882684 \\
I \ldots\ldots\ldots\ \underline{23\ \ 27\ \ 42\ .5} \ldots\ldots\ldots & \text{sin.} & \underline{9.6000333} \\
2)\ \underline{126\ \ 43\ \ 10\ .2} & 2) & \underline{19.5083597} \\
\tfrac{1}{2} S \ldots\ \ 63\ \ 21\ \ 35\ .1 \ldots\ldots\ldots\ldots & & 9.7541798 \\
\underline{34\ \ 35\ \ 44} \ldots\ldots\ldots & & (M = 34^{\circ}\ 35'\ 44'') \\
\tfrac{1}{2} S + M \ldots\ 97\ \ 57\ \ 19\ .1 \ldots\ldots\ldots & \text{sin.} & 9.9958003 \\
\tfrac{1}{2} S - M \ldots\ 28\ \ 45\ \ 51\ .1 \ldots\ldots\ldots & \text{sin.} & \underline{9.6823306} \\
 & 2) & \underline{19.6781309} \\
(\text{sin. } 43^{0}\ 39'\ 26''.3) \ldots\ldots & & 9.8390654
\end{array}$$

Hence, the complement of latitude is . . 87° 18′ 52″.6
and the latitude, nearly . . . . 2 41 7 . 3.

Longitude. Sept. 28th.

Δ	87°	$18'$	$52''.6$	..... sin.	9.9995168
I	23	27	42.5	..... sin.	9.6000333
δ	103	15	27.7		19.5995501
2)	214	2	2.8		
$\frac{1}{2}$ sum	107	1	1.4	....	9.9805574
$\frac{1}{2}$ sum $-\delta$	3	45	33.7	....	8.8166798
				(20 added)	38.7972372
					19.5995501
				2)	19.1976871
					9.5988435

which is the logarithmic sine of $383^\circ\ 23'\ 38''.6$; therefore longitude $+\ 90^\circ$ .... = 766 47 17.2
and (reject^g. 12 signs) the long. = 316 47 17.2 = $10^s\ 16^\circ 47' 17''.2$.

Latitudes and Longitudes deduced from the Nautical Almanack.

Since these latitudes and longitudes are expressed in the Nautical Almanack, for apparent noon and midnight, it is necessary to know the time of the passage of the Moon,

Sun's epoch for 1811,	9^s	10°	$14'$	$10''.5$				
Mean motion to Sept. 27,	8	26	8	20.7				
Mean longitude on 27,	18	5	22	31.2	in time	12^h	21^m	$30^s.08$
Mean motion for 1 day	0	0	59	8.333				
Mean longitude on 28,	18	6	21	39.5	in time	12	25	26.63

but equation of the equinoxes in right ascension is $-.26$.

Hence, on 27th sidereal time (see p. 702,)	20^h	11^m	$24^s.25$
Sun's mean longitude from true equinox	12	21	29.82
Approximate time	7	49	54.43
Acceleration, (see p. 526,)..................	0	1	16.98
Mean time of transit of Moon's first limb	7	48	37.45

On the 28th, Sidereal time, (see p. 703,) ……	21^h	12^m	$31^s.66$
Sun's mean longitude reckoned from true equinox	12	25	26.37
Approximate time, nearly ……………………	8	47	5.3
Acceleration …………………………………	0	1	26.35
Mean time of transit of Moon's first limb ……	8	45	38.95

But these are the mean times: the apparent times may be obtained by adding to them the equations of time. Now the equation of time proportional to $7^h\ 48^m\ 37^s$, on Sept. 27th, is $8'\ 54''$ subtractive of apparent time, and Sept. 28th, $9'\ 14''.6$. Hence, the times are

on the 27th, $7^h\ 57^m\ 31^s.45$; $\therefore$ x (see p. 699,) = .66322

on the 28th, 8 54 53.55, and x ………… = .74298.

Moon's Latitudes.	d'.	d''.	d'''.
27th, Noon 4° 3′ 36″			
	− 27′ 0″		
Midnight. . 3 36 36		− 3′ 46″	
	− 30 46		+ 24″
28th, Noon 3 5 50		− 3 22	
	− 34 8		+ 29
Midnight. . 2 31 42		− 2 53	
	− 37 0		+ 37
29th, Noon 1 54 41		− 2 16	
	− 39 17		
Midnight. . 1 15 24			

Hence, for the

	Twenty-seventh.	Twenty-eighth.
a =	$4°\ 3'\ 36''$ ……………………	$3°\ 5'\ 50''$
d' =	$-27\ \ 0$ ……………………	$-34\ \ 8$
d'' =	$-\ 3\ 46$ ……………………	$-\ 2\ 53$
d''' =	$+\ 0\ 24$ ……………………	$+\ 0\ 37$
x =	.66322 ……………………	.7429
$\frac{x-1}{2}$ =	$-.16839$ ……………………	$-.128547$
$\frac{x-2}{3}$ =	$-.4456$ ……………………	$-.419031.$

Hence, the latitudes =, respectively,

$$\left.\begin{array}{r} 4^\circ\ 3'\ 36'' \\ -17\ \ 54 \\ +\ 0\ \ 25.24 \\ +\ 0\ \ \ 1.19 \end{array}\right\} = 3^\circ\ 46'\ 8'' \qquad \left.\begin{array}{r} 3^\circ\ 5'\ 50'' \\ -25\ \ 21.47 \\ +16\ \ \ 0.52 \\ +\ 1\ \ \ 0.48 \end{array}\right\} = 2^\circ\ 40'\ 46''.5$$

Moon's Longitudes.	d'.	d''.	d'''.
27th, Noon $9^s\ 27^\circ\ 2'\ 32''$			
	$7^\circ\ 7'\ 14''$		
Midnight... 10 4 9 46		$5'\ 30''$	
	7 12 44		$-23''$.
28th, Noon 10 11 22 30		5 7	
	7 17 51		-28
Midnight... 10 18 40 21		4 39	
	7 22 30		-40
29th, Noon 10 26 2 51		3 59	
	7 26 29		
Midnight... 11 3 29 20			

Hence, for the

	Twenty-seventh.	Twenty-eighth.
a =	$9^s\ 27^\circ\ 2'\ 32''$	$10^s\ 11^\circ\ 22'\ 30''$
d' =	7 7 14	7 17 51
d'' =	5 30	4 39
d''' =	$-$ 23	$-$ 40

* and Moon's longitudes =

$$\left.\begin{array}{r} 9^s\ 27^\circ\ 2'\ 32'' \\ +\ 4\ \ 43\ \ 21.2 \\ -\ \ 36.855 \\ -\ \ \ 1.144 \end{array}\right\} = 10^s\ 1^\circ\ 45'\ 15''.2 \text{ on 27th,}$$

$$\left.\begin{array}{r} 10^s\ 11^\circ\ 22'\ 30'' \\ +\ \ 5\ \ 25\ \ 16.9 \\ -\ \ 26.64 \\ -\ \ \ 1.6 \end{array}\right\} = 10^s\ 16^\circ\ 47'\ 18''.6 \text{ on 28th,}$$

* In order to place the whole of the detail under the eye of the student, we subjoin the arithmetical computation. What is here effected by

If we now exhibit, under one point of view, the results obtained from observations, and those results that are computed from the Nautical Almanack, we shall have

	Transit of Moon's Limb, Mean Time.	Moon's Latitude from Observation.	Moon's Latitude from Tables.	Error of Table.
1811, Sept. 27,	$7^h\ 48^m\ 37^s.45$	$3°\ 46'\ 28''.4$	$3°\ 46'\ 8''$	$-20''.4$
28,	8 45 38 .95	2 41 7 . 3	2 40 46.5	− 20 . 8
1812, Nov. 18,	12 5 19 . 9	4 58 20	4 58 33.2	+ 13 . 2
		Moon's Longitude from Observation.	Moon's Longitude from Tables.	Error of Table.
1811, Sept. 27,		$10^s\ 1°\ 45'\ 22''.8$	$10^s\ 1°\ 45'\ 15''.2$	$-7''.6$
28,		10 16 47 17 . 2	10 16 47 18.6	+ 1 . 4
1812, Nov. 18,		2 0 0 28	2 0 0 18.8	− 9 . 2*

---

by the *differential theorem*, might have been, and in practice is, effected, but less accurately, by Tables of second differences,

L. x .... 9.8216628 .... 9.8216628 } 19.04798

L. 7° 7′ 14″ ... 4.4088164 L. $\frac{x-1}{2}$ 9.2263163 } L. $\frac{x-2}{3}$ ... 9.64894

4.2304792 L. 5′ 30″ 2.5185139 L. 23″ 1.36173

1.5664930 0.05865

No. = 4° 43′ 21″.2 No. = − 36″.855 No. − 1″.1446

L. x .... 9.8709339 .... 9.8709339 } 18.97999

L. 7° 17′ 51″ ... 4.4194766 L. $\frac{x-1}{2}$ 9.1090629 } L. $\frac{x-2}{3}$ 9.62225

4.2904105 L. 4′ 39″ 2.4456042 L. 40″ 1.60206

1.4256010 0.20430

No. = 5° 25′ 16″.9 No. = 26″.64 No. = 1″.6

* See Note in opposite page.

Results like those that have been just obtained* serve, as we have before observed, a double purpose: they become tests of the accuracy of the Lunar Tables, and means of correcting them. It is obvious how they perform the first office. The mode of performing the second has also been already explained in Chapter XXI. The Moon's place, previously to its insertion in the *Ephemerides of England*, &c. is computed from the Lunar Tables on certain conditions, as they may be called: that is, the mean epoch, the mean motion, the equation of the centre, the longitude of the apogee, and the equations expounding the modifications of the Sun's disturbing force, &c. are all assumed of certain magnitudes: which magnitudes may be erroneous: all, perhaps, in slight degrees, some certainly erroneous: since, otherwise, the Moon's computed place ought to agree with the observed, the observations being supposed to be exact. Although, in correcting the Tables, we may be more assured of the exactness of some of the *elements* than of others, yet it is the safer and the more scientific plan to suppose them all erroneous: and to form equations such as

$$a.dL + b.dm + c.dE + f.dp + \&c. = C,$$

in which dL, dm, &c. shall represent the variations or errors of the longitude, equation of the centre, &c. and C shall be such a quantity as we have just deduced in p. 708, and there represented,

---

* The results do not exactly agree with the results obtained by the computers of the Nautical Almanack, who, by order of the Board of Longitude, and for the purpose of ascertaining the relative accuracy of the several Lunar Tables, have compared the Greenwich Observations, from 1783 to 1819, with the Moon's longitudes and latitudes set down in the Nautical Almanack, and in the *Connoisance des Tems*. The disagreements are found amongst the latitudes: which may arise from the Moon's parallaxes being computed from different Tables, or from Tables constructed on different *oblatenesses* of the Earth. *Some* differences must occur, since in the comparisons, the Moon's places, at the times of the transits of its limbs, were deduced by means of the Tables of second differences, which cannot give results so exact, (we are speaking of arithmetical exactness) as the differential theorem is able to give.

according to the case by $-7''.6$, $+1''.4$, $-9''.2$, &c. In order to deduce the values of the errors of the elements we must form, at least, as many equations as there are supposed errors: but in practice, for reasons already assigned in Chapter XXI, a great number of equations are selected and combined together to form one equation. If the *variations of the elements* are in number 10, 10 sets of equations must be formed, and then the values of the variations or errors, or, under a different name, the *corrections* of the elements of the Tables, must be deduced by the ordinary but laborious process of elimination. By such means the present Lunar Tables have been advanced to their present state of perfection.

We must now pass on to other matters: and those will next claim our attention, which are connected with, and depend on, the lunar theory. Of such sort are eclipses and the methods of computing, at assigned times, the distances of the Moon from the Sun and certain fixed stars. Both subjects are of considerable extent, intricacy, and practical utility, since both, with different degrees however of accuracy, may be made subservient to the determination of the longitudes of places.

By the latter term we mean, in the most general sense, any points on the Earth's surface, whether such are permanent land-stations, or the temporary places of vessels at sea. For the determination of the longitudes of places of the latter description, lunar eclipses are of no use: and indeed, of but small use in fixing the longitudes of land-stations: not, however, from any defect in the lunar theory, but from the practical uncertainty of marking the times when the *phases* of an eclipse commence and terminate. Lunar eclipses might be excluded from a work, the scope of which should be strictly limited to subjects of merely practical utility. A wider range, however, has already been taken in the present Treatise; and, acting on a like plan, we will, in the next Chapter, treat of Lunar Eclipses: which are certainly phenomena of great interest, of celebrity in the History of

Astronomy, and of importance in settling certain of the lunar elements *.

* The uncertainty of the *time* of an eclipse, to the amount of a minute of time, vitiates the determination of the longitudes of places. But an error of that magnitude would be but of little consequence, when the happenings of eclipses, distant from each other by several centuries, are employed in fixing such an element of the lunar theory, as the Moon's mean motion.

CHAP. XXXV.

On Eclipses of the Moon.

IN Chapter IV, a lunar eclipse was shewn to arise from such an interposition of the Earth between the Moon and Sun, as to cause the shadow of the Earth to fall on part, or on the whole, of the Moon's disk.

This prescription of circumstance is necessary: since an opaque body, interposed at a certain distance between the Sun and Moon, does not necessarily cause an eclipse: for instance, if the diameter of the interposed body should be below a certain magnitude, its shadow would not reach the Moon.

The existence, therefore, of eclipses depends on the relative magnitudes of the Sun and Earth, supposing the mutual distances of the Sun, Earth, and Moon, to be assigned.

The Moon being in opposition, and at her mean distance, the apparent diameters of the Sun and Earth, seen from the Moon's centre, are $31' 59''.08$, and $1^{\circ} 55' 8''$. Now, at the extremity, or conical point of the Earth's shadow, the apparent diameters of the Sun and Moon are the same. The Moon, therefore, must be considerably nearer to the Earth than the extremity of the Earth's shadow: or, what amounts to the same, the length of that shadow must be greater than the Moon's distance from the Earth. By computation, it is found to be four times as great.

The eccentricity of the Moon's orbit being very small, equal only to 0.0548553, it would follow, if the above result, relative to the length of the shadow, were established for any distance of

the Moon from the Earth, that in all distances the shadow would extend far beyond the Moon. In fact by an easy computation we have the following results :

	Length of Axes of Shadow.
☉ in perigee	212.896 rad. ⊕
at mean distance	216.531
in apogee	220.238.

Hence, the least length of the shadow is more than 212 radii of the Earth, whereas the Moon's distance from the Earth never exceeds 64 radii.

Hence it appears a lunar eclipse must always happen whenever the Earth is *interposed* between the Sun and Moon; understanding, by such expression, the Earth's centre to lie in a line joining the centres of the Sun and Moon. In this latter situation of the three bodies, the Moon is in opposition. In such *kind* of opposition, an eclipse must always happen, and there would be only that kind, if the plane of the Moon's orbit coincided with that of the ecliptic.

The Moon's orbit being inclined to the ecliptic, and, opposition meaning nothing more, than the difference, in longitude, of a semi-circle, or of 180^0, the Moon may be in opposition, and still either directly above or below the right line joining the centres of the Sun and Earth; and, consequently, may either be above or below the conical shadow, the axis of which lies in the direction of the above-mentioned line.

Since the inclination of the Moon's orbit, (see p. 661,) is about $5^0\ 9'$, if the Moon in opposition should be either in its greatest northern or southern latitude, that is, either $5^{\circ}\ 9'$ above or below the ecliptic, no eclipse can take place, since the greatest section of the Earth's shadow at the Moon never exceeds $64'$. But, in the next succeeding opposition, after the lapse of a synodic period, the Moon cannot be again in her greatest latitude, since, the synodic period being greater than the sidereal, the Moon would, on that account, have approached the ecliptic, even supposing the nodes to have been stationary. But the nodes, instead of being stationary, are, during a synodic period, regressive

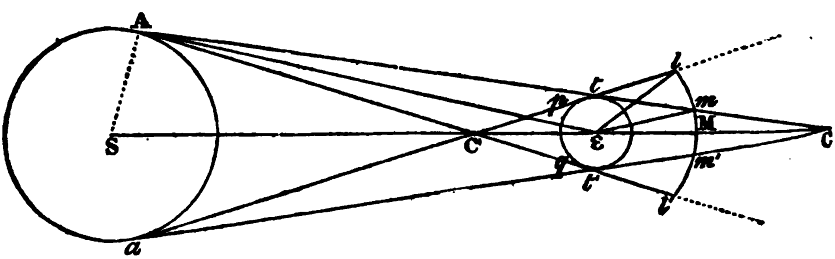
to the amount of 1° 35′. For this reason, then, as well as for the one just stated, the Moon approaches the ecliptic. In succeeding oppositions, the Moon, by the operation of both causes, would approach nearer and nearer to the ecliptic, till at length an opposition would occur, in which the Moon would be either, exactly, or very nearly, in its node: and if in its node, then it would be in the ecliptic, and in such case, an eclipse must happen.

An eclipse may happen, if the Moon be *near* to the node of her orbit; the least degrees of proximity are called the *Lunar Ecliptic Limits*.

These limits are easily determined from the inclination of the Moon's orbit, the Moon's apparent diameter, and the apparent diameter of a section of the Earth's shadow at the Moon. The two former conditions may be supposed to be known by previous methods, (see pp. 661, &c.) and it is the latter only that now requires to be investigated.

Apparent Diameter of a Section of the Earth's Shadow at the Moon.

Let S represent the Sun's centre, E the Earth's, and let the circles described round the centres S, E represent sections of those bodies. Draw AtC, $at'C$, tangents to the circular sections



of the Sun and Earth, and the triangular space included within tC, $t'C$, will represent the section of the conical shadow of the Earth. Let mMm' be part of the Moon's orbit, then the section of the Earth's shadow at the Moon is mMm', and its apparent

semi-diameter at the Earth, which we have to estimate, is the angle mEM*.

$$\angle mEM = \angle Emt - \angle ECm,$$
$$= \angle Emt - (\angle AES - \angle EAt).$$

Let $\angle\ Emt$, the angle subtended at the Moon by the Earth's radius, or the Moon's horizontal parallax, be denoted by.... P,

$\angle\ AES$, the Sun's apparent semi-diameter, by .... $\frac{D}{2}$,

$\angle\ EAt$, the angle subtended by the Earth's radius at the Sun, or the Sun's horizontal parallax, by .... p.

Hence,

The apparent semi-diameter of ⊕'s shadow $= p + P - \frac{D}{2}$.

Hence, the distance of the centres of the Moon and of the Earth's shadow, when the Moon's disk just touches the shadow, will be the preceding expression plus the Moon's apparent semi-diameter $\left(\frac{d}{2}\right)$, that is,

$$p + P - \frac{D}{2} + \frac{d}{2}.$$

If we take $P = 57' \ 1''$, $p = 8''.8$, and $\frac{D}{2} = 16' \ 1''.3$, we shall have

The mean apparent semi-diameter of ⊕'s shadow $= 41' \ 8''.5$, which is nearly three apparent semi-diameters of the Moon.

* We have, more than once, adverted to the necessary defect which diagrams in Astronomy are subject to, in representing distances and magnitudes according to their true proportion in nature. The Figure in the preceding page is an instance of it. The Earth's radius is there made not less than one-third of the Sun's, whereas it is about $\frac{1}{110}$th part. But, if it had been so drawn, we should have had a most inconvenient diagram, in which it would have been difficult to discern the lines and angles, which are the subjects of investigation.

Hence, since the Moon in the space of an hour moves over a space nearly equal to its diameter, the Moon may be entirely within the shadow, about two hours, or a total eclipse may endure that time.

In order to find the greatest value of the preceding expression, we must take the greatest parallax of the Moon, and the least of the Sun: for, since there is a constant ratio between the Sun's horizontal parallax and his apparent semi-diameter, the latter will be the least when the former is: and although in the expression the parallax is additive, yet its diminution below its mean or even its greatest quantity is trifling, relatively to that of its apparent diameter.

Hence, since the ☽ 's greatest horizontal parallax is $1^{\circ}\ 1'\ 29''$
and the ☉ 's least semi-diameter 15 45.48
the corresponding parallax of the ☉ 0 8.6

We have, nearly,
the greatest semi-diameter of the ⊕'s shadow .... $= 45'\ 52''$,
and the diameter .... $= 1^{\circ}\ 31'\ 44''$.

Precisely after this manner, and by the same formula, namely, $\left(p + P - \frac{D}{2}\right)$ may the apparent diameters of the Earth's shadow be computed, for other distances of the Sun and the Moon. Thus,

		Apparent Diameter of ⊕'s Shadow.
☉ in perigee.	☽ in apogee	$1^{\circ}\ 15'\ 24''.3036$
	at mean distance	1 23 2.31
	in perigee	1 30 40.3164
☉ at mean distance.	☽ in apogee	1 15 56.8656
	at mean distance	1 23 34.872
	in perigee	1 31 12.8784
☉ in apogee.	☽ in apogee	1 16 28.2936
	at mean distance	1 24 6.3
	in perigee	1 31 44.3064

3

In p. 714, there is given an expression for the length of the Earth's shadow, in terms of the Earth's radius obtained from the value $\frac{D}{2} - p$, of the angle Ect; thus

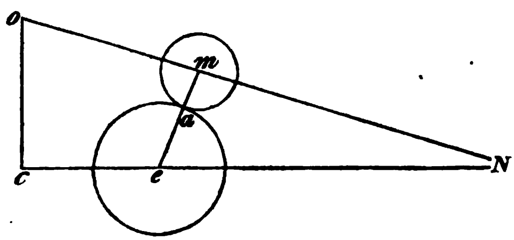
$$Ec = \frac{Et}{\sin. \angle Ect} = \frac{\text{rad. } \oplus}{\sin. \left(\frac{D}{2} - p\right)}.$$

Since there is a constant ratio (see p. 651,) between the Sun's semi-diameter and horizontal parallax, (which ratio is that of the radius of the Sun to the radius of the Earth, and in numbers, as 110 : 1 nearly), the denominator of the preceding fraction may be expressed either, in terms of the semi-diameter, or of the parallax ; thus,

$$\text{Length of shadow} = \frac{\text{rad. } \oplus}{\sin. (109\, p)},$$

$$\text{or} = \frac{\text{rad. } \oplus}{\sin. \frac{109\, D}{220}}.$$

But to return to the investigation of the extreme cases in which eclipses can happen. To the greatest apparent semi-diameter of the Earth's shadow (see p. 714,) add the greatest apparent semi-diameter of the Moon, and the result will be the greatest apparent distance of the Moon's centre from the ecliptic, at



which an eclipse can happen. Thus, in the Figure, if Ne be part of the ecliptic, Nm part of the Moon's orbit, e the centre

of a section of the Earth's shadow; if we take (see p. 716,) ea in its greatest value, equal to $45' \, 52''$, and ma, the greatest apparent semi-diameter of the Moon, $= 16' \, 45''.5$, then me, $= 62' \, 37''.5$, is the greatest distance of the Moon at which an eclipse can happen. If the distance be greater, there can be no eclipse, if less, and less within certain limits, there may or may not be an eclipse; its happening depending on the relative proximities of the Earth to the Sun and Moon.

The *ecliptic limit* Ne, corresponding to the greatest value of me, may be thus computed:

By Naper's Rules,

$$\text{rad.} \times \sin. me = \sin. Ne \times \sin. \angle eNm;$$

$\therefore$ taking $me = 62' \, 38''$, and the inclination of the Moon's orbit, (what it generally is, in these circumstances,) equal to $5^\circ \, 17'$, we have

$$
\begin{aligned}
10 + \log. \sin. 62' \, 38'' & \ldots\ldots\ldots\ldots\ldots\ldots \; 18.2605076 \\
\log. \sin. 5^\circ \, 17' & \ldots\ldots\ldots\ldots\ldots\ldots \; 8.9641697 \\
\therefore \log. \sin. Ne & \ldots\ldots\ldots\ldots\ldots\ldots \; \underline{9.2963379} \\
\therefore Ne = & \ldots\ldots\ldots\ldots\ldots\ldots \; 11^\circ \, 25' \, 40'', \text{ nearly.}
\end{aligned}
$$

The species of eclipse represented in the above Figure, where the two circular sections of the Moon and shadow are in contact, is called an *Appulse*.

The opposition of the Moon must have happened soon before this appulse, if the direction of the Moon's motion be supposed from m towards N. For, the Moon moving more quickly * than the Sun, and consequently, than the centre (e) of the shadow, cannot long have quitted a point o, such that the corresponding position of the centre of the shadow would be at c. And in these positions of the Moon and shadow, the former is in opposition.

* The diurnal motions of the Moon and Sun are respectively $13^\circ \, 10' \, 35''.027$, and $59' \, 8''.33$.

In the computation of eclipses there are several expedients employed for abridging its labour. Eclipses are to be expected when the Moon is near her node, and in opposition. But the labour of a direct and formal computation may frequently be spared, by roughly ascertaining certain limits, beyond which, it is useless to expect an eclipse. Thus, as we have seen in the preceding page, if Ne be greater than $11^0\ 26'$, no eclipse can happen. But Ne is the difference of the true longitudes of the centre of the ⊕'s shadow and of the ☽ 's ☊ at the time of the appulse; the time of appulse differs a little from the time of true opposition, and therefore, for two causes, from the time of mean opposition. The mean longitude of the centre of the Earth's shadow differs from the true longitude, by reason of the equation of the centre, and other small equations. If therefore, we compute the *mean* longitude of the Earth's shadow at the time of *mean* opposition, it will differ from the longitude of e, (see Fig. p. 717,) at the time of appulse for three causes; the difference, of the times of appulse and of true opposition, of the times of mean and true opposition, and of the mean and true longitudes. But, notwithstanding these sources of inequality, the consequent error in the value of Ne computed, from the mean longitude of the Earth, and for the time of mean opposition, is within certain limits; and accordingly M. Delambre states that, if Ne be $> 12^0\ 36'$, there cannot be an eclipse, if $< 9^0$, there must be one. Between 9^0, and $12^0\ 36'$, the happening of the eclipse is doubtful, and the doubt must be removed by a more exact calculation. The time of mean opposition may be computed from the Tables of the Sun and Moon. But, the computation is facilitated by means of a Table of *Epacts*. The *Epact for a year*, meaning the Moon's age at the beginning of the year, the age commencing from the last *mean* conjunction; and the *Epact for any month*, meaning the Moon's age at the beginning of the month, supposing the age to have begun from the beginning of the year. Delambre in his Astronomical Tables has given a new method of computing the probable times of the happening of eclipses. (See Vince, vol. III. Introduction, p. 56.)

In the preceding explanations we have supposed an eclipse to begin when the Moon enters the Earth's shadow at m'. A spec-

tator at the Moon in any point within m' and m, (see Fig. p. 714,) would, by reason of the intervention of the Earth, be unable to see any part of the Sun's disk. But, before and after this *eclipse*, properly so called, the Moon's light would be obscured; or, what amounts to the same thing, the spectator, on the Moon's surface, previously to being entirely deprived of the Sun's light, would lose sight of portions of his disk. In order to determine, when this obscuration first begins, and when it ends, draw two tangents $AC'ql'$, $aC'pl$, to the Sun and Moon; then, the moment the Moon enters $l'l$, part of the Sun's light is stopped; or, a spectator at the Moon situated any where between $l'm'$ sees part only of the Sun's disk. Entering $m'm$, the spectator loses sight of the Sun entirely; emerging from $m'm$, he regains, in his progress through ml, the sight of successively greater portions of the disk, and finally, emerging from ml, he again sees the full orb of the Sun.

The space included within the lines pl, ql', is the section of what is, properly enough, denominated the *Penumbra*; and its angle is $lC'l'$.

Angle of the Penumbra.

$$\angle AC'S = \angle AES + \angle EAC',$$
$$= \odot\text{'s apparent semi-diameter} + \odot\text{'s hor. parallax},$$
$$= \frac{D}{2} + p.$$

Hence, may be deduced,

The Apparent Semi-diameter of a Section of the Penumbra at the Moon's Orbit.

$$\text{For, } \angle lEC = \angle ElC' + \angle EC'l$$
$$= \text{☽'s hor. par}^{x}. + \frac{D}{2} + p$$
$$= P + p + \frac{D}{2}.$$

From this formula, as in the case of the umbra (p. 716,) the several values of the apparent semi-diameter of the penumbra,

corresponding to certain positions of the Sun and Moon, may be computed.

Since the apparent semi-diameter of the Moon's penumbra is

$$P + p + \frac{D}{2},$$

the distance of the centres of the Moon and shadow, when the Moon first enters the penumbra, is

$$P + p + \frac{D}{2} + \frac{d}{2};$$

d representing the Moon's apparent diameter.

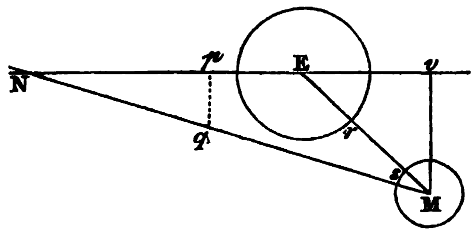
In the preceding investigations we have supposed the cones of the umbra and penumbra to be formed by lines drawn from the Sun and touching the Earth's surface. This, probably, is not the exact case in nature; for, the apparent diameter of the Earth's shadow is found, by observation, to be somewhat greater than what would result from the preceding formula. This circumstance is, with great appearance of probability, accounted for, by supposing those solar rays, that, from their direction, would glance by and rase the Earth's surface, to be stopped and absorbed by the lower strata of the atmosphere. In such a case, the conical boundary of the Earth's shadow would be formed by certain rays exterior to the former and would be larger.

This is not the sole effect of the atmosphere in eclipses; but, another, totally of a different nature, results from it. Certain of the Sun's rays, instead of being stopped and absorbed, are bent from their rectilinear course, by the refracting power of the atmosphere; so as to form a cone of faint light interior to that cone which has been mathematically described as the Earth's shadow. The effect of this, or the phenomenon of which the preceding statement is presumed to be the explanation, is a reddish light visible on the Moon's disk, during an eclipse.

We will now proceed to shew how the time, duration and magnitude, of a lunar eclipse, may be computed.

Let NqM represent part of the Moon's orbit, vEN the ecliptic, N the node.

Suppose the Moon's place of opposition to be q, p being the corresponding place of the centre of the Earth's shadow, and



the latter to describe Ep, whilst the Moon's centre describes Mq. Let also

$m =$ ☽ 's horary motion in longitude,
$n =$ ☽ 's motion in latitude,
$s =$ ⊙ 's (or, the shadow's centre's) motion in longitude,
$\lambda =$ ☽ 's latitude when in opposition at q,
$t =$ time from q to M,
$c =$ distance of M from E (ME);

then, in the time t, the ☽ 's motion in longitude $= mt\ (vp)$,
in latitude $= nt\ (Mv - pq)$
the ⊙ 's motion in longitude $= st\ (Ep)$;

consequently, $Mv = pq + nt = \lambda + nt$, and $Ev = pv - Ep = mt - st$;

$$\therefore c^2\ (ME^2) = Mv^2 + Ev^2 = (\lambda + nt)^2 + (mt - st)^2,$$

which expression expanded produces a quadratic equation, of which t is the quantity to be determined, and the value of which will depend on that of c; or, if we assign to c such values as belong to the different *phases* of an eclipse, the results will be intervals of time between the happening of such phases, and the time of opposition, which latter time may be computed from the Tables of the Sun and Moon.

If in the preceding expression for t^2, we substitute, after expansion tan. θ instead of $\dfrac{n}{m - s}$, there will result

$$n^2 t^2 + 2\lambda n \sin.^2 \theta . t = (c^2 - \lambda^2) \sin.^2 \theta,$$

and if from this, by the Rule for the solution of a quadratic equation, we deduce the value of t, we shall have

$$t = \frac{1}{n}\left[-\lambda \sin.^2 \theta \pm \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}\right],$$

from which expression, as it has been stated, may be deduced values of the time corresponding to any assigned values of c.

For instance, if we wish to determine the time from opposition, at which the Moon first enters the Earth's penumbra, we must assume (see p. 721,)

$$c = P + p + \frac{D}{2} + \frac{d}{2}.$$

t has two values corresponding to the same value of c, the second of which will denote the time at which the Moon quits the penumbra. If we wish to determine the time at which the Moon enters the umbra, we must assume, (see p. 7~~21~~,)

$$c = P + p + \frac{d}{2} - \frac{D}{2}.$$

If we wish to determine the time when the whole disk has just entered the shadow, we must subduct d from the preceding value, and make

$$c = P + p - \frac{d}{2} - \frac{D}{2},$$

and similarly for other phases.

The two values (t', t'') of t are

$$t' = \frac{1}{n}\left[-\lambda \sin.^2 \theta + \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}\right],$$

$$t'' = \frac{1}{n}\left[-\lambda \sin.^2 \theta - \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}\right],$$

which values can never equal each other, except the quantity under the radical sign, that is, $c^2 - \lambda^2 . \cos.^2 \theta = 0$;

in which case the value of t, namely $-\frac{\lambda \sin.^2 \theta}{n}$, represents the *middle* of the eclipse, the distance (c) of the centres being $\lambda \cos. \theta$.

This value (λ cos. θ) of c corresponding to the middle of the eclipse, is the least distance, or, the nearest approach of the centres of the Moon and shadow. For, if by the rules for finding the maxima and minima of quantities, we deduce from the expression, p. 723, l. 3, the value of t, it will be found equal to $-\frac{\lambda \sin.^2 \theta}{n}$.

The nearest approach of the centres being known, the magnitude of the eclipse is easily ascertained. Thus, on the supposition that λ cos. θ is less than the distance $\left(P+p+\frac{d}{2}-\frac{D}{2}\right)$ at which the Moon's limb just touches the shadow, some part of the Moon's disk is eclipsed; and the portion of the diameter of the eclipsed part is

$$P+p+\frac{d}{2}-\frac{D}{2}-\lambda \cos.\ \theta.$$

The portion of the diameter of the non-eclipsed part, is the Moon's apparent diameter (d) minus the preceding expression, and, therefore, is

$$\lambda \cos.\ \theta+\frac{d}{2}+\frac{D}{2}-P-p.$$

If this expression should be equal nothing, the eclipse would be *just* a total one. If the expression should be negative, the eclipse may be said to be *more than* a total one, since the upper boundary of the Moon's disk would be below the upper boundary of the section of the shadow: and the distance of the two boundaries would be the preceding expression.

The preceding formulæ for the parts eclipsed, which are parts of the Moon's diameter, are usually expressed in twelfths of that diameter; which twelfths are, with no great propriety of language, called *Digits*. Thus, if the part eclipsed should be 24′ 52″, the Moon's diameter being 33′ 18″; then, the part eclipsed $=\frac{24' \, 52''}{33' \, 18''} \times \overset{\text{Digits.}}{12} = \overset{\text{Digits.}}{8.96}$.

By p. 723, the second root of the quadratic, or

$$t'' = -\frac{1}{n}[\lambda \sin.^2 \theta + \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}],$$

which is negative with respect to the other value t'; that is, if the first be previous to opposition, the latter is subsequent to it: hence the whole duration of that part of the eclipse which takes place between equal values of the distance of the centres is the sum of the two times, and therefore =

$$t' + (-t'') = \frac{2}{n} \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)}.$$

If in this expression we substitute that value of c, which is $P + p + \frac{d}{2} - \frac{D}{2}$, (see p. 723,) the quantity

$$\frac{2}{n} \sin. \theta \sqrt{(c^2 - \lambda^2 \cos.^2 \theta)},$$

denotes the time from the Moon's first entering, to her finally quitting the shadow or *umbra*. And, if we substitute for c, $P + p + \frac{d}{2} + \frac{D}{2}$, (see p. 723,) the resulting expression will denote the whole time of an eclipse, from the Moon's first entering till her finally quitting the *penumbra*.

EXAMPLE.

Of the Eclipse, which happened on March 17, 1764, *it is required to calculate the beginning, middle, and the end; also the number of Digits eclipsed.*

By the Lunar and Solar Tables it appears that the epoch, or the time of true opposition, happened on the 18th of March 1764, at $0^h\ 6^m\ 12^s$, mean solar time at Paris (reckoned from midnight).

By the above-mentioned Tables the following numerical results were obtained.

☽'s lat. at the time of opposition λ =	38′ 42″ N.	
☽'s horary motion in latitude ... n =	−3 26	(lat. decreasing)
☽'s horary motion in longitude .. m =	37 23	
☉'s horary motion in longitude .. s =	2 29	
☽'s apparent diameter .... d =	33 18	
☽'s corresponding hor^l. parallax P =	61 0	
☉'s apparent diameter .... D =	32 10	
☉'s corresponding hor^l. parallax p =	0 9.	

Hence, (see p. 722,)

$$\tan.\,\theta = \frac{n}{m-s} = -\frac{3'\,26''}{34'\,54''} = -\frac{206}{2094};$$

$$\therefore \theta = -5^0\,37'\,6''.5.$$

Hence, (see p. 724,) the middle of the eclipse, or,

$$-\frac{\lambda\,\sin.^2\theta}{n} = \frac{2322}{206} \times \sin.^2(5^0\,37'\,6''.5) = 6^m\,29^s.$$

This is the time reckoned from the epoch of opposition, which is March 18, $0^h\ 6^m\ 12^s$, consequently, the middle of the eclipse was March 18, $0^h\ 12^m\ 41^s$. Now, in order to find the times when the Moon first entered and when it finally quitted the shadow, we must first compute (see p. 723,) the corresponding values of c, and accordingly we have

$$c = \frac{d}{2} - \frac{D}{2} + p + P = 61'\,43'',$$

or, adding (see p. 721,) $1'\,40''$ for the effect of the Earth's atmosphere,

$$c = 63'\,23'',$$

which value being substituted in

$$-\frac{\lambda\,\sin.^2\theta \pm \sin.\,\theta\,\sqrt{(c^2 - \lambda^2\cos.^2\theta)}}{n},$$

the two resulting values (t'', t') of t are

(end of eclipse) $t'' = 6^m\ 29^s + 1^h\ 26^m\ 8^s = \quad 1^h\ 32^m\ 37^s$

(beginning) $t' = 6 \quad 29 \; - \; 1 \quad 26 \quad 8 \; = -1 \quad 19 \quad 39$

and consequently, the duration of the eclipse . . . $2^h\ 52^m\ 16^s$.

Since $t' = -1^h\ 19^m\ 39^s$ is negative, the commencement of the eclipse happened before the time of opposition, therefore, at Paris, it happened $1^h\ 19^m\ 39^s$ before March 18, $0^h\ 6^m\ 12^s$, that is, on March 17, $22^h\ 46^m\ 33^s$, and the eclipse terminated $1^h\ 32^m\ 37^s$ after the time of opposition March 18, $0^h\ 6^m\ 12^s$, that is, on March 18, $1^h\ 38^m\ 49^s$.

Since the preceding times are computed, according to the usage of French Astronomers, from midnight, and since, at the time of opposition, the Moon was nearly on the meridian, it is

plain that the whole of this eclipse must have been seen at Paris, and could not have been seen on the hemisphere opposite to that, on which Paris is situated.

The distance of the centres corresponding to the middle of the eclipse, and to the greatest *phase*, that is, to the greatest quantity of eclipsed disk, or

$$\lambda \cos.\ \theta = \ldots\ldots\ldots\ldots\ldots\ldots\ 38'\ 31''.$$

The eclipsed part, or

$$\frac{d}{2} - \frac{D}{2} + p + P - \lambda \cos.\ \theta = \ldots 23'\ 12'',$$

or (see p. 721,), accounting for the effect of atmosphere, 24′ 52″, and expressed in digits $= \overset{\text{Digits.}}{12} \times \dfrac{24'\ 52''}{33'\ 18''} = \overset{\text{Digits.}}{8.96}$.

In deducing the equation that involves the time (t) we supposed the Moon to describe the space Mq, whilst the centre of the shadow described Ep: and, expressed by means of the horary motions, the line pv was $= mt$*, and the line, which is the difference of Mv and pq, was $= nt$. According to this notation, therefore, the tangent of the inclination of the Moon's orbit $\left(\text{which} = \dfrac{Mv}{Nv}\right) = \dfrac{nt}{mt} = \dfrac{n}{m}$. Now the Moon approaches the shadow for two reasons, one of which is its motion in latitude, (nt), the other the *excess* ($mt - st$) of its motion in longitude above that of the shadow. Hence, its approach to the shadow would evidently be the same, if we suppose the centre of the shadow to be *quiescent*, the Moon to move with its proper motion in latitude (nt), and besides with an imaginary proper motion, in longitude, equal to the relative one, $mt - st$; with such an hypothesis the equation (see p. 722,)

$$c^2 = (\lambda + nt)^2 + (m - s)^2\, t^2,$$

would equally result, and the same conclusions relative to t, &c.

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* The Reader must observe that mt, nt, &c. are not lines like pq, &c. but the products of two algebraical symbols, m, t and n, t.

would also equally result. In this case, since we suppose the shadow to be at rest and the two motions of the Moon to be nt, and $(m - s)\,t$, the Moon must move towards the shadow along an imaginary orbit, the tangent of whose inclination would be $\frac{nt}{(m - s)\,t}$, or $\frac{n}{m - s}$, an inclination greater therefore than that of the real orbit.

This imaginary orbit, (which originates by a species of translation of the equation involving t,) has, for the purpose of graphically representing the phases of an eclipse, been invented by Astronomers, and been termed the Moon's *relative* Orbit. If we prolong the line pq below q, by a quantity equal to $n \times t$, so that the whole line, beginning from p, may be equal to $\lambda + nt$ $(\lambda = pq)$ and then, from the extremity of the prolonged line, draw a line parallel to pv, towards M, and equal to $(m - s)\,t$, and lastly, join p and the extremity of the line parallel to pv; the joining line will represent a portion of the relative orbit, and be equal to ME (c).

The *relative* orbit is a mere mathematical fiction, convenient enough for representing the phases of an eclipse, but not essential to their computation, as the very fact of the preceding computations, made without reference to it, sufficiently proves. If, however, by independent reasonings, it be established and laid down as the basis of investigation, then may all the preceding results relative to the duration and quantity of an eclipse be obtained. It may not be improper to note, that the artifice of computation which substitutes tan. θ instead of $\frac{n}{m - s}$, when geometrically exhibited, introduces the *relative* orbit.

In the preceding computations of the duration, &c. of a lunar eclipse, we have supposed the motion of the Sun in longitude, and the motions of the Moon in longitude and latitude to be uniform. This, during the short continuance of an eclipse, is nearly, but not exactly, true. The error of the supposition, however, may be corrected by means of the Lunar and Solar Tables, which give the true motions of the Sun and Moon for every

instant of time, and then the eclipse may be computed to the greatest exactness.

Since the computation of eclipses, (especially, of solar,) is attended with considerable difficulties, it is natural to search for expedients that may lessen them. Now, an eclipse depends on two circumstances, the syzygy of the Moon, and the proximity to the node of its orbit. The first circumstance, whether it be an opposition or a conjunction, recurs after a synodic period, or, 29 days. But, at the end of this period, the proximity of the Moon to the node of its orbit cannot be the same, in degree, as it was at the beginning. It must, according as the Moon is approaching or receding from the node, be less or greater. This arises from the *regression* of the nodes. But, the nodes still regressing, before they have performed a circuit of the heavens, an opposition or conjunction must happen, in which the Moon would be either exactly, or very nearly, at the same distance from the node, as it was at the beginning of the period. If, for the sake of illustration, we suppose the synodic period to be 30 days, and the Sun after quitting the node of the Moon's orbit, to return to the same after 330 days, then at the end of this latter period, and after eleven lunations, if the Sun and Moon should have been in conjunction, or opposition, at the beginning, they would be again so, and besides the Moon would be in the same degree of proximity to the node. If, however, the return of the Sun to the node should not be performed exactly in 330 days, but in 330 days 12 hours, then at the end of 661 days, after two revolutions with respect to the node and 60 lunations, the Moon would be in syzygy with the Sun, and at the same distance from the node, as it was at the beginning. Now, if the Moon, at different periods, be in syzygy with the Sun, and at the same distance from the node, the same phases of an eclipse must be always seen at those periods (supposing the mutual distances of the Moon, Sun, and Earth, not to alter). Hence, an eclipse computed for one period would serve for other periods, and, eclipses could be predicted; since, after the lapse of a certain number of days, they would recur.

A lunation, and the Sun's period with regard to the node of the Moon's orbit, are not of the values, which, in the preceding

illustration, we have supposed them to be. The former is $29^d\ 12^h\ 44^m\ 2^s.8$, (29.530588) the latter $346^d\ 14^h\ 52^m\ 16^s.032$ (346.61963). But, with these true values, the period of the recurrence of the Moon to the same position, relatively to the Sun and the node of its orbit, is to be determined on the same principles, which, indeed, are those which have been previously used on the occasion of the transits of *Venus* and *Mercury* over the Sun's disk, (see p. 613.). We must find two numbers in the proportion of 29.530588 to 346.61963: if not exactly, nearly so, employing the method of continued fractions. Now two numbers, nearly so, are 19 and 223; the Moon's node, therefore, after 223 lunations has, relatively to the Sun, returned 19 times to the same position. And accordingly at the end of 223 lunations, that is, of 18 years 11 days *, there are the same conditions requisite for an eclipse, as at the beginning; after such interval, then eclipses, solar as well as lunar, will recur, and in the same order. If we know, therefore, previous, we can predict subsequent, eclipses.

This simple method of predicting eclipses was known to the antient Astronomers. It, however, is not exact, since 19 to 223, is only an approximate ratio: even were it exact, still the lunar inequalities, the periodical and secular, would prevent the Moon from being at the end of $18^y\ 11^d$, or of $36^y\ 22^d$, &c. precisely at the same distance from the node, as at the beginning.

The method, however, may, with advantage, be used for ascertaining, very nearly, the happening of eclipses; after which, the exact times may be calculated by means of the Astronomical Tables.

By means of the period of 223 lunations, called by the Chaldean Astronomers, the *Saros*, eclipses may be predicted; but, independently of this, there is, for finding directly those syzygies at which eclipses may happen, the method of *Astronomical Epacts*, (see p. 7[illegible]).

* More exactly, $18^y\ 10^d\ 7^h\ 43^m$, or $18^y\ 11^d\ 7^h\ 43^m$, accordingly as four or five leap years happen in the interval of 223 lunations.

CHAP. XXXVI.

On Solar Eclipses.

An eclipse of the Sun, is caused by the interposition of the Moon between the Sun and Earth; in consequence thereof, the whole, or part of the Sun's light is prevented from falling on certain parts of the Earth's surface.

A spectator, deprived of the whole of the Sun's light, is involved in the Moon's shadow; deprived of part, in the penumbra.

A material circumstance of distinction exists between lunar and solar eclipses: the former are seen, at the same time, by every spectator who sees the Moon above his horizon. The latter may be seen by different spectators at different times; or may be seen by one spectator and not by another. The passage of the Moon's shadow across the Earth's surface, during a solar eclipse, has been properly likened to that of the shadow of a cloud.

In the case of the Moon, it was shewn, that, if that body were within certain limits of distance from the node of her orbit, an eclipse must happen in opposition; because, (see p. 712,) the shadow of the Earth, in all distances of the Moon and Sun, extends far beyond the lunar orbit. The length of the Moon's shadow must be determined as that of the Earth's has been, on the same principles and by similar formulæ. But, the result, in certain respects, will be different. The Moon's shadow will never extend far beyond the Earth, and sometimes will fall short of it. Hence, the happening of a solar eclipse will depend not solely on the ecliptic limits, but also on the relative distances of the Sun, Moon, and Earth.

In order to determine the length of the Moon's shadow, we may use the Figure of page 714.

$$\text{Now, by p. 717, } CE = \frac{Et}{\sin. \angle ECt},$$

$$= \frac{Et}{\sin. (\angle AES - \angle EAt)}.$$

In this case E must represent the Moon, and accordingly $\angle AES$, which is the apparent semi-diameter of the Sun seen from *the Moon*, is equal to

$$\text{apparent semi-diameter } \odot \text{ seen from } \oplus \times \frac{\text{dist. } \odot \text{ from } \oplus}{\text{dist. } \odot \text{ from } ☽},$$

and the angle EAt is the Sun's horizontal parallax belonging to the *Moon*, and equal, therefore, to

$$\odot\text{'s horizontal parallax for } \oplus \times \frac{☽\text{'s rad.}}{\oplus\text{'s rad.}} \times \frac{\text{dist. } \odot \text{ from } \oplus}{\text{dist. } \odot \text{ from } ☽}.$$

Hence, calling the radii of the Moon and Earth, r, R, and the distances of the Sun from the Moon, and Earth, k, K respectively, there results

$$\text{length of Moon's shadow} = \frac{r}{\sin. \left(\frac{D}{2} \times \frac{K}{k} - p\frac{rK}{Rk}\right)}$$

$$= \frac{r}{\sin. \left\{\left(\frac{D}{2} - p\frac{r}{R}\right)\frac{K}{k}\right\}}$$

$$= \frac{r}{\sin. \left\{\left(\frac{D}{2} - p\frac{r}{R}\right)\frac{P}{P-p}\right\}}$$

$$\text{For, since } p = \frac{R}{K}, \text{ and } P = \frac{R}{K-k}, \quad \frac{K}{k} = \frac{P}{P-p}.$$

By means of this formula, we have

	Length of Shadow.	☽'s Dist.
⊙ in apogee, ☽ in perigee	59.730	55.902
⊙ in perigee, ☽ in apogee	57.760	63.862

And this latter case is one of those mentioned in p. 731, and in which the Moon's shadow never reaches the Earth.

The formula for the length of the Earth's shadow has been adapted so as to express the length of the Moon's shadow. Similar alterations may be applied to the other formulæ. For instance, (see p. 715,)

the appa^t. semi-diam. of ⊕'s shadow $= \angle Emt - (\angle AES - \angle EAt)$.

Now we have already shewn (p. 732,) that

$$\angle AES - \angle EAt = \left(\frac{D}{2} - p\frac{r}{R}\right)\frac{P}{P-p},$$

and $\angle Emt$, (the Moon being at E, and the Earth at M,) equals the ☽ 's apparent semi-diameter $\left(\frac{d}{2}\right)$.

Hence,

the appa^t. semi-diam^r. of ☽ 's shadow $= \frac{d}{2} - \left(\frac{D}{2} - p\frac{r}{R}\right)\frac{P}{P-p}$

$$\left(\text{since see p. 651, } \frac{d}{2P} = \frac{r}{R}\right) = \frac{d-D}{2} \times \frac{P}{P-p}.$$

Hence, when the Moon's apparent diameter (d) equals the Sun's (D), the apparent semi-diameter of the Moon's shadow is equal nothing; or, the vertex of the conical shadow just reaches the Earth.

When the Moon's apparent diameter (d) is less than the Sun's (D), the expression for the apparent diameter of a section of the Moon's shadow is negative; in other words, the shadow never reaches the Earth.

In a similar manner may the formulæ for the penumbra of the Earth be transformed, and adapted to the case of the Moon.

In order to find the distance of the centres of the Moon's shadow and of the Earth, when the Earth's disk just touches the section of the Moon's shadow, we must add to the expression, l. 13, the apparent semi-diameter of the Earth, seen from the Moon, which, in other words, is the Moon's horizontal parallax (P). Hence

$$\text{distance} = P + \frac{D-d}{2} \times \frac{P}{P-p}.$$

From this expression the solar ecliptic limits may be computed, precisely as the lunar were (see p. 718,) and they will be found equal to $17^\circ\ 21'\ 27''$.

The same diagram and formulæ, as we have seen, apply equally to solar as to lunar eclipses; and, to a spectator placed in the Moon, our solar eclipses must appear, precisely, as lunar eclipses appear to us; the fictitious spectator might also compute the duration, and magnitude, of an eclipse caused by the shadow of the globe on which he is placed, by processes like those which have already been used, (p. 722,) in the case of lunar eclipses. The forms of the resulting equations, and the steps of the process, would be the same in each case. It would be only necessary to make such slight alterations as we have already made. And, under this point of view, there is no difference between lunar and solar eclipses. The computation of the one is as easy as that of the other. But, still the fact is, the subject of solar is much more difficult than that of lunar eclipses. There is then some material circumstance of difference between them, which it is now necessary to point out.

In the preceding computations relative to lunar eclipses, no consideration was had of any particular parts of the Moon's disk which might either be covered by, or approach within a certain distance of, the Earth's shadow. In the ingress, for instance, merely the time of contact was determined, and nothing said concerning the position of the point of contact relatively to any fixed point in the Moon's equator. The lunar latitude and longitude of the point of contact is a matter of indifference to the observer on the Earth's surface. But, to an observer at the Moon, the case is quite different: to such an one, the eclipse does not begin when the Earth's shadow comes in contact with the Moon's disk, but when it begins to obscure his station. Now, in the predicament of this fictitious observer at the Moon, during what to us is a lunar eclipse, is an observer at the Earth during a solar eclipse. It is necessary for him to know when, and how long, the shadow

of the Moon will obscure a station of an assigned longitude and latitude.

Solar eclipses then are more difficult of computation because more is required to be done in them, than in lunar eclipses. If in the investigation of the latter, there had been solved a problem, in which it was required to determine the time when a particular point on the Moon's surface was eclipsed, then from such solution we should possess the means of determining, what it is essential to determine, in solar eclipses.

The method, however, of computing lunar eclipses (given in pp. 722, &c.) may be adapted to solar; and in such a manner as to determine the times of the happening of the latter at an assigned place. This we will endeavour to explain.

First, that method may (making such substitutions as have already been made in pp. 722, &c.) be employed in computing the time and duration of a solar eclipse with reference to the *whole disk* of the Earth; that is, the eclipse being supposed to begin at the first contact between the Moon's shadow and any part of the Earth, and to end at the last contact.

At any time (t) included within the duration (T) of such an eclipse, we are able to compute the apparent distance of the centres of the Sun and Moon, supposing the spectator to be placed in the centre of the Earth. The problem is precisely the same as the one in p. 722, relative to a lunar eclipse. Corresponding to the time t, the Solar and Lunar Tables, will furnish us with the longitude of the Sun, the longitude and latitude of the Moon, &c.; such quantities in fact, as λ, m, ρ, &c.; and, involving these quantities precisely as they were in pp. 722, &c., an equation exactly similar to the one of p. 722, would result: and from its solution, since t is supposed to be given, c would result; but if c be assigned, then is t the resulting quantity.

If, instead of a spectator in the Earth's centre, we suppose one on the surface, in what respects and degree ought the conditions of the preceding problem to be changed? The latitudes and longitudes (l, λ), computed for the former spectator, cannot belong to the latter, because angular distances (and such are

latitudes and longitudes) seen from the centre are not the same as when seen from the surface. They differ however solely by *parallax*. If therefore the true longitudes and latitudes at any time be diminished by parallax, the resulting longitudes and latitudes (l', λ') will belong to a spectator on the Earth's surface, for the same time. These latter being substituted as in page 722, the equation

$$n^2 t^2 + 2\lambda' n \sin.^2 \theta \times t = (c^2 - \lambda'^2) \sin.^2 \theta,$$

will express the relation between t and c.

In finding therefore the time, at which, the apparent distance of the centres of the Sun and Moon should be of an assigned magnitude, or in finding the magnitude for an assigned time, the chief thing required to be done, is to diminish the angular distances, which the Astronomical Tables furnish us with, by the effects of parallax in the directions of those angular distances.

The angular distances, as we have seen (p. 735,) are measured along the circles of latitude and longitude. What we require then, are formulæ for computing the *parallaxes in longitude and latitude*. The investigation of such formulæ is the chief object of the ensuing Chapter.

That Chapter is on the *Occultation of fixed Stars by the Moon*. A subject which, equally with solar eclipses, requires the aid of formulæ for computing the parallax in longitude and latitude. The investigation of those formulæ might have been introduced into the present Chapter, but it was judged right to defer it to the next, because its subject may *mathematically* be viewed in the light of the simplest case of a solar eclipse. For, if from this last we make abstraction of all the ordinary phenomena, the two cases are similar. In the one, we have to find the apparent distance of the centres of the Sun and Moon; in the other, the apparent distance of the centre of the Moon and a fixed star. In each we must take the latitudes and longitudes from the Tables, and then correct such for parallax; but the latter case is somewhat the more simple, because it is necessary to compute the parallax in latitude and longitude for one body only, namely, the Moon; the other, the fixed star, having no parallax.

There is a third phenomenon, *The Transit of an inferior Planet over the Sun's Disk*, which is nearly similar to an occultation and a solar eclipse in its general circumstances, and is exactly so in its mathematical conditions. In the two latter phenomena, the Moon by its interposition obscures the light of the Sun, or suddenly extinguishes that of the star: in the former, the planet successively darkens parts of the Sun's disk; this effect then, like an occultation, is a species of eclipse. But, without any forced analogies or violation of the proprieties of language, it is a sufficient reason for classing these phenomena together, that it is *mathematically convenient* so to do. To each, the same equations and formulæ apply; and, as we shall hereafter perceive, they may all be employed in attaining the same object, the determination of the longitudes of places.

The next Chapter will put us in possession of the means of computing the apparent distance of the centres of the Sun and Moon. If that distance be the sum of the semi-diameters of those bodies, their disks will be just in contact, and the corresponding time will be that of the beginning or the end of an eclipse. Such, considering the practical use of solar eclipses in determining the longitudes of places, is the essential problem; and to that we shall restrict ourselves: still, it must not be forgotten, it is only one out of many that may be proposed on the same subject.

The times of the beginnings of solar eclipses can be exactly noted: which is the circumstance which gives them utility and distinguishes them from lunar. In order therefore that the observer may be prepared to note the times of the phases of an eclipse, he ought to know them approximately at least, by previous computation. This he may do by computing, for the several times included within the whole duration of the eclipse, the apparent distances of the centres of the Sun and Moon: and, then, from such results he may determine nearly (which is all he wants) the time when the distance shall be equal the sum of the semi-diameters of those bodies.

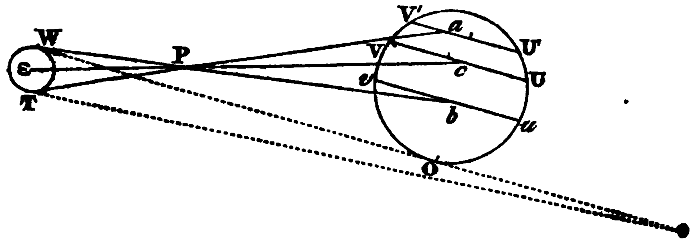
CHAP. XXXVII.

On the Occultation of fixed Stars by the Moon.

PARALLAX enters as a condition into almost all Astronomical calculations; because we agree to reckon, from the centre of the Earth, observations which we must make on its surface. The parallax in its greatest value (the horizontal,) being the greatest angle under which the Earth's radius can be seen from an heavenly body, is less, the more distant the body. Fixed stars are so distant that they have no parallax, or, at the most, a very small one. Were the Moon equally distant, her centre, or any point of her disk, would be seen at the same angular distance from a fixed star, whether the Earth's centre or its surface were the spectator's place. If her disk therefore were in contact with a fixed star, the contact would be seen, at the same instant of time, by an imaginary spectator in the Earth's centre, and by all spectators (to whom the Moon should be visible) on its surface. The same instant of time, however, would be differently reckoned by different spectators, according to the situation of their meridians. If 3^h were the time of observation at Greenwich, the time might be 7^h at a place to its east, or might be noon at a place to its west. And, in this case, the mere differences of the *reckoned* times of the happening of the phenomenon would be the angular distances of the several meridians, or the differences of *the longitudes* of the stations of the several observers.

The Moon, by reason of its great relative proximity, is more affected by parallax than any other heavenly body. Suppose in the Figure (which is intended subsequently to illustrate the transit

of *Venus*) $V'VOU$, &c. to be the Moon's disk, $W\epsilon T$ the Earth *, then a spectator at W would see a star ✻ in apparent contact with the point O in the Moon's disk, and (if the Moon's centre be supposed moving towards WO) in the instant of time immediately



previous to an occultation. A spectator at T would see the star ✻ separated from the Moon's disk; a spectator in ϵ, the Earth's centre would also see it separated but by a less angle. To these latter spectators the instant of contact, immediately preceding an occultation, would not have arrived. Hence, it is plain, that the *absolute time* of an occultation would be different to different observers; and, accordingly, the mere difference of the *reckoned* times of the happening of the phenomenon, would not, in all cases, give the difference of the longitudes of the places of observation. Account must also be made of that difference in the absolute time, which would be nothing, were it not for the effects of parallax.

The effects of parallax in longitude and latitude are usually computed by a process of considerable length, involving several subordinate ones. These latter, being distinct steps in the investigation, may be proposed as independent problems. And, on such occasions, authors have been accustomed so to treat a complicated process. They resolve it into its parts, and propose such for solution under the form of problems, and towards the beginnings of their treatises. The object in view, in this arrangement, is the accommodation of the student, who, it is intended, should thus separately subdue the parts of a formidable calcula-

* P and the lines VU, $V'U'$, &c. are of no use in the present illustration.

tion. But, in this case, he must be content to learn the solutions of problems, without discerning the objects of their application. He must take them on trust, and consider that, although not of independent and immediate, they may be of subsidiary and future, use.

In the present instance it is intended to resolve the process for computing the parallax in longitude and latitude into its several parts; previously to propose such parts as problems for solution; and then to proceed immediately to their use and application. On this plan, therefore, we are required to find

The right ascension of the mid-heaven, *or of the Medium Cœli.*

The altitude of the *Nonagesimal.*

The longitude of the Nonagesimal.

1st. *The Right Ascension of the Mid-Heaven.*

The right ascension of the *mid-heaven* has been already explained (see p. 527.). It is, at any assigned time, the right ascension of a point of the equator on the meridian at that time, or, should a star be then on the meridian, it is the right ascension of such star. In like manner should the Sun, either the true, or the imaginary mean, Sun, then the true right ascension of the former, or the mean longitude of the latter, would be the right ascension of the mid-heaven. Suppose, the star, or the Sun, to have passed the meridian and to be to the west of it, then the right ascension of the *Mid-heaven* must be the right ascension of the star, or of the Sun, plus the angular distance of the star or Sun from the meridian, that is, plus the *hour* or *horary angle* (see p. 10,) of the star or Sun. If the true Sun be used in the computation, the right ascension of the mid-heaven will be the

☉'s true right ascension + true time from meridian . . . . (*A*).

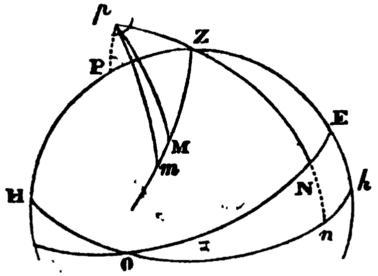
If the mean Sun, then the right ascension required is

☉'s mean longitude + mean time.

The Altitude of the Nonagesimal.

The *Nonagesimal* is that point of the ecliptic, which, at any assigned time, is the highest above the horizon. If *Hh* be the

horizon, ONE a portion of the ecliptic, and if ON be taken $= 90^{0}$, the point N is the nonagesimal, and its height is Nn;



Nn being the continuation of a vertical circle passing through N and the zenith Z.

Nn the height of the nonagesimal is (see *Trig.* p. 129,) the measure of the spherical angle EOH, the inclination of the ecliptic to the horizon.

$$pN\,(= \text{a quadrant}) = pZ + ZN,$$
$$\text{also } Zn\,(= \text{a quadrant}) = Nn + ZN;$$
$$\therefore\ pZ = Nn,$$

or, pZ is equal to the height of the nonagesimal and measures the inclination of the ecliptic to the horizon.

In order to find pZ, take P the pole of the equator, then, in the triangle PpZ, we have

PZ the co-latitude of the place,

Pp the obliquity of the ecliptic,

$\angle\, pPZ = 270^{0} -$ right ascension of the Mid-heaven.

Since the right ascension of E is the same as the right ascension of the Mid-heaven.

This then is that case of oblique spherical triangles, in which, from two sides and an included angle, it is required to find the third side; a problem of the same kind as that of the latitude of a star to be determined from its right ascension and north polar distance (see p. 159,) and which we shall similarly solve by the aid of a subsidiary angle (θ), (see *Trig.* p. 170).

Assume then θ such, that

$$\tan.^2 \theta = \frac{\sin.\ \text{obl}^y. \times \cos.\ \text{lat.} \times \text{ver. sin.}\ (90^\circ + \text{Æ of mid-heaven})}{\text{ver. sin. (co-latitude} - \text{obliquity)}},$$

then, $\text{ver. sin.}\ pZ = \text{ver. sin. (co-lat.} - \text{obliquity)} \times \sec.^2 \theta$*

$$\text{or, } \sin. \frac{pZ}{2} = \sin. \frac{1}{2}\ (\text{co-lat.} - \text{obliquity}) \times \sec.\ \theta,$$

and in logarithms,

$$\log.\ \sin. \frac{pZ}{2} = 10 + \log.\ \sin. \frac{1}{2}\ (\text{co-lat.} - \text{obliquity}) + \log.\ \sec.\ \theta.$$

The complement of the altitude (pZ) of the nonagesimal is ZN, and is sometimes called the *Latitude of the Zenith.*

Longitude of the Nonagesimal.

p, P being the poles of the ecliptic and the equator, the arc pP, if continued, must pass through the solstitial point; therefore, the longitude of P is 90°; and the longitude of N (the longitude of the nonagesimal) is

the longitude of P plus the angle PpN ($= PpZ$).

Now,

$\sin.\ Pp\hat{Z} = \text{cosec. height of nonagesimal} \times \sin.\ pPZ \times \cos.\ \text{lat.}$

or, (see *Trig.* p. 159,)

$$\cos.^2 \tfrac{1}{2}\, pPZ \,.\, \sin.\ pZ \,.\, \sin.\ pP$$

$$= \sin. \tfrac{1}{2}\, (pP + pZ + PZ) \,.\, \sin. \tfrac{1}{2}\, (pP + pZ - PZ),$$

from either of which expressions PpZ may be computed.

From the right ascension of the mid-heaven have been found the height and longitude of the nonagesimal; from these latter we may proceed to, what indeed are the chief objects of search, the parallaxes in longitude and latitude.

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* Examples to these several methods will be given under that belonging to the general problem of 'the distance of two bodies.'

Parallax in Longitude.

Let M be the true place of an heavenly body, m its apparent place depressed, in a vertical circle ZMm, by the effect of parallax, (see Chap. XII,) then the parallax in longitude is the angle Mpm, the measure of which, since Mm is small, is very nearly the fluxion, or the differential of the angle ZpM: and such we shall assume it to be. Now, let

L, l, be the latitudes of M, m, ($= 90^\circ - pM$, $90^\circ - pm$,)
K, k the angles ZpM, Zpm,
h, (pZ) the height of the Nonagesimal,
p, the common parallax, P ($= p$. sec. alt.) the horizontal,
a, the parallax in longitude; δ the parallax in latitude,
Z, z, the zenith distances ZM, Zm.

Then, by *Trigonometry*, p. 157, we have

$$\cot. z . \sin. h = \cot. k . \sin. \angle pZm + \cos. h . \cos. \angle pZm.$$

Of this equation take the *differential* or fluxion, and, since $\angle pZm$ is constant, and dk or $\dot{k} = a$, and dz, or $\dot{z} = p$, there results

$$p . \frac{\sin. h}{\sin.^2 z} = a . \frac{\sin. \angle pZm}{\sin.^2 k}.$$

But,

$$\sin. pZm = \sin. k \times \frac{\sin. pm}{\sin. Zm} = \sin. k \times \frac{\cos. l}{\sin. z};$$

$$\therefore a, \text{ the parallax in longitude, } = \frac{p}{\sin. z} \times \frac{\sin. h . \sin. k}{\cos. l},$$

$$= P \frac{\sin. h . \sin. k}{\cos. l} \text{ (very nearly).}$$

In this expression $k = K + dk = K + a$; $\therefore a$, the quantity sought, is contained in the formula that is meant to express its value. This is a frequent case in which there is an appearance of arguing in a circle. In order to evade such arguing we must approximate to the value of a, by supposing, in the first case, k to equal K: thus, first find a value (e) of a from this expression

$$a\ (e) = P \frac{\sin. h . \sin. K}{\cos. L},$$

then investigate a nearer value of a, from

$$a = P \cdot \frac{\sin. h \cdot \sin. (K + e)}{\cos. L},$$

and, if this last value be not sufficiently accurate, the above process must be repeated.

Parallax in Latitude.

By a formula similar to that which we have just used, and which differs from it only, in the circumstance of the angle k being used for pZm, l for z, &c., we have

in $\triangle\ Zpm$, $\tan. l \sin. h = \cot. pZm \cdot \sin. k + \cos. h \cdot \cos. k$,

in $\triangle\ ZpM$, $\tan. L \sin. h = \cot. pZm \sin. K + \cos. h \cdot \cos. K$,

eliminate, from these two equations, $\cot. pZm$, and there results

$$\sin. h(\tan. L. \sin. k - \tan. l. \sin. K) = \cos. h(\sin. k \cos. K - \cos. k. \sin. K)$$

$$= \cos. h \times \sin. (k - K).$$

Now, $k - K = a$, and $\sin. (k - K) = \sin. a = a$ (nearly) $=$ $P \dfrac{\sin. h \cdot \sin. k}{\cos. L}$: substituting $\therefore$ and dividing by $\sin. h \times \sin. k$,

$$\tan. L - \tan. l \frac{\sin. K}{\sin. k} = P \frac{\cos. h}{\cos. L};$$

$$\therefore \tan. L - \tan. l = P \frac{\cos. h}{\cos. L} - \tan. l \left(1 - \frac{\sin. K}{\sin. k}\right)$$

$$= P \frac{\cos. h}{\cos. L} - \frac{\tan. l}{\sin. k} (\sin. k - \sin. K).$$

$$\text{Now, } \tan. L - \tan. l = \frac{\sin. (L - l)}{\cos. L \cdot \cos. l},$$

$$\text{and } \sin. k - \sin. K = 2 \cdot \cos. \left(\frac{k + K}{2}\right) \sin. \left(\frac{k - K}{2}\right),$$

and since, $k - K = a$, $\dfrac{k + K}{2} = K + \dfrac{a}{2}$: substitute, and

$$\frac{\sin. (L - l)}{\cos. L \cdot \cos. l} = P \frac{\cos. h}{\cos. L} - \frac{2 \tan. l}{\sin. k} \cdot \left\{\cos. \left(K + \frac{a}{2}\right) \sin. \frac{a}{2}\right\}.$$

But sin. $(L - l) = \sin. dl = \sin. \delta = \delta$, nearly, and sin. $\frac{a}{2} = \frac{a}{2}$

$$= \frac{P \sin. h \sin. k}{2 \cos. L};$$

$$\therefore \delta, \text{ the par. in lat.}, = P \cos. h. \cos. l - P. \sin. h \sin. l \times \cos. \left(K + \frac{a}{2}\right)^*$$

This expression, since $l = L - \delta$, is under the same predicament as the former one, (p. 743,) and must be treated in the same manner; that is, we must find a value of δ by supposing $l = L$, and then a nearer value. Since the Moon's latitude is never very large, and at the time of an eclipse (for computing which the above expressions are useful) is always very small, (and consequently sin. l is very small) we may assume, as a first step in the approximation,

$$\delta = P \cos. h \cos. L \; (= f \text{ suppose,})$$

and then the second step may be made by computing δ, from

$$\delta = P \cos. h \cos. (L - f) - P \sin. h \sin. (L - f). \cos. \left(K + \frac{a}{2}\right)$$

and the investigation continued will give more exact values of δ, the parallax in latitude †.

The formulæ for computing the parallaxes in longitude and latitude have been deduced by, what has technically been called, *the Method of the Nonagesimal.* This method, of no recent invention, naturally suggested itself, as Lalande observes, to the mind of Kepler. For, parallax takes place in a vertical circle, therefore, if the heavenly body were situated in a vertical circle, such as $pZNn$ passing through N the nonagesimal point, the effect of parallax, in such a circle, would be nothing in longitude but would take place, altogether, in latitude; since ON, the

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* See *Mem.* Gottingen, tom. II, p. 168; where Mayer has given, very nearly, the same expressions; also Lalande, tom. II, p. 305. Edit. 3.

† The expressions for the parallaxes in right ascension and declination may easily be deduced from the preceding processes. We must then consider p to be the pole of the equator.

ecliptic, is perpendicular to pZN. Again, if the Moon, always near to the ecliptic at the time of an eclipse, should also be near to the nonagesimal, then the greater its altitude the less would be the parallax in latitude, (see Lalande, tom. II, p. 291.)

Distance of the Moon and a Star at the time of an Occultation.

Computing by the preceding formulæ the parallaxes, we must apply them, with their proper signs, to the true longitudes and latitudes furnished by the Tables, or by observation, and the results will be the apparent longitudes and latitudes of the centre of the Moon and of the star. Suppose these to be l, l', k, k', respectively; then, in order to find the distance (D), we have (in a triangle such as Mpm, Fig. p. 741), the two sides $90^0 - l$, $90^0 - l'$ (analogous to Mp, mp), and the included angle, $k - k'$ (analogous to Mpm); and D is the side opposite to the angle $k - k'$: therefore, (*Trig.* pp. 139, 172, &c.),

$$\cos. D = \cos. l . \cos. l' \cos. (k - k') + \sin. l . \sin. l',$$

and substituting for $\cos. D$, &c. $1 - 2\sin.^2 \frac{D}{2}$, &c. there results

$$\sin.^2 \frac{D}{2} = \sin.^2 \left(\frac{l - l'}{2}\right) + \cos. l . \cos. l' . \sin.^2 \left(\frac{k - k'}{2}\right),$$

whence D may be deduced, and most conveniently, by means of a subsidiary angle, (see the page just referred to).

The preceding method is not confined to the case of an occultation, but is equally applicable to the finding of the distances of the Sun and Moon during a solar eclipse, and of the Sun and an inferior planet during a transit. And, in all the cases, since the distances are small, a more simple formula for computing D may be introduced. For, D may be considered as the hypothenuse of a right-angled triangle, the sides of which are $l - l'$, and $(k - k) \cos. l$*, in which case

* For $k - k'$ is the arc on the great circle, $(k - k').\cos. l$, on the parallel; for instance, in Fig. p. 9, if $ab = \angle\, aPb\ (k - k')$ $ss' = ab . \cos. sb = (k - k') \cos. sb$.

$$D^2 = (l - l')^2 + (k - k')^2 . \cos.^2 l$$
$$= (l - l')^2 \left\{1 + \left(\frac{k - k'}{l - l'}\right)^2 \cos.^2 l\right\};$$
$$\therefore D = (l - l') . \sec. \theta,$$
$$\text{making } \tan. \theta = \frac{k - k'}{l - l'} . \cos. l.$$

The latter expression for the value of D is easily deducible from the former, by substituting in the former $\frac{D}{2}, \frac{l - l'}{2}$, &c. instead of their sines, which may be done with inconsiderable error, by reason of the smallness of those angles, during the contiguity of the Moon and star, &c.

The first term of the expression for $\sin.^2 D$, (see p. 746,) is $\sin.^2 \left(\frac{l - l'}{2}\right)$. In which expression l, l', are the apparent latitudes, therefore if δ, δ', were the parallaxes, and Δ the difference of the true latitudes, we should have

$$l - l' = \Delta + \delta - \delta'.$$

Suppose now one of the bodies (that to which the latitude l' belongs) to have no parallax in latitude, but the other to have a parallax equal to $\delta - \delta'$, then, still as before,

$$l - l' = \Delta + (\delta - \delta'),$$

and a similar result will hold good with regard to $\sin.^2 \frac{k - k'}{2}$; therefore, if the coefficient of this latter term, instead of being $\cos. l . \cos. l'$, were a constant quantity a, for instance, (or involved merely the *difference* of the parallaxes), the distance D would result precisely of the same value $\sin.^2 D$ from the expression

$$\sin.^2 \frac{D}{2} = \sin.^2 \frac{l - l'}{2} + a . \sin.^2 \frac{k - k'}{2},$$

if, instead of assigning to each body its proper parallax, we suppose one to be entirely without, and *attributed* to the other an imaginary parallax in latitude and longitude, equal to the difference

of the real parallaxes. And in this case, the rule given by Astronomers, (see Lalande, 434, tom. II, and *Cagnoli*, p. 463,) would be proved to be true. Since, however, the coefficient cos. l. cos. l', is not a constant quantity such as a, but [since it equals $\frac{1}{2}$ cos. $(l - l') +$ cos. $(l + l')$], involves, besides the difference, the *sum* of the parallaxes, the rule is not perfectly exact. It, however, is nearly so, since $\sin.^2 \dfrac{k - k'}{2}$, which is multiplied into cos. l. cos. l', is a very small quantity.

We have spoken of the general case of the Problem, when the distance of the centres of two heavenly bodies is to be found. But, if we speak of each particular case, then we must say, the rule is slightly inaccurate in a solar eclipse and in a transit, but exact in an occultation, since one of the bodies, the fixed star, is devoid of parallax.

The *Distance of the Centres* is the last step in the mathematical process belonging to the subject of the occultation of a fixed star by the Moon; and, since the process is somewhat complicated, we will endeavour to illustrate it, and its subordinate methods, by an Example.

Required the apparent Distance of Antares *from the Centre of the Moon at the instant of Immersion, which was observed at Paris in April* 6, 1749, $13^h\ 1^m\ 20^s$, *Apparent Time**.

(1.) *Right Ascension of the Mid-Heaven.*

Convert the time into degrees and take from the Tables the Sun's longitude, and we have (see p. 740,)

$$\text{Æ of Mid-heaven } (A) = 15^0\ 58' + 195^0\ 29'$$
$$= 211^0\ 18'$$

Since, $15^0\ 58' = \odot$'s Æ,

and $195\ \ 20 = 13^h\ 1^m\ 20^s$.

* Lalande, tom. II, pp. 437, &c.

(2.) *Altitude of the Nonagesimal,* (*see* 1st *Form*, p. 742,)

log. sin.	23°	28′	22″	(obliquity)	9.60022*
cos.	48	38	50	(lat. cor. see p. 329,)	9.82000
ver. sin.	301	18	0	(90° + *A*)	9.68167
					29.10189
ver. sin.	17	52	48	(co-lat. – obliquity)	8.68395 . . . . (*a*)
					20.41794 = 2 log. tan. θ
2 sec.	58	16	54	(θ)	20.55845
				(*a*)	8.68395
				20 + log. ver. sin. pZ =	29.24240

∴ pZ (h), the altitude of the nonagesimal, is 34° 23′ 9″.

(3) *Longitude of the Nonagesimal,* (*see Form*, p. 742.) †.

pZ (h)	34°	23′	9″	. . . . log. sin. =	9.75186
Pp	23	28	22	. . . . sin.	9.60022
PZ	41	21	10	. . . . (*b*)	1935208
sum . . =	99	12	41		
½ sum	49	36	20.5	. . . . log. sin.	9.88172
½ sum – PZ	8	15	10.5	. . . . sin.	9.15697
				(20 added)	39.03869
				(*b*)	19.35208
				2 log. cos. PpZ =	19.68661

∴ PpZ = 91° 36′ 30″, and consequently, (see p. 742,) the longitude of the nonagesimal = 181° 36′ 30″.

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* Five decimals are sufficient: more, such is the nature of the process, would not add to the accuracy of the result.

† The angle PpZ being nearly 90°, is the reason, why it is expedient to use the second, (see p. 742,) of the formulæ, which, in the first instance, gives only half the angle PpZ. For a more full explanation of this point, consult *Trig.* Chap. V.

Hence, since by the Lunar Tables the longitude of the Moon was 245° 31′ 42″.4, K, or the Moon's distance from the nonagesimal, (see Fig. p. 741,)

is 245° 31′ 42″.4 − 181° 36′ 30″ = 63° 55′ 12″.

(4.) *Parallax in Longitude*, (see p. 743,)

log. 0° 57′ 16″.2 (P, from Tables)	3.53608	sum =
log. sin. 34 23 9 (h)	9.75186	13.28890
Ar.com. cos. 3 47 58.7 (L ☽'s true lat.)	0.00096	
sin. 64 10 * ($K + a$)	9.95427	
(rejecting 10)	3.24317 = log. 29′ 10″	

∴ ϵ, or the first approximate value of a, is 29′ 10″, and

$K' + \epsilon = 64° \ 24' \ 22''$,

log. sin. 64° 24′ 22″ ($K + \epsilon$)	9.95515
Sum (see p. 744,) rejecting 10	3.28890
(rejecting 10)	3.24405 = log. 29′ 14″.1;

∴ a, the parallax in longitude, is 29′ 14″.1.

(5.) *Parallax in Latitude*, (see p. 744.)

Computation of the first part of the expression,

log. P	3.53608	sum = 13.45267
log. cos. 34° 23′ 9″ (h)......	9.91659	
cos. 3 47 58.7 (L)	9.99903	
(rejecting 20)	3.45170 = log. 47′ 9″; ∴ 47′ 9″	

is the first approximate value of δ.

Again,

log. cos. 4° 35′ 7″.7 ($L + \delta$)	9.99861
log. P + log. cos. h....	3.45267
(rejecting 10)	3.45128 = log. 47′ 6″.7, 2d value of δ.

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* K (see l. 4,) = 63° 55′ 12″, and, since a is some small quantity, it is *conjecturally* taken, in the first trial, equal to 14′ 48″, which added to K, makes $K + a = 64° \ 10'$.

Computation of the second part of the expression,

log. $P \times$ sin. h (see p. 745, l. 3.) 3.28794

log. cos. $64^\circ\ 9'\ 49''\ \left(K + \frac{a}{2}\right)$ 9.63929

sin. 4 35 8.7 (☽ 's latitude) 8.90283

(rejecting 20) 1.83006 = log. $1'\ 9''$

Since the Moon's latitude was south, this last part ($1'\ 9''$) of the parallax in latitude must be added; consequently, the whole parallax in latitude (δ) = $47'\ 6''.7 + 1'\ 9'' = 48'\ 15''$, nearly. Hence, applying the parallaxes thus found to the true longitude and latitude,

☽ 's apparent long. = $245^\circ\ 31'\ 42''.4 + 29'\ 14''.1 = 246^\circ\ 0'\ 56''.5$

☽ 's apparent lat. = 3 47 58.7 + 48 15 = 4 36 13.7.

(6.) *Apparent Distance of the Moon and Antares,* (see p. 747.)

Long. of *Antares* (k') . . $246^\circ\ 16'\ 19''.2$ . . lat. (l') $4^\circ\ 32'\ 10''.5$

☽ 's longitude (k) . . . . 246 0 56.5 . . lat. (l) 4 36 13.7

$k' - k$ . . . . 0 15 22.7 . . $l - l'$ . . 0 4 3.2

$\therefore$ log. cos. $4^\circ\ 34'\ 12''\ \left(\frac{l + l'}{2}\right)$ * 9.9986171

log. . . . . 0 15 22.7 . . . . 2.9650605

Ar. comp. log. 0 4 3.2 . . . . 7.6140364

10.5777140 = log. tan. θ

log sec. 75 11 21 (θ) 10.5923906

Ar. comp. log. 0 4 3.2 . . . . 7.6140364

log. $951''.38$ = 2.9783542

therefore the distance required is $15'\ 51''.38$.

By the preceding process the *apparent* distance of a fixed star and of the Moon's centre has been found at the instant of *occul-*

* $\frac{l+l'}{2}$ used instead of l.

tation. A process, almost entirely the same, will give the distance of the Sun and Moon in a solar eclipse, and the distance of the Sun, and of an inferior planet, during the *transit* of the latter across the Sun's disk. The *difference* in the processes is pointed out in the Rule of p. 748: which Rule directs us to *suppose* one body to be devoid of parallax, and the other to be invested with a parallax, equal to the difference of the parallaxes of the two bodies.

The above process, as it stands, is rather long and would have been much more so, had we deduced from Tables, the Moon's real longitude and latitude. But we, in fact, know the latter quantities from the Nautical Almanack, or may deduce them by interpolation. The computers of *occultations*, are so enabled to abridge their labours. The utility of such labours will be more fully explained in a subsequent Chapter: but we will not dismiss the present without giving to the students a slight idea of the principle and manner of using the result of the preceding computations.

The Moon's latitude and longitude (see p. 746,) are computed for the instant of time, at which the star is on the Moon's disk. When the *time is given* we can, from the Lunar Tables, or from the results from those Tables registered in the Nautical Almanack, compute directly, or by interpolation, the Moon's latitude, longitude, and semi-diameter. But, since the Nautical Almanack, (confining our views to its results) is computed for Greenwich, we cannot, should the occultation be observed at Cambridge, determine the time at the former place, except we know how much it is to the *west* of the latter place. For instance, an occultation is observed at Cambridge, at 11^h: the Moon's latitudes are expressed in the Nautical Almanack for *Greenwich*, noon and midnight: we must not, therefore, by interpolation, compute the latitude corresponding to 11^h, but the latitude to 11^h minus corresponding the time due to the difference of the longitudes of Greenwich and Cambridge. The determination, however, of such *difference* is one of the special uses of the problem. The thing, therefore, requisite to be known in the process of solution, is the result of such process. We must,

therefore, assume some quantity as the difference, and compute, agreeably to such assumption, the Moon's latitude and longitude: thence, as it is pointed out in the preceding pages, we compute the distance of the Moon's centre, and of the star on its disk: such distance is the Moon's semi-diameter. But we can also determine the Moon's semi-diameter, by interpolating between the values expressed in the Nautical Almanack, for noon and midnight, its value corresponding to 11^h minus the assumed time of the difference of the longitudes of Greenwich and Cambridge. Should that difference be assumed, as it probably will be, erroneously, the two values of the semi-diameter compared together will not agree. The quantity of their disagreement will become an index of the error of the original assumption, and the means of amending it: and, by repetition of process, of completely correcting it.

By computing the parallaxes in longitude and latitude, we have, in the preceding pages, deduced the Moon's apparent longitudes and latitudes from her true, and thence the apparent distance of the Moon from the star. If we reverse the process, we may deduce the true distance of the Moon and star: and some authors make the same use of the true, as, according to the above explanation, may be made of the apparent, (see Vince, vol. I. pp. 334, &c.)

CHAP. XXXVIII.

On the Transits of Venus and Mercury over the Sun's Disk.

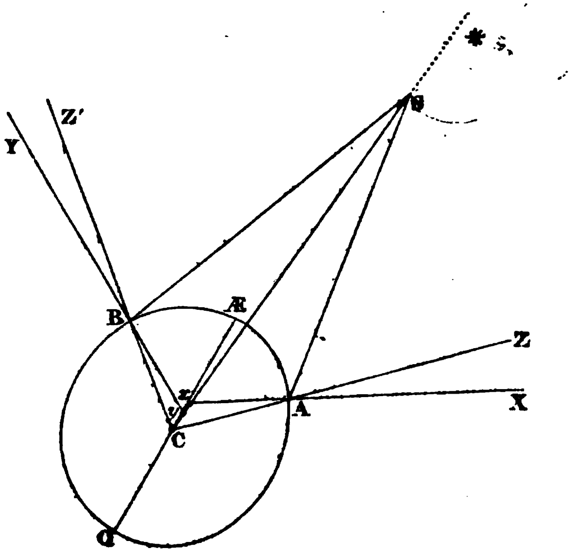
We have already stated in p. 736, that the phenomena of eclipses, occultations, and transits are very nearly alike in their general circumstances, and exactly alike in their mathematical theories. In those theories, the essential problem to be solved is the apparent angular distance of two heavenly bodies, in apparent proximity to each other, when viewed by a spectator on an assigned station on the Earth's surface.

In an eclipse and occultation, the Sun's parallax is supposed to be known: were it supposed to be known in a transit, there would be an additional circumstance of similarity between its theory and the theories of the former phenomena: for, they would have the same object, and would equally serve to the determination of the longitudes of places. And, in point of fact, this is the present state of the case. One transit of *Venus* has already answered the special purpose of determining the parallax of the Sun, and future transits may be used, either to confirm the accuracy of that determination, or for the general purposes which *eclipses*, in their extended signification, (see p. 736,) are made subservient to.

It is the object of the present Chapter to explain the use that *has been made* of the transit of *Venus*; or, to shew the special use of that phenomenon in determining the important element of the Sun's parallax.

The Sun's parallax is the angle subtended at the Sun by the Earth's radius; which angle can be found, if another subtended

by a chord, lying between two known places, can. And to find this latter angle is the object of the method given in Chap. XII, pp. 325, &c. If we refer to that Chapter we shall find the angle ASB to be the object of investigation. Now, in its instrumental measurement, an error of three or four seconds may be committed; which, in the case of the Moon, the parallax of which



is about 1°, is of little consequence, but a probable error of that magnitude in the case of the Sun, the parallax of which is less than nine seconds, would render the result of the method so uncertain, as entirely to vitiate it.

Retaining the principle of the method, Astronomers have sought to correct its error, by *computing*, instead of instrumentally measuring, an angle such as ASB, or an angle from which it may be immediately deduced.

Suppose, for the sake of illustration, S to be a point in Venus's disk, and BS continued to be a tangent to the Sun's disk: then the direction of a line AS would be to the left of the Sun's disk. In other words, the moment of contact or ingress would have arrived to a spectator at B, but not to a spectator at A. It would,

however, arrive some minutes after, when by the retrograde motion (see p. 556,) of *Venus*, the line AS, always a tangent to the disk of *Venus*, should become one to that of the Sun. Suppose AS, in this latter direction (to the right of its present position) to intersect BS produced in some point S' situated in the Sun's disk: then, the angle SAS' is proportional to the time elasped between the contacts at B and A: which time is known from observation and the ascertained difference of longitudes of the places B and A: suppose it t, and let h be the horary approach of *Venus* to the Sun (about $240''$); then,

$$1 : t :: h : ht, \text{ which is equal to the angle } SAS',$$

which is by these means *computed*.

SAS' being known, $SS'A$, or $AS'B$, may be determined from the known ratio between SA and SS'.

The preceding is a very imperfect description of the method that was actually used in the problem of the transit of *Venus*. But it shews the principle of the method and the reason of its superior accuracy: for, since the time of contact can be observed to be within three or four seconds, or since the limit of the error in time is about three seconds, and since the excess of the horary motion of *Venus* above the Sun's is $240''$, that is, $4''$ in 1^m, or $\frac{1''}{15}$ in 1^s, an error of 6^s (3^s at each place of observation) would only cause an error of $\frac{6''}{15}$ in the estimation of the angle SAS', and an error in the estimation of $SS'A$, (on which the parallax depends) less in the proportion of SA to SS', that is, in the case of *Venus*, of one to two and a half nearly.

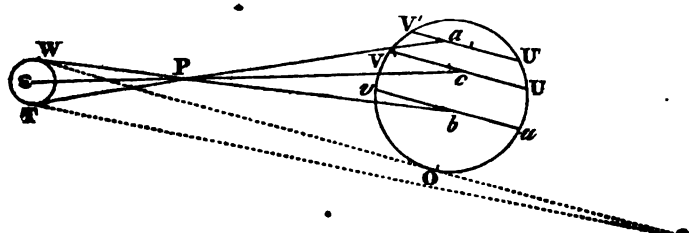
The imperfection of the method, as it has been described, consists in this; that it requires to be known, what it is very difficult to determine, the difference of the longitudes of the places A and B. For, t is the difference of *actual* or *absolute time*, which depends on the reckoned time at each place of observation, and the difference of the longitudes of those places. If the contact was observed at Greenwich at $3^h\ 40^m$, and

at a place 15° east of Greenwich, at 4^h 41^m, the difference in absolute time would be only 1^m; since 1^h, in the reckoned time, is entirely due to the difference of the meridians. We shall, however, in the subsequent pages, see a method of getting rid of the imperfection which we have just noted.

The longitude of the Cape of Good Hope, which had been long the station of an European Colony, and where the transit of 1761 was observed, was known to a considerable degree of accuracy. That of Otaheite, where it was expedient to observe the transit of 1769, was not known. And, from the difficulty of ascertaining with sufficient precision this nice condition of the longitude, Astronomers, by modifying their process of calculation, have got rid of it entirely. Instead of observing the ingress, they observe the duration of the transit, and from the difference of durations, at different places, deduce the difference of the parallaxes of *Venus* and the Sun, and then the Sun's parallax.

The difference in the durations of transits does not amount to many minutes. To make it as large as possible, it is expedient so to select the places of observation, that, at one, the duration should be accelerated, at another, retarded beyond the *true time* of duration; which true time is supposed to be that which would be observed at the Earth's centre.

If P were *Venus*, ϵ the Earth, W a place towards the north pole (Wardhus for instance) and T (Otaheite) towards the south, and $V'V$, &c. the Sun's disk, then the *true* line of transit, seen from the centre ϵ would be VU: from W, $v\,u$ would be the line; from T, $V'U'$. If T should be the *true duration* of the transit,



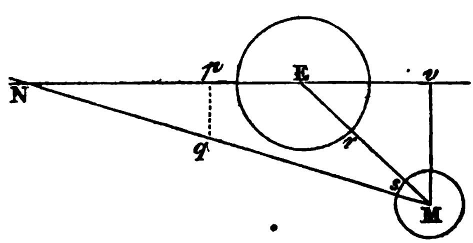
or the time of describing VU, then the time of describing $v\,u$ nearer to the Sun's centre than VU, would be $T + t$: of describ-

ing $V'U'$ more remote than VU from the Sun's centre, $T-t'$: and, accordingly, the difference of the durations of the transits seen from T and W, would be $T+t-(T-t')=t+t'$. This, as it is plain, is entirely the effect of parallax, and, as it is also plain, the effect is compounded of the parallaxes of Venus and the Sun: since changes in the distances of P and of the Sun will produce changes in the dimensions of the lines $V'U'$, vu.

We will now proceed to treat the subject mathematically, and to deduce, by means of a simple equation, the *difference* of the parallaxes of Venus and the Sun. That difference being determined, the values of both the parallaxes may be deduced by means of Kepler's law relative to the periods of planets, and their distances from the Sun.

In the subsequent mathematical process we shall have a proof of what we have more than once asserted, namely, the similarity of the mathematical theories of eclipses, occultations, and transits. For, T, $T+t$, $T-t'$ will be computed by means of the formula employed in Chap. XXXV. The only difference in the computation of T and of $T+t$ consists in assuming in the former, the angular distances seen from the Earth's centre and given by the Astronomical Tables, and in the latter, those angular distances *corrected* for the effects of parallax in longitude and latitude.

In the above-mentioned formula, the time and the apparent angular distance of two heavenly bodies were involved. And the diagram employed on that occasion will suit the present*. Instead of E and M representing the centres of the Earth's shadow and



* The same diagram will serve for an occultation, M being the Moon, and E the star.

the Moon, let them represent the centres of the Sun and Venus; then, *EM* will represent the distance of their centres previous to a transit, or after one: and, the Tables of the Sun and of the planets, will, as in an eclipse (see p. 725,) furnish us with quantities analogous to λ, m, n, &c. Suppose then, at the time of conjunction,

♀'s lat. .... λ ..... horary motion in lat. ..... n
♀'s long. ...... l ...... horary motion in long. ..... m
☉'s horary motion in long. ..... s.

If we form an equation, precisely as the one in p. 722, was formed, we shall have

$$n^2t^2 + 2\lambda nt \,.\, \sin.^2\theta = (c^2 - \lambda^2) \,.\, \sin.^2\theta,$$

$$\text{whence, } t = \frac{1}{n}\left[-\lambda \sin.^2\theta \pm \sin.\theta \sqrt{(c^2 - \lambda^2 \cos.^2\theta)}\right],$$

t being the time from conjunction, and c the distance of the centres.

In this equation substitute, instead of c, the sum of the apparent semi-diameters of the Sun and *Venus*, and the resulting time will be that of the first or last *exterior contact*: substitute the difference, and the resulting time will be that of the first or last *interior contact*. The duration of a transit is the difference between the times of the last and first exterior contacts, and is to be found exactly as the duration of an eclipse was in pp. 726, &c.

The times which we have mentioned, as resulting from the preceding equation, would be noted by a spectator in the Earth's centre: they belong to the points V, U, and the line VU. But to a spectator at T, for instance, the contact instead of at V would appear to take place at V'; and, it would appear to happen at a time, different from (T') the computed time of its happening at V, at $T' + t'$, for instance, t' being a small quantity and entirely the effect of parallax.

The latitudes and longitudes of *Venus* and the Sun continually altering, those quantities at the time $T' + t'$ from conjunction would be different from what they were at the time T': their change would be proportional to t'. The time T' being computed from the preceding equation, the corresponding latitudes and

longitudes may be taken from the Tables, or may be easily computed from their values at the time of conjunction. At this latter time, we have supposed the latitude of *Venus* to be λ. It is convenient for us to use that symbol (λ) to denote the latitude at the time T' of contact; let also the corresponding longitudes of *Venus* and the Sun be l, l'; and the horary motions m, n, s: then (see p. 722,) at the time t' from contact,

$$♀\text{'s long.} \ldots\ldots l + mt' \quad ♀\text{'s lat.} \ldots\ldots \lambda + nt',$$
$$\odot\text{'s long.} \ldots\ldots l' + st'.$$

And accordingly, the distance of the centres (such as EM) would be the hypothenuse of a right-angled triangle, of which the sides, are, respectively, $(l + mt') - (l' + st')$, and $\lambda + nt'$.

These angular distances belong to the centre of the Earth; but when they are diminished, as in the case of an occultation, (see p. 746,) by the parallaxes in longitude and latitude, they are made to belong to a spectator on the Earth's surface. Let the parallaxes in longitude be a, a'; in latitude δ, δ'; then, the sides of the right-angled triangle are

$$(l + mt' - a) - (l' + st' - a'), \text{ and } \lambda + nt' - \delta + \delta',$$
$$\text{or } l - l' + (m - s)t' - (a - a'), \text{ and } \lambda + nt' - (\delta - \delta').$$

The hypothenuse is the distance of the centres. But, the time is that at which a contact of the limbs of the Sun and *Venus* is seen; if the contact therefore be an *internal* one, (when the whole of *Venus's* disk is just within the Sun's), the distance will be the difference of the semi-diameters of *Venus* and the Sun: let it equal Δ, then,

$$\Delta^2 = \left[l - l' + (m - s)t' + a' - a\right]^2 + (\lambda + nt' + \delta - \delta')^2.$$

In which expression, $a - a'$, $\delta - \delta'$, and t' are very small quantities; rejecting therefore their squares and products in the expression expanded;

$$\Delta^2 = (l - l')^2 + 2(l - l') \times (m - s)t' - 2(l - l') \times (a - a')$$
$$(a) \qquad + \lambda^2 + 2\lambda nt' - 2\lambda(\delta - \delta').$$

But, since by hypothesis, (see l. 6,) l, l', &c. are the longitudes, &c. at the time of contact seen from the centre, we have

$$\Delta^2 = (l - l')^2 + \lambda^2,$$

thence deducing t' from (a),

$$t' = \frac{(l - l')(a - a') + \lambda(\delta - \delta')}{(l - l')(m - s) + \lambda n}.$$

In this expression, l, l', λ, m, s, n, are to be computed from the Tables, and the parallaxes in longitude and latitude (a, a', δ, δ') are to be computed from the expressions in pages 743, &c. that is, if P, P' represent the horizontal parallaxes of *Venus* and the Sun,

$$a = \frac{P \,.\, \text{sin.}\, h \,.\, \text{sin.}\, k}{\text{cos. lat. } ♀},\quad a' = \frac{P' \,.\, \text{sin.}\, h \,.\, \text{sin.}\, k'}{1},$$

$$\delta = P \,\text{cos.}\, h \,.\, \text{cos. } ♀\text{'s app. lat.}$$

$$- P \,\text{sin.}\, h \,.\, \text{sin. } ♀\text{'s app. lat.} \times \text{cos.} \left(\frac{K + \delta}{2}\right),$$

$$\delta' = P' \,\text{cos.}\, h \text{ (since } ⊙\text{'s apparent latitude is nearly} = 0.)$$

At the time of a transit, *Venus's* latitude is very small, and her longitude is nearly equal to that of the Sun, the coefficients of P, P', therefore, in the expressions for a, a', and for δ, δ', must be nearly equal. Let these coefficients be a, a', b, b' respectively, then

$$t' = \frac{(l - l')(aP - a'P') + \lambda(bP - b'P')}{(l - l')(m - s) + \lambda n};$$

or, since $aP - a'P' = a'(P - P') + (a - a')P$, and $(a - a')P$, as well as $(b - b')P$, are very small quantities and may be neglected, we have

$$t' = \frac{a'l - a'l' + \lambda b'}{(l - l')(m - s) + \lambda n} \times (P - P').$$

From this equation, if t' should be known from observation, $P - P'$, the excess of the parallax of *Venus* above that of the Sun, (which is the object of investigation,) could be determined. We must consider, therefore, by what means t' may be ascertained.

The Astronomical Tables, from which the quantities, l, l', &c. are supposed to be taken, are computed for Greenwich. At

such a place, let the time of the conjunction of *Venus* and the Sun be T; then, at any place to the west of Greenwich and distant by a longitude $= M$ (expressed in time), the reckoned time, at which the conjunction would be seen from the centre of the Earth, would be $T - M$; the time of internal contact, seen also from the centre, would be $T - M + T'$; and the time, at which the contact would be seen from the place of observation (whose longitude is M) would be

$$T - M + T' + t'.$$

Now, the observer, by means of his regulated clock, is able to note this time; suppose it H', then

$$t' = H' - T + M - T', \text{ and consequently,}$$

$$H' - T + M - T' = \frac{a'l - a'l' + \lambda b'}{(l - l')(m - s) + \lambda n} \times (P - P')$$

$= f(P - P')$, f representing the coefficient of $P - P'$.

From this equation $P - P'$ could be determined, if M, the longitude of the place, were known. We must, however, for the reasons alledged in p. 757, seek to dispense with that condition. This is simply effected by observing the *last* interior contact, that is, the one immediately preceding the egress of *Venus's* disk from the Sun. Let the quantities analogous to T', H', and belonging to this last contact be T'', H'', and the coefficient of $P - P'$ (analogous to f) be f'; then,

$$H' - T + M - T' = f(P - P')$$
$$H'' - T + M - T'' = f'(P - P'),$$

consequently,

$$H' - H'' - (T' - T'') = (f - f')(P - P') \ldots . (h)$$

$$\text{and } P - P' = \frac{H' - H'' - (T' - T'')}{f - f'}.$$

This expression is deduced by observing at the same place the times of ingress and egress. If we take a second place of observation, then there will result an equation similar to (h), such as

$$H_{\prime} - H_{\prime\prime} - (T' - T'') = (f_{\prime} - f_{\prime\prime})(P - P'),$$

and subtracting this from the former (h),

$$(H' - H'') - (H_{\prime} - H_{\prime\prime}) = [(f - f') - (f_{\prime} - f_{\prime\prime})] \times (P - P')(h')$$

whence, we have the value of $P - P'$, obtained from the difference of the *durations* of the transit*.

The parallax is inversely as the distance; but, by observation and the Planetary Theory, (see Chap. XVII,) the ratio of the distances of the Earth from *Venus* and the Sun, is known, and therefore the ratio of P *to* P'; let it be as $g : 1$, and let the coefficient of $P - P'$ in (h') be q, the left hand side being $= A$; then

$$(g - 1)\,q\,P' = A,$$

$$\text{and } P' = \frac{A}{q\,(g - 1)}.$$

This is the value of P' when the Sun is at some distance ρ from the Earth. At the mean distance (1)

$\odot$'s horizontal parallax (nearly his mean) $= \rho P'$.

The preceding formula, applied to the transit of *Venus* which happened in 1769, would give

$$P - P' = \frac{1416}{65.72962} \times 1'' = 21''.5428.$$

And the Astronomical Tables, at the epoch of the observations, gave

$\oplus$'s distance from $\odot$ (ρ) 1.01515
♀'s distance from $\odot$ 72619

$$\text{and therefore } g - 1 = \frac{72619}{28896}, \text{ and}$$

$$P' \text{ the Sun's parallax} = 21''.5428 \times \frac{28896}{72619} = 8'.5721$$

* This last operation, although unnecessary in the preceding simple statement, is not so in practice: since, by means of it, the errors of the Tables introduced into the calculation as arbitrary quantities are got rid of.

and (see p. 763, l. 14,) the ☉'s hor. par. $= 8''.5721 \times 1.01513 = 8''.7017$*

In the fraction $\frac{1416}{65.72962}$ $(= P - P')$ the numerator is obtained from observations on the times of contact. If that numerator had been $1416 - 65.72962$, the quotient, instead of being $21''.5428$, would have been $20''.5428$. In other words, a difference of $65^s.72962$, made in noting the times of the transit,

* The equation (see p. 763,) for determining the difference of the parallaxes of Venus and the Sun, was obtained by observing, at *different* places, the *differences* of the durations of the transits. The transit of 1767, was observed at several places, and an exact result was endeavoured to be obtained, by taking the *mean* of several results. The following are the results and their mean according to M. Delambre:

	Sun's Parallax.	Difference of Parallaxes.
Taïti, (Otaheité) Wardhus	8.7094	21''.561
Taïti, Kola	8.5503	21 .166
Taïti, Cajanebourg	8.3863	20 .762
Taïti, Hudson's Bay	8.5036	21 .066
Taïti, Paris and Petersburgh	8.7780	21 .730
California, Wardhus	8.6160	21 .330
California, Kola	8.3880	20 .765
California, Cajanebourg	8.1636	20 .208
California, Hudson's Bay	8.1521	20 .284
California, Paris and Petersburgh	8.7155	21 .576
Hudson's Bay, Wardhus	9.1266	22 .592
Hudson's Bay, Kola	8.4589	20 .941
Hudson's Bay, Cajanebourg	8.1730	20 .233
Hudson's Bay, Paris and Petersburgh	9.2491	22 .897

Here the mean of the first 5 results is, nearly, ... 8''.59
of the next 5 .... 8 .41
of the next 4 .... 8 .75
of all .... 8 .57.

would have produced an error of one second only in the difference of the parallaxes, and consequently, an error in the Sun's parallax less in the ratio of 28896 to 72619, or (of 2 to 5 nearly). Or, what amounts to the same thing, it would have required an error in time equal to $164^s \left(= 65.7 \times \frac{5}{2}\right)$ to have produced an error of $1''$ in the value of the Sun's parallax.

The special Astronomical use of the transit of *Venus* is, as it has been observed, the determination of the Sun's horizontal parallax. But, that important element being once determined, the transit of an inferior planet, even with regard to its use and object, may be made to enter the class of eclipses and occultations, and, like them, be made subservient to the determination of the longitudes of places.

That a transit may be adapted to this latter purpose, is evident from the equation of p. 762, namely,

$$H' - T + M - T' = f.(P - P'),$$

for in that, if $P - P'$ be supposed to be known, M, the longitude of the place of observation, is the only unknown quantity.

Transits, however, are phenomena of such rare occurrence, that their use, in this latter respect, is very inconsiderable*.

The fixed stars, the Sun, the planets, and the Moon, with their peculiar and connected theories, have already been treated of. There is another class of heavenly bodies, called *Comets*,

* The transit of Mercury was used by M. Kohler to determine the longitude of Dresden, see *Phil. Trans.* 1787, p. 47: and by Short to determine the difference of longitudes of Paris and Greenwich, (see *Phil. Trans.* 1763, vol. LIII, p. 158.). M. Delambre, however, and properly, says 'Le mouvement relatif est si lent et les observations de l' entreé et de la soireè sont en consequence si peu susceptibles de precision qu'on ne doit recourir a ce moyen que faute d' autres' (*Mem. Inst.* tom. II, p. 442,) see also *Phil. Trans.* vol. LIII, pp. 30, and 300: also vol. LII: *Mem. Acad. Paris,* 1761: *Phil. Trans.* No. 348, p. 454, (Halley's account) and Horrox's *Venus in Sole visa.*

which ought not to be passed over. Yet their strictly mathematical theory is so difficult, that, instead of attempting to put the Student in possession of it, we shall content ourselves with acquainting him with some of its general circumstances, and with referring him to ampler sources of information.

CHAP. XXXIX.

On Comets.

Comets are bodies occasionally seen in the heavens, with ill-defined and faint disks, and usually accompanied with a *coma* or stream of faint light in the direction of a line drawn from the Sun through the Comet.

Comets resemble the Moon and planets in their changes of place amongst the fixed stars: but, they differ from them in never having been observed to perform an entire circuit of the heavens. There are also other points of difference; the inclinations of the planes of their orbits observe not the limits of the Zodiac, as the planes of the orbits of the Moon and planets do; and, the motions of some of them are not according to the order of the signs.

Comets, like planets, move in ellipses, but, of such great eccentricity, that thence has arisen a ground of distinction, and Comets are said to *differ* from planets, because they move in orbits so eccentric. The eccentricities of those that have been observed have been found so great, that parabolas would nearly represent them.

What are called the elements of a Comet's orbit are less in number than those of a planet's, being only five. It is impossible from the observations made, during one appearance of a Comet, to compute the major axis of its orbit and its period, and consequently the area described by it in a given time: what Astronomers seek to compute, and what they with difficulty compute, are the perihelion distance; its place, or longitude; the epoch of that longitude; the longitude of the ascending node, and the inclination of the orbit.

The elements of the orbits of planets are capable of being determined by observations made on the meridian: by longitudes and latitudes computed from right ascensions and declinations. Comets, however, require observations of a different kind: by the rotation of the Earth they are brought on the meridian, but, (from their proximity to the Sun whilst they are visible,) not during the night, when alone the faintness of their light does not prevent them from being discerned. They must therefore be observed *out* of the meridian; and, in that position, the differences between their right ascensions and declinations and those of a known contiguous star must be determined.

It is difficult to make these latter observations with accuracy by reason of the doubtful and ill-defined disk of the Comet; and a small error in the observations will materially affect the elements of the orbit.

The rigorous solution of the problem of the elements of a Comet's orbit requires three observations only. But then the solution is attended with so many difficulties, that in this, as in other like cases, Astronomers have sought, by the indirect methods of trial and conjecture, to avoid them. If, (and this case always happens) more than three observations are obtained, the redundant ones are employed in correcting and confirming previous results.

The periodic time, as we have observed, cannot be determined from observations during one appearance of a Comet. If known, it can only be so, by recognising the Comet during its second appearance. And the only mode of recognising a Comet, is by the identity of the elements of its orbit with those of the orbit of a Comet already observed. If the perihelion distance, the positions of the perihelion and of the nodes, the inclination of the orbit, are the same or nearly so, we may presume, with considerable probability, that the Comet we are observing, has been previously in the vicinity of the Sun; and that, after moving round by the aphelion of its oval orbit, it has again returned towards its perihelion distances.

Comets not having been formerly observed with great accuracy, it so happens, that the period of one alone, that of the Comet

observed in 1682, 1607, and 1531, is known to any degree of certainty. Its period is presumed to be about 76 years. Assuming the Earth's mean distance to be unity, the perihelion distance of the Comet was 0.58, and the major axis of the orbit 35.9. The inequalities which are noted in its period are supposed to arise from the influence of some disturbing forces*.

The chief business of the present Treatise, hitherto, has been with calculations founded on observations made on the meridian. But, there are many important processes dependent on angular distances observed *out* of the meridian: such, for instance, as those for ascertaining the latitude and longitude of a ship at sea. The nature of the observations, in these cases, requires a peculiar instrument; which, besides being adapted to the measuring of angular distances out of the meridian, may be held in the hand of the observer, and used by him, even when he becomes unsteady by the motion of the vessel. The description and use of such an instrument will be explained in the ensuing Chapter.

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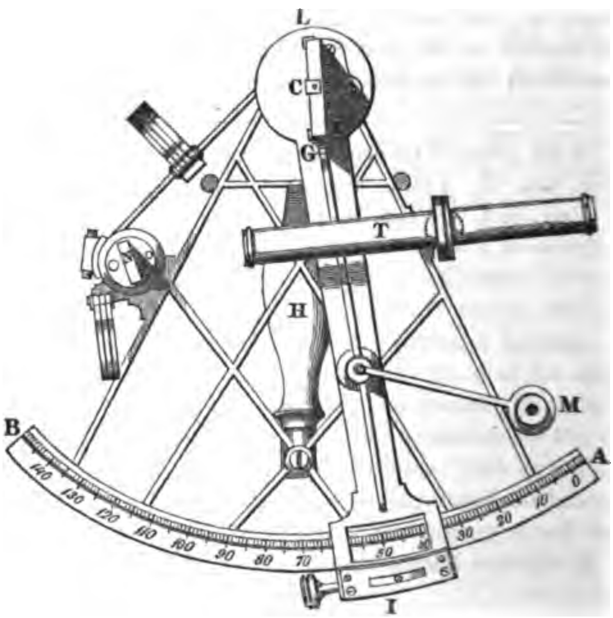
* On the subject of Comets, see Laplace, *Mec. Celeste*, Liv. II, p. 20, &c. Biot, tom. III, Add. p. 186, Englefield: Cagnoli, p. 429, Newton, *Arith. Univ.* Sect. 4, Chap. II, Prob. 30.

CHAP. XL.

ON THE APPLICATION OF ASTRONOMICAL ELEMENTS AND RESULTS, DEDUCED FROM MERIDIONAL OBSERVATIONS, TO OBSERVATIONS MADE OUT OF THE MERIDIAN.

On Hadley's Quadrant and the Sextant.

THE larger figure is intended to represent a *Sextant*, as it is usually fitted up, with its handle *H*, the telescope *T*, the micro-



scope *M* moveable about a centre, and capable of being adjusted so as to *read off* the divisions on the graduated limb *AB*. The

less Figure is intended as a sketch of the larger and for the purpose of explaining its properties.

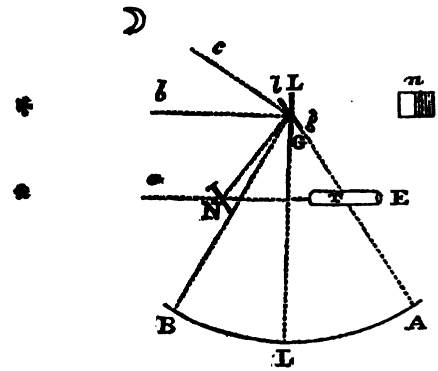
LCG and *N* (in the large Figure) must be supposed to represent the edges of two plane reflectors; the planes of which are perpendicular to the plane of the instrument in which the graduated limb and the connecting bars lie. The upper part of the reflector *N*, which is fixed, and called *the Horizon glass*, is transparent and free from quicksilver, as in *n* (in the small Figure) which is represented as *N* appears when viewed through the tube of the telescope *T*. The other reflector *LCG* (*the index glass*) is attached to the limb and index *I*, and with them moveable round a centre placed near *C*. Now, the instrument is so constructed that, when the index *I* is moved up to *A* and points to *o* on the graduated arc, the planes of the two reflectors *LCG* and *N* are exactly parallel to each other. In the small Figure, *lg* represents this position of *LG*.

In this position of the index *I* and the reflector *LG*, if the eye at *E* (small Figure) look through the upper part of the horizon glass at *N*, and perceive a distant object such as a star (✱), it will also perceive the image of the same star reflected from the under and silvered part of *N*. For, by hypothesis, the reflectors are parallel: and since the star is extremely distant, two rays from it (*a N*, *bg*) falling on *N* and *LG* must be parallel; therefore the latter ray, after two reflections, the first at *LG*, the second at *N*, must proceed towards the eye in the direction of *a N* produced.

Suppose now, the eye still looking through the telescope at the same object (✱), the index *I*, the limb *GI*, and with them the reflector *LCG*, to be moved from *A* towards *B* (*LGI* is their position in the small Figure); in this case the star ✱ can no longer be seen by two reflexions, but some other object such as the ☽ may: and if so, two objects, the ✱ and ☽, would be seen nearly in contact; the former in the upper part of the horizon glass *N*, the latter on the lower silvered part.

In consequence then of this translation of the index *I* from *A*, where it was opposite *o*, to another position between *A* and *B*;

two objects (✻ and ☽) inclined to each other at a certain angle (bgc in small Figure) are brought into contact. If, therefore, the arc moved through (AI in the small Figure) bore any relation to the angular distance of the two objects, and we could ascertain such relation, we should, in such case, be able by measuring AI, or by reading off its graduations, to determine the angular distances of the two observed objects. This relation we will proceed to investigate.



In the first position (LG) both the direct and the reflected rays from ✻ are seen in the direction of the telescope (T); the direct ray from ✻ is always seen in the same direction. But, in the new position, the reflected ray (in order that ☽ may be seen) must also be seen in that direction; therefore, the ray must come from the under part of N in the same direction: and therefore, since N is fixed, the ray must always be incident on N in the same direction, and consequently be *always reflected* from LCG in the same direction. What we have then, to determine is reduced to this. *To find the inclination of two incident rays, such, that the position of the reflector being changed (from* LG *to* lg *for instance,) each shall be reflected into the same direction.*

Let the first incident ray (and consequently the reflected ray) be inclined to the reflector at an angle $= A$: let the reflector be moved through an angle $= \theta$, and towards the reflected ray: (for instance, from the position gl to GL in the small Fig.), then the angle between the reflected ray and the plane in its new position $= A - \theta$ between the first incident ray and the plane ...... $= A + \theta$.

But, by the laws of reflection, the second incident ray must form with the reflector, an angle equal to that which the reflected ray does; an angle, therefore, $= A - \theta$. Now, the difference between the angles which the incident rays form with the same position of the plane, is no other than the inclination of the incident rays, equal, therefore, to

$$(A + \theta) - (A - \theta), \text{ or, } 2\theta.$$

This is the important principle in the construction of the instrument. For, suppose the arc AB to be one-sixth part of a circle, and the index I, when the two objects are seen in contact, be one-third of the way between AB; then, the inclination of the two reflectors (for the reflector N is always parallel to the first position lg) would be one-third of one-sixth of 360° or 20°: and, accordingly, the angular distances of the two objects would be 40°. Instead of dividing AB into a number of degrees proportional to its magnitude (60° for instance, if $AB = \frac{1}{6}$th circumference), it is usual to divide it into *twice* that number. In such a graduation the number of degrees, minutes, &c. intercepted between *o* and the index will at once determine the angular distance of the two objects.

The objects must be brought into contact: in the case of a star and the Moon, the former must be made just to touch the limb of the latter: in the case of the Sun and Moon, their two limbs must be made to touch.

For the sake of illustration, we have supposed the two objects to be a star and the Moon: and, in practice, those are frequently the observed bodies. But, the instrument is capable of measuring the angular distance of any two objects lying in any plane: the Sun and Moon, for instance, and in such cases there are certain darkened glasses, near to N, and between N and L (see Fig.) contrived for the purpose of lowering the Sun's light to that of the Moon's, or the Moon's to that of a star's.

The uppermost and lowest points in the disks of the Sun, or of the Moon, may be considered as two objects; therefore, their distances, which are the diameters of the Sun and the Moon, may be measured by the described instrument. Instead of the points in the direction of a vertical circle, we may observe two opposite points

in an horizontal direction: and, accordingly, we can measure the horizontal diameters of the Sun and Moon.

If we make a star, or the upper or the lower limb of the Sun or Moon, to be one object, and the point in the horizon directly beneath to be the other, we can measure their angular distance, which, in these cases, is either the *altitude* of the star, or the altitude either of the upper or the lower limb of the Sun and Moon. In this observation, the horizon is viewed through the upper part of the reflector *N*, which is the reason why that is called the *horizon*-glass. At sea, where the horizon is usually defined with sufficient accuracy, the altitude of the Sun or of a star may be taken, by the above method; but at land the inequalities of the Earth's surface oblige us to have recourse to a new expedient, in the contrivance of what is called an *Artificial*, sometimes a *False Horizon*. This, in its simplest state, is a basin either of water, or of quicksilver: to the image of the Sun or other object seen therein we must direct the telescope *T*, and view it directly through the upper part of *N*, and then move, backwards, or forwards the limb and index, till by the double reflexion, the upper or the under limb of the reflected Sun is brought into contact, or exactly made to touch the under or the upper limb of the image of the Sun seen in the *Artificial Horizon*. The angle shewn by the instrument is double either of the altitude of the Sun's upper or under limb: subtract or add the Sun's diameter, divide by two, and the result is the altitude of the Sun's centre: all other proper corrections, instrumental as well as theoretical, being supposed to be made.

It is evident from the preceding description, that the plane of the instrument must be held in the plane of the two bodies, the angular distance of which is required: in a vertical plane, therefore, when altitudes are measured; in an horizontal, when, for instance, the horizontal diameters of the Sun and Moon are to be taken. In the management of the instrument, this adjustment of its plane, or the holding it in the plane of the two bodies, is the most difficult part.

The instrument is to be held by the handle *H*, and generally is, in the left hand of the observer: his right being employed in

moving and adjusting the index, its connected limb, and the reflector *LCG*. Its great and eminent advantage is, that it does not require to be fixed, nor that the observer using it should himself be steady. It is the chief instrument in Nautical Astronomy: since by its means alone, the position of a vessel at sea may be determined.

The instrument represented and described in this Chapter is, the *sextant*: which is an improvement on the *quadrant*, called, from its inventor, *Hadley's Quadrant* *. Besides these, on the same principle, but of better contrivance, is the *reflecting circle* †: also, Borda's *reflecting repeating* circle, on the principle of Mayer's. (See *Mem. Gottingen*, tom. II, also *Tabulæ Motuum*, &c. 1770).

We subjoin two instances of the uses of the sextant.

Angular Distance of the Sun's Centre, and of the Horizon (at Sea,) or (see p. 774,) *Altitude of the Sun's Centre.*

Alt. ⊙'s lower limb	49°	$10'$	$0''$
(*a*) ⊙'s semidiameter	0	15	51
	49	25	51
‡ Refrac. (Chap. X.)	0	0	43
true alt. ⊙'s centre	49	25	8

Distance of eastern and western limbs, or ⊙'s horizontal diameter	$31'$	$42''$
(*a*) ⊙'s semi-diameter	15	51

Altitude of the ⊙*'s Centre, by means of the Artificial Horizon,* (see p. 774,)

By inst. ⊙'s upper limb	100°	$2'$	$47''$
Apparent altitude	50	1	23.5
(*b*) ⊙'s semi-diameter	0	15	50
	49	45	33.5
Refraction	0	0	43
True alt. ⊙'s centre	49	44	50.5
⊙'s horizontal diameter		$31'$	$40''$
(*b*) ⊙'s semi-diameter		15	50

* Described in the *Phil. Trans.* Year 1738, No. 420, p. 147.

† Invented by Mr. Troughton: for a description of it, see Rees' *Encyclopedia*, new edit. Art. *Circle*.

‡ The Nautical Tables of Refraction include within their results the correction for the Sun's parallax.

The sextant (using that as the generic name of like instruments) is, as it has appeared, a secondary instrument, but capable of performing, in an imperfect degree indeed, several astronomical operations. It measures, and generally, angular distance. It affords us, therefore, the means of determining the latitude of a place, from the meridional altitude of the Sun or a star, since such *meridional* altitude is the angular distance of the horizon and star when on the meridian. From two observed altitudes, one of which is meridional, and the declination of the observed body, we are able, by computation, to determine the *time* of the other observed altitude. From the same data the azimuth of the observed body may be determined. By means of the observed distance, between a star and the Moon, we derive a method (a thing hereafter to be explained) of determining the *longitude* of a place. So that, as it has been said, the sextant is itself and alone a sort of portable Observatory, capable of performing many astronomical operations, but all imperfectly. This would naturally be expected on the ground, that an instrument of general uses cannot be excellent when employed in special ones.

The succeeding Chapter will contain several methods adapted to the uses of the sextant, and to the uses of instruments performing like operations.

CHAP. XLI.

On the Mode of computing Time and the Hour of the Day; by the Sun; by the Transit of Stars; by equal Altitudes; by the Altitude of the Sun or of a Star.

WE will preface the methods that ought to be considered, perhaps, as the special objects of this Chapter, with some that are adapted to observations made on the meridian.

Transit of the Sun over the Meridian.

When the Sun's centre is on the meridian, it is *true* or *apparent* noon. It can be determined to be there, by means of a transit instrument. With this, observing the contacts of the Sun's western and eastern limbs with the middle vertical wire, note, by means of the clock, the interval of time, and half that interval added to the time of the contact of the western, or subtracted from that of the eastern, will give the time at which the Sun's centre is on the meridian. For greater accuracy, the times of contact of the Sun's limbs with the vertical wires to the right and left of the middle one may be noted, (see pages 96, &c.)

The time thus determined is *apparent* noon; in order to deduce the *mean* time, apply the *Equation of time*, (see Chap. XXII.). For instance, the *equation* on Nov. 8, 1808, is stated in the Nautical Almanack to be $-\ 16^m\ 3^s.7$, therefore, when the Sun's centre on that day was on the meridian, the mean solar time was $12^h - 16^m\ 3^s.7$, or $11^h\ 43^m\ 56^s.3$; 12^h being supposed to denote the time when the centre of the *mean* Sun is on the meridian.

Transit of a fixed Star; of the Moon; of a Planet over the Meridian.

The mean Sun leaves a meridian and returns to the same in 24^h, describing $360° 59' 8''.3$; $59' 8''.3$ being the increase of its mean right ascension in that time. Since the mean Sun, by its definition, moves equably, the time from mean noon must be always proportional to the Sun's distance from the meridian. If a star, then, were on the meridian, the time would be proportional to the Sun's angular distance from the star; it would be proportional, therefore, to the difference of the right ascensions of the star and the Sun, at the time when the star is on the meridian.

The Sun's right ascension in the Nautical Almanack is expressed solely for noon, that is, when the Sun's centre shall be on the meridian of Greenwich; and such right ascension continually increasing, will be greater when the star comes on the meridian, and the Sun is more to the west, than it was at noon. In the interval between the transits of the Sun and star, the former will have moved to the east, and towards the latter, by an increase of right ascension proportional to the interval. The angular distance therefore of the star and Sun, or the difference of their right ascensions, when the former is on the meridian, is

$$*\text{'s } Æ - \odot\text{'s } Æ \textit{ (at preceding noon)} - \text{increase of } \odot\text{'s } Æ,$$

and to this angular distance is the time proportional.

The time from noon is nearly proportional to the $*$'s right ascension $-$ $\odot$'s right ascension at noon; therefore, the increase of $\odot$'s right ascension is nearly proportional to that angle. If a therefore denote the increase of the Sun's right ascension in 24^h, we have the time =

$$*\text{'s } Æ - \odot\text{'s } Æ - \frac{D}{24} \times a,$$

$$\text{(making } D = *\text{'s } Æ - \odot\text{'s } Æ.)$$

EXAMPLE.

A Star in Capricorn *whose* Æ $= 20^h\ 30^m\ 7^s$ *was on the Meridian at Greenwich, Nov.* 8, 1808. *Required the time.*

✱'s Æ	20^h	30^m	7^s
By Naut. Alm. ☉'s Æ (noon of Nov. 8.) ..	14	53	52 *
✱'s Æ − ☉'s Æ (*D*)	5	36	15
☉'s Æ Nov. 9............	14	57	53.5
8.............	14	53	52
a =..........	0	4	1.5

* The Sun's right ascension is expressed in time, the Moon's in degrees, and to be expressed in the hours, minutes, &c. of *sidereal time*, must be converted into such at the rate of 15° for 1^h; for $\frac{24}{360} = \frac{1}{15}$.

For facilitating this operation and its reverse, appropriate Tables are provided; but, it may be, nearly with as much ease, effected by dividing and multiplying by 4. Thus, to convert $7^h\ 21^m\ 56^s.21 = 7^h\ 21^m\ 56^s\ 12'''$ into degrees, &c. begin with the minutes, and take the fourth of them, then, of the seconds, &c. reckoning the minutes of the quotient as degrees, the seconds as minutes, &c. thus:

$$\begin{array}{rrrr} & 4)21^m & 56^s & 12''' \\ \hline & 5^\circ & 29' & 3'' \\ \text{But } 7^h = & 105 & & \\ \hline & 110 & 29 & 3 \end{array}$$

For the reverse operation, multiply by 4, reckoning the seconds of the product as thirds, the minutes as seconds, &c.

$$\text{Thus} \ldots\ldots\ldots\ldots 36^\circ\ 8'\ 34''\ 30''' \ldots\ldots\ldots\ldots (36^\circ = 30 + 6 = 2^h + 6^\circ)$$

$$\begin{array}{r} 4 \\ \hline 2^h\ 24^m\ 34^s\ 18'''\ 0 \end{array}$$

or dividing $18'''$ by 6 to reduce it to a decimal, the product is $2^h\ 24^m\ 34^s.3$.

The reasons of the two operations are these; in the first we ought to multiply by 15, or, which is the same thing, by $\frac{60}{4}$; therefore we may divide by 4 and dispense with the multiplication by 60, by merely *raising the denomination* of the quotient; for $60 \times 1'' = 1'$. In the second case, we ought to *divide* by 15, or which is the same thing

∴ apparent time =

$$5^h\ 36^m\ 15^s - \frac{5^h\ 36^m\ 15^s}{24^h} \times 4^m\ 1^s.5 = 5^h\ 35^m\ 19^s.3,$$

and the mean time =

$$5^h\ 35^m\ 19^s.3 - 16^m\ 2^s \text{ (the equation of time)} = 5^h\ 19^m\ 17^s.3.$$

Since the increase of the Sun's mean Æ is $59'\ 8''.3$ in 24 hours, a meridian of the Earth describes, in that time, $360^\circ\ 59'\ 8''.3$; therefore, it describes 360° in $24^h \times \frac{360^\circ}{360^\circ\ 59'\ 8''.3}$, or in $23^h\ 56^m\ 4^s.09$. This is the time of the Earth's rotation, or the length of a sidereal day, expressed in mean solar time. If the Sun, therefore, and a Star were together on the meridian on a certain day, on the succeeding one, the Star would return sooner, or more quickly, to the meridian by $3^m\ 55^s.9$ of mean solar time. On this account, stars are said to be *accelerated*. The *acceleration* on mean solar time, therefore, when the Star and Sun are distant by 360°, or by 24 *of sidereal time,* is $3^m\ 55^s.909$; when distant by 180°, or by 12^h *of sidereal time,* it is $1^m\ 57^s.955$; when distant by 60°, or 4^h, it is $39^s.388$, and generally the *acceleration* is

$$\frac{\ast\text{'s Æ} - \odot\text{'s Æ}}{24^h} \times 3^m\ 55^s.909\,^*.$$

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thing, we may *multiply* by $\frac{1}{15}$ or $\frac{4}{60}$; therefore, we may multiply solely by 4, and dispense with the division by 60 by merely *lowering the denomination* of the product; for $\frac{1'}{60} = 1''$.

* Twenty-four sidereal hours = $23^h\ 56^m\ 4^s.092$ of mean solar time,

and, $23^h\ 56^m\ 4^s.092$ (= $23^h.93447$) : 24 : 24 : $24^h.065709$,

in other words,

24 mean solar hours = $24^h\ 3^m\ 56^s.55$ of sidereal time.

Now *acceleration* for 24^h is	3^m	$55^s.909$
3^m	0	0.491
56^s	0	0.153
.55...........................	0	0.15
	3	56.558

and $3^m\ 56^s.55$ deduced from $24^h\ 3^m\ 56.55$ leaves 24^h of solar time, as it ought to do.

This is only another mode of expressing the rule given in p. 778; instead of the increase of the Sun's mean right ascension, in 24 hours of mean solar time, we there took the real increase between two apparent noons.

There are* Tables constructed for the *Acceleration of stars on mean solar time*, which render the computation of the hour, by means of the transit of a fixed star, very easy; the rule is,

$$\text{the time} = \ast\text{'s } Æ - \odot\text{'s } Æ - \text{acceleration.}$$

Thus, in the former instance,

✱'s Æ	20^h	30^m	7^s
Nov. 8. ⊙'s mean Æ	15	9	57.3
	5	20	9.7
Acceleration	0	0	52.3
Mean time	5	19	17.4

The right ascensions of the Sun and of the stars, are always expressed in sidereal time; and care must be taken to distinguish the hours, minutes, &c. of that time, from the hours, minutes, &c. of mean solar time. If we subtract, from an angle expressed in the symbols of sidereal time, the *acceleration*, the remainder is expressed in mean solar time. Thus,

A star is to the east of the meridian $30^0\ 30'$, or	2^h	$2'$	$0''$
The acceleration, or the Sun's motion in $2^h\ 2'$	0	0	19.99
	2	1	40.01

therefore in $2^h\ 1^m\ 40^s.01$ of mean solar time, the star will be on the meridian.

The time is proportional to a *less* angle than the difference of the right ascension of the star and the Sun; or, stars are *accelerated*, because the Sun, in the interval between his transit and that of the star, moves towards the latter. In the case of the Moon then, the time must be proportional to a *greater* angle than the difference of the Sun's right ascension on the preceding noon, and the Moon's; or, the Moon must be *retarded;* because, in

* Zach's Tables *d'Aberration*, &c. Tab. XXVI.

the interval between the transit of the Sun and that of the Moon, the latter, by its greater motion in right ascension, has increased its angular distance from the former. It would be easy, as in the former case, to compute the hour from the Moon's transit over the meridian, (or what is the same thing, to find the hour of the Moon's transit), but instead of it, we will give a formula applicable to all cases:

Let the increment of ☉'s Æ in 24^h be . . . . a
of a ✳, or of the ☽, or of a planet . . . . A.

Let also the difference between the right ascension of the heavenly body and that of the Sun at *the preceding noon*, expressed in sidereal time, be . . . . . t

then, if $a = A$, the hour of transit will be proportional to t
if $a > A$, ... to some less angle . . $t - \tau$
if $a <$ . . . . to some greater . . . $t + \tau$.

Hence in the first case, which can only happen with a planet, the time of transit is proportional to t; that is, if the Sun's right ascension when on the meridian be $30^\circ\ 30'$, or $2^h\ 2^m$, less than that of the planet, the latter will be on the meridian at $2^h\ 2^m$ of solar time.

In the second case, $a > A$

$$24 : a - A :: t - \tau : \tau; \quad \therefore \tau = t \times \frac{a - A}{24 + a - A}.$$

In the third case $a < A$

$$24 : A - a :: t + \tau : \tau; \quad \therefore \tau = t \times \frac{A - a}{24 + a - A}.$$

Hence, in the second case, the time of transit $= t - t \times \dfrac{a - A}{24 + a - A}$

in the third, $= t + t \times \dfrac{A - a}{24 + a - A}$, or, $t - t \times \dfrac{a - A}{24 + a - A}$

therefore, in both cases,

$$\text{the time of transit} = t\left(1 - \frac{a - A}{24 + a - A}\right)$$

$$(\text{expanding}) = t\left\{1 - \frac{a - A}{24} + \left(\frac{a - A}{24}\right)^2 - \left(\frac{a - A}{24}\right)^3 + \&c.\right\}$$

Hence in the case of a fixed star, when $A = 0$, the time of

$$*\text{'s transit} = t - \frac{at}{24} + \left(\frac{a}{24}\right)^2 t - \&c.$$

in which the two first terms (which are sufficient) give the rule of computation used in p. 778, l. 28.

In the case of the Moon, $a = A$; therefore all the terms are additive, and

$$\text{the time of ☽'s transit} = t + \frac{A-a}{24}\,t + \left(\frac{A-a}{24}\right)^2 t + \&c.$$

In the case of a planet, a may be less or greater than A; if equal, then the time of transit $= t$, as before, p. 782, l. 13.

There is one case which has not been mentioned, that in which a planet is *retrograde* (see Chap. XXIII.). In this case, the approach of the Sun and planet is greater than that of the Sun and a star, and the same approach, as if, instead of the Sun having a motion in right ascension equal to a, we suppose it endowed with a motion equal to $a + A$; substituting therefore in the form, p. 783, l. 29, $a + A$ instead of a time of the planet's

$$\text{transit} = t - \frac{a+A}{24}.t + \left(\frac{a+A}{24}\right)^2 .t - \&c.$$

When the planet is *stationary*, its hour of passage is evidently the same as that of a fixed star which has the same right ascension.

Example.

Let it be required to find the time of the Moon's passing the Meridian of Greenwich, June 13, 1791.

June 14, ☽'s Æ . .	15^h	43^m	32^s	☉'s Æ . .	5^h	30^m	38^s	
13, ditto	14	42	32	ditto	5	26	29.1	
	1	1	0 = A		0	4	8.9 = a	
June 13, ☽'s Æ . .	14	42	32	A	1	1	0	
☉'s Æ . .	5	26	29.1	a	0	4	8.9	
	9	16	2.9 = t		0	56	51.1 = $A-a$	

$$\therefore t \ldots\ldots \quad 9^h\ 16^m\ 2^s.9 = \left\{\begin{array}{l}\text{approx. time}\\ \text{of ☽ 's transit.}\end{array}\right.$$

$$t.\frac{A-a}{24},\ \text{or}\ \frac{9^h\ 16^m\ 2^s.9}{24} \times 56^m\ 51^s.1 \quad 0\ \ 21\ \ 57$$

$$9\ \ 37\ \ 59.9 = \left\{\begin{array}{l}\text{more cor-}\\ \text{rect time.}\end{array}\right.$$

$$t\left(\frac{A-a}{24}\right)^2 \ldots\ldots\ldots\ldots \quad 0\ \ \ 0\ 49.8$$

$$9\ \ 38\ \ 49.7 = \left\{\begin{array}{l}\text{still more}\\ \text{corr}^t\text{. time.}\end{array}\right.$$

This last result (in apparent time) is sufficiently exact for Astronomical purposes *.

The second additional term $21^m\ 54^s.7 = \dfrac{9^h\ 16^m\ 29^s}{24^h} \times 56^m\ 51^s.1$, is evidently the *proportional part* † of $56^m\ 51^s.1$, corresponding to $9^h\ 16^m\ 29^s$; the third additional term, $49^s.8$, =

$$\left(\frac{A-a}{24}\right)^2.t = \frac{A-a}{24} \times \frac{A-a}{24}.t = \frac{21^m\ 54^s.7}{24^h} \times (A-a)$$

$= \dfrac{21^m\ 54^s.7}{24^h} \times 56^m\ 51^s.1$ is evidently the *proportional part* of $56^m\ 51^s.1$, corresponding to the time $21^m\ 54^s.7$. This is the explanation of the rule, as it is sometimes given by Astronomers, which directs us to find a first, and a second proportional, and to add them to the approximate time of the Moon's transit, in order to find a more correct time. (See Nautical Almanack, 1811, pp. 154, 155. Also Wollaston's *Fasciculus*, Appendix, p. 76.)

The hour, or the mean solar time, may be determined or computed from the transit of a fixed star; and, much more exactly, than from the transit of the Moon or of a planet. With regard therefore to these two latter, the object of the preceding methods

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* See in pp. 702, 705, &c. the time of the Moon's transit, found from the observed sidereal time of the transit of its limb.

† Tables are computed for facilitating these operations.

is to determine from Astronomical Tables, the times of their transits, or passages over the meridian, rather than the hour of the day from the transits.

Time determined by the Sidereal Clock.

If we can determine the time from the transit of a fixed star, it is an immediate inference that we can determine it from the sidereal clock. For, the clock is regulated by the observed transits of stars, and when regulated, we may suppose it always to indicate the right ascension of some imaginary star: Thus,

July 1, 1790, time by sidereal clock	13^h	20^m	15^s
☉'s mean longitude (by Tables)	6	54	35.86
	6	25	39.14
* *Acceleration* (Maskelyne, Tab. XXI.)......	0	1	3.1
Mean solar time	6	24	36.04

The preceding computations of transits †, &c. have been made for Greenwich, for which place our Astronomical Tables, and the Nautical Almanack are constructed. For any other place, we must account for the difference of longitude. Thus, to find, on July 9, 1808, the Sun's right ascension at noon, at a place 35^0 (2^h 20^m) east of Greenwich, we have only to find the Sun's right ascension 2^h 20^m previous to noon time at Greenwich: which is easily done by subtracting from the right ascension at noon the proportional increase of right ascension in 2^h 20^m: thus,

July 10, ... ☉'s Æ	7^h	17^m	$48^s.5$
9, ... ditto	7	13	43.2
Increase of Æ in 24^h		4	5.3
Proportional increase in 2^h 20^m = ...		0	33

∴ Sun's Æ, at noon, at the required place, = 7^h 17^m $15^s.5$.

* The *Acceleration* is the Sun's mean motion in right ascension, and by this latter title it is called by Maskelyne in the Table referred to. See Wollaston's *Fasciculus*, Appendix, p 69.

† See another Example in pp. 705, 706, &c.

A similar method must be used to find the Moon's right ascension, or longitude, &c. at noon, at any given place, with this difference, however, that the change of right ascension will not be simply proportional to the time, but must be computed more exactly by the differential method and series $\left(a + xd' + x \cdot \frac{x-1}{2} d'' + \&c.\right)$ See *Trigonometry*, p. 259, also pp. 706, &c. of this Work.

We now proceed to the methods of determining the time, by observations made out of the meridian.

The Method of equal, or of corresponding, Altitudes.

The principle of the method is this: before noon, if the Sun be the body to be observed, note its altitude and the time, and wait till the Sun, in the afternoon, descends to an equal altitude; half the time elapsed between the two observations is, nearly, the distance of each observation from noon.

The same process is to be used with a star or planet: half the sum of the times between two equal altitudes observed, respectively, in the east and west, is, in time, the star's passage of the meridian; exactly the passage of the star, very nearly that of the planet.

The sole condition respecting altitudes mentioned in the preceding description is their equality. The corresponding altitudes, therefore, may be taken at any distance from the meridian. Hence, if we had ten altitudes in the east, and ten corresponding ones in the west, half the sum of the times for each pair would be the star's passage over the meridian: and, accordingly, one-twentieth of the sum of the times would be the *mean* time of it.

In this operation, as before when only one pair of altitudes is employed, the result is only *nearly* true, if the observed body be the Sun or a planet: since, in either case, the declination is changed during the interval of the observations.

With regard to the instruments necessary to the above opera-

tions, a sextant may be used, in default of better instruments, or when, as would be the case at sea, fixed instruments cannot be used. But the *better* instruments are astronomical quadrants, (see pp. 58, &c.) declination circles, repeating circles, or any of that class which are furnished with movements in azimuth, and will serve as *equal altitude* instruments. With any instrument of such sort, properly adjusted, *clamp* the telescope at a certain graduation of the limb of the instrument, and a little above what, probably, may then be the star's altitude, (the star being supposed to be in the east). Turn the instrument towards the star, and note the time when it passes through the middle point of the horizontal wire, in the field of the telescope (the point *a* in the figure of p. 58.). Note also the time when the star, after having passed the meridian, descends to (*a*), the middle point of the horizontal wire. Half the interval, as it has been already said, is the sidereal time of the star's passing the meridian. But in order to procure a *mean* result (see p. 786,) repeat the first operation (l. 6, &c.) after the telescope shall have been elevated through a certain number of graduations, 20′ for instance. The second observation being made, make a third, fourth, &c. the telescope, at each, being raised through 20′. When the star shall have passed the meridian, go through the same operations, but in an inverse order. For instance, Lacaille who constantly deduced his time from *corresponding altitudes*, made the following observations of the star Arcturus.

Altitudes.	Times East and West.	Sum of Times.	Times of Transit.
$43^{0}\ 10'$	$10^{h}\ 55^{m}\ 47^{s}$ 17 11 55.5	$28^{h}\ 7^{m}\ 42^{s}.5$	$14^{h}\ 3^{m}\ 51^{s}.25$
43 30	10 57 57 17 9 45.5	28 7 42.5	51.25
43 50	11 0 7.5 17 7 35	28 7 42.5	51.25
44 10	11 2 18.5 17 5 24.5	28 7 43	51.5
		112 30 50.5	
		14 3 51.31	

Here the least hour-angle from one pair of observations is $14^h\ 3^m\ 51^s.25$, the greatest $14^h\ 3^m\ 51^s.5$, and the mean of 4 pairs of observations is $14^h\ 3^m\ 51^s.31$.

If the telescope of the instrument be furnished with a micrometer, having a wire moveable but always preserving its parallelism to the horizontal wire (to hf in the figure of p. 58,) two observations may be made at each position of the telescope, one when the star is bisected by the moveable wire, the other when it is bisected by the horizontal. The object of this is to procure a greater number of results, in order to deduce a truer *mean* result.

The following Table, from Lacaille, contains the observations made with the horizontal wire, and the subsidiary observations made with the moveable one.

Altitudes.	Star's Time in the East.	Star's Time in the West.	Sums of Times.
43° 10′	$10^h\ 55^m\ 47^s$	$17^h\ 11^m\ 55^s.5$	$28^h\ 7^m\ 42^s.5$
	51.5	50.5	42
30	57 57	9 45.5	42.5
	58 2	40.5	42.5
50	11 0 7.5	7 35	42.5
	12	30	42
44 30	2 18.5	5 24.5	43
	23	20	43

mean . . . . $28^h\ 7^m\ 42^s.5$

sidereal time of star's passing the meridian 14 3 51.25.

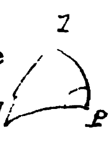
Here the mean time of the star's passage over the meridian, is $14^h\ 3^m\ 51^s.25$, instead of $14^h\ 3^m\ 51^s.31$ as it was in p. 787.

If we examine the preceding Table, the greatest time of transit from a single pair of observations is, (regarding only the seconds,) $51^s.5$, the least $51^s.0$. Lacaille, therefore, could rely on determining, by his method and with his instrument, the time of the star's transit to within a quarter of a second.

In the preceding illustration the star Arcturus was the body observed. Should the Sun or a planet be the object, then instead

of noting the time of bisection, as it is called, we must note the time of contact of the upper or lower limb with the horizontal wire. But this is not the only circumstance of difference. The Rule itself (see p. 786.) must be altered, since, from the change of declination during the observations of two corresponding altitudes, half the sum of times cannot be exactly the sidereal time of the Sun's, or planet's passage of the meridian.

This point is easily explained. Suppose the Sun's north declination to be increasing. In such a case the Sun, after passing the meridian, will be *longer* in descending to the *corresponding* altitude in the west, than it was in ascending from the eastern altitude to the meridian. Half the interval, therefore, would have the effect of throwing the meridian too much to the west, or, of retarding the time of transit. What remains then is to investigate a correction of the time dependent on the change of declination.



In a triangle ZPS, where Z is the zenith, P the pole, S the Sun, the angle ZPS measures the time $\left(\frac{t}{2}\right)$ from noon, and by *Trigonometry*, p. 139,

$$\cos.\frac{t}{2}\times\sin.ZP\times\sin.PS=\cos.ZS-\cos.ZP\times\cos.PS.$$

Now, $\frac{t}{2}$ being the exact time from noon, if PS remain constant, we have to ascertain the variation in $\frac{t}{2}$, from the variation in PS: for that purpose, it will be sufficient to deduce the proportion between the *differentials* or *fluxions* of these quantities; accordingly, taking the differential of the above equation,

$$-\frac{dt}{2}.\sin.\frac{t}{2}.\sin.ZP\sin.PS+d\,(PS)\cos.PS\cos.\frac{t}{2}.\sin.ZP=$$
$$d.(PS)..\sin.PS\cos.ZP,$$

or putting $\frac{dt}{2}=\epsilon$, $d\,(PS)=\delta$, and reducing,

$$\epsilon=\delta\left(\tan.\text{decl}^{n}.\times\cot.\frac{t}{2}-\tan.\text{lat}.\times\text{cosec}.\frac{t}{2}\right).$$

$$\text{or} = \frac{\delta}{\sin. \frac{t}{2}} \left(\tan.\ \text{decl}^{n}. \times \cos. \frac{t}{2} - \tan.\ \text{lat.}\right)$$

$$= \frac{\delta}{\sin. \frac{t}{2}} \left(\tan.\ \text{lat.} - \tan.\ \text{decl}^{n}. \times \cos. \frac{t}{2}\right),$$

if the declination, during the observations, should decrease.

As this operation of corresponding altitudes is an useful one, and of frequent occurrence, M. Zach has enabled us (see Nouvelles *Tables d' Aberration*, &c. pp. 29, &c.) to compute the correction ϵ by means of two Tables. The two Tables are constructed from the above formula thus modified. Let H be the latitude, D the Sun's declination, and let δ, instead of denoting the change of declination during half the interval of the observations, denote the daily change: instead of δ, therefore, we must write $\frac{\delta}{24} \times \frac{t}{2}$.

If also $\frac{t}{2}$ is to be expressed in hours and parts of an hour, we must write $\sin 15^{\circ} \times \frac{t}{2}$, instead of $\sin. \frac{t}{2}$, &c. So that ϵ, expressed in time, becomes

$$\epsilon = \frac{\delta}{360^{\circ}} \times \frac{\frac{t}{2}}{\sin.\ 15^{\circ} \times \frac{t}{2}} \left(\tan.\ H - \tan.\ D \,.\, \cos.\ 15^{\circ} \,.\, \frac{t}{2}\right)$$

$$= \frac{\delta}{360^{\circ} \,.\, \sin.\ 15^{\circ}} \cdot \frac{\sin.\ 15^{\circ}}{\sin.\ 15^{\circ} \,.\, \frac{t}{2}} \cdot \frac{t}{2} \,.\, \tan.\ H$$

$$- \frac{\delta \tan.\ D}{36 \,.\, \tan.\ 150^{0}} \cdot \frac{\tan.\ 150^{0}}{10 \,.\, \tan.\ 15^{\circ} \,.\, \frac{t}{2}} \cdot \frac{t}{2},$$

$$\text{make } a = \frac{\delta}{360^{\circ} \,.\, \sin.\ 15^{\circ}},$$

$$\tan. \alpha = \frac{\sin. 15^0}{\sin. 15^0 . \frac{t}{2}} . \frac{t}{2},$$

$$b = - \frac{\delta \tan. D}{36 . \tan. 150^0},$$

$$\tan. \beta = \frac{\tan. 150^0}{10 . \tan. 15^0 . \frac{t}{2}} . \frac{t}{2},$$

and

$$\epsilon = a \tan. \alpha \tan. H + b . \tan. \beta.$$

Here α, β depending on $\frac{t}{2}$ (half the interval of the observations) are taken from the same Table (Tab. XVIII.) the argument of which Table is $\frac{t}{2}$, and a and b depending on the Sun's declination (and, therefore, on the Sun's longitude) are taken from a second Table (Tab. XIX.) the argument of which is the Sun's true longitude.

Thus, suppose with a sextant we took a double altitude

$(76^0\ 50')$ at $9^h\ 47^m\ 50^s$ A. M.
and 3 0 14.5 P. M.
then since 2 12 10

is the distance of the first observation from noon,

$\frac{1}{2}(5^h\ 12^m\ 14^s.5)$
or 2 36 7.25

is half the interval $\left(\frac{t}{2}\right)$ of the observations; entering then Tab. XVIII. with the argument $2^h\ 36^m\ 7^s.25$, we obtain

$$\alpha = 46^0\ 55'\ 16'',$$
$$\beta = 10\ \ 30\ \ 5,$$

and entering Tab. XIX. with $5^s\ 4^0\ 33'\ 55''$, which, nearly, is the Sun's longitude for August 28th, 1822, we have

$$a = 13''.726,$$
$$b = 10.295.$$

Hence, Falmouth being the place of observation (the latitude of which is $50^{\circ}\ 8'$), we have

log. tan. $46^{\circ}\ 55'\ 16''$	10.0292440		
log. tan. 50 8	10.0782398		
log. $13''.726$	1.1375440		
	1.2450278	. . No. +	$17''.58$
log. tan. $10^{\circ}\ 30'\ 5''$	9.2679669		
log. $10''.295$	1.0126264		
	1.2805933	. . No. —	1 .908
			15 .67

This (+ $15''.67$) then is the correction to be added to $\frac{1}{2}(9^{h}\ 47^{m}\ 50^{s} + 15^{h}\ 0^{m}\ 4^{s}.5)$, or $12^{h}\ 23^{m}\ 57^{s}.25$, in order to have the time of apparent noon, which accordingly is

$$12^{h}.\ 24^{m}\ 12^{s}.92.$$

This is the result from one pair of corresponding altitudes: but, as soon as one observation is made, preparation is made for another by advancing (see p. 787,) the limb of the telescope on the limb of the instrument, 10 or 20 minutes: for instance, in the example from which the above times were taken, the second double altitude was 77°, and the times before and after noon were, respectively,

(see p. 791,)	9^{h}	48^{m}	$31^{s}.5$
and (adding 12^{h})	14	59	24 .5
the half, or time from noon	12	23	58
the correction computed as above . .		+	15 .67
∴ the time from noon	12	24	13 .67.

As in the case of the observed times of the corresponding altitudes of a star, the *mean* of all the results is to be taken as the true result. All the observations are subjoined.

Place, Falmouth: Time, August 28, 1822.				
Double Altitudes.	Times A. M.	Times P. M.	Corrections.	Times of Apparent Noon.
76° 56′	9^h 47^m 50^s	3^h 0^m $4^s.5$	15″.67	12^h 24^m $12^s.93$
77 0	48 31.5	2 59 24.5	.67	13.67
10	49 10.5	58 45.5	.65	13.65
20	49 49.4	58 5.5	.64	13.09
30	50 30.6	57 26	.62	13.92
40	51 10.5	56 47	.60	14.50
50	51 50.2	56 6.5	.58	13.93
78 0	52 31	55 25	.56	13.56
5	52 50	55 6	.55	13.55
10	53 11	54 46.5	.54	14.29
80 0	10 0 41.2	47 16	.40	14.0
10	1 23.2	46 34	.39	13.99
20	2 4.4	45 51	.38	13.08
40	3 28.8	44 30	.33	14.73
50	4 10.5	43 47	.31	14.06
81 0	4 52.8	43 4.6	.32	13.72
		Mean		12^h 24^m $13^s.792$

We have given instances of a star and the Sun: the method will also apply, with equal facility, to a planet. The second Table (XIX.) of M. Zach cannot indeed be used because its argument is the Sun's longitude, but it is easy to dispense with it by computing the change of the planet's declination in 24 hours.

Thus,

$$\epsilon = \frac{\delta}{360^\circ . \sin. 15^\circ} \tan. a . \tan. H - \frac{\delta}{36 . \tan. 150^\circ} \tan. D . \tan. \beta,$$

in which ϵ can be computed, if δ be known.

Example.

April 8, 1809, Mars was observed at Florence, and the following were the conditions:

latitude of Florence, or	$H =$	$43^\circ\ 46'\ 40''$
south declination of Mars, or	$D =$	$5\quad 9\quad 40$
diurnal change of declination, or	δ	$+6\quad 38$
half the interval of observation, or	$\frac{t}{2}$	$4^h\ 10^m$.

Hence,

log. $398''$ ($= 6'\ 38''$).	$= 2.5998831$	
log. $\frac{1}{360 \cdot \sin. 15^\circ}$	8.0307013	
log. tan. $50^\circ\ 33'\ 40''$ (α)	0.0848395	
log. tan. $43\quad 46\quad 40$ (H).	9.9814658	$-$
	0.6968897	No. $4''.97$.

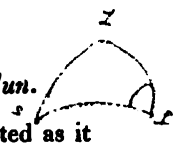
Again,

log. $398''$	2.5998831	
log. $\frac{1}{36 \cdot \tan. 30^\circ}$	8.6822581	
log. tan. $7^\circ\ 8'\ 16''$ (β).	9.0976954	
log. tan. $5\quad 9\quad 40$ (D).	8.9557974	
	9.3356340	No. $-0''.22$
	the correction	4.75.

Since the change of the Sun's declination may be had from the Nautical Almanack, a calculation, exactly similar to the preceding, will apply to the corresponding altitudes of the Sun, and be equally simple with the one of p. 791, from which, indeed, it does not much differ.

The above method of determining the time from *corresponding altitudes* is the best of all methods, when we are not provided with a fixed and adjusted transit instrument. It is, as M. Zach observes, capable of great exactness, and is independent of the

rectification of the instrument. It requires the aid solely of a chronometer, sufficiently good to mark the times during an interval of 5 or 6 hours. Those astronomical elements, such as the latitude of the place, the altitude of a star, its right ascension, &c. which are requisite to be known in the following methods, need not be known in this.



Time determined from an observed Altitude of the Sun.

The altitude of the Sun is to be observed and corrected as it was in page 775; then, we have to find the angle ZPS (h), from ZS ($90^\circ - A$) thus determined, from the Sun's north polar distance (p) given by the Tables, and from the latitude (L) of the place, known or previously determined by observation. Then by *Trig.* pp. 139, &c. making $h = ZPS$, we have cos. h

$$= \frac{\cos. ZS - \cos. ZP \times \cos. PS}{\sin. ZP \,.\, \sin. PS} = \frac{\sin. A - \sin. L \,.\cos. p}{\cos. L \,.\, \sin. p};$$

$$\therefore 2 \,.\sin.^2 \frac{h}{2} = 1 - \cos. h = \frac{\cos. L \,.\sin. p + \sin. L \,.\cos. p - \sin. A}{\cos. L \,.\, \sin. p}$$

$$= \frac{\sin. (p + L) - \sin. A}{\cos. L \,.\, \sin. p}$$

$$= \frac{2}{\cos. L \,.\sin. p} [\cos. \tfrac{1}{2}(p + L + A) \sin. \tfrac{1}{2}(p + L - A)],$$

and, in logarithms, $\quad 2 \log. \sin. \frac{h}{2} = 20 +$

$$\log. \cos. \tfrac{1}{2}(p + L + A) + \log. \sin. \tfrac{1}{2}(p + L - A) - \log. \cos. L - \log. \sin. p.$$

EXAMPLE.

The Sun's Altitude being 39° 5′ 28″; *his North Polar Distance, from Nautical Almanack,* 74° 51′ 50″, *and the Latitude of Place,* 52° 12′ 42″; *it is required to deduce the Time.*

$$
\begin{array}{llllll}
 & L = 52^\circ & 12' & 42'' & \ldots\ldots \cos. = & 9.7872806 \\
 & p = 74 & 41 & 50 & \ldots\ldots \sin. = & \underline{9.9846660} \\
 & A = \underline{39} & \underline{5} & \underline{28} & & \underline{19.7719466} \ldots (a) \\
\text{sum} & 166 & 10 & & 20 & \\
\frac{1}{2}\,\text{sum} & 83 & 5 & \ldots\ldots \cos. = & 9.0807189 & \\
\frac{1}{2}\,\text{sum} - A & 43 & 59 & 32 \ldots \sin. = & \underline{9.8417102} & \\
 & & & & 38.9224291 & \\
 & & & (a) & \underline{19.7719466} & \\
 & & & 2) & \underline{19.1504825} &
\end{array}
$$

$$\log.\sin.\frac{h}{2} = 9.5752412 = \log.\sin.\ 22^\circ\ 5'\ 20''\tfrac{1}{3}$$

$$\therefore h = 44^\circ\ 10'\ 40''\tfrac{2}{3} = (\text{in time})\ 2^h\ 56^m\ 43^s,\ \text{nearly}.$$

This is the time for Greenwich; for any other place, we must correct p, taken from the Nautical Almanack, by adding to it, or subtracting from it, the change in the Sun's north polar distance, proportional to the difference of longitude between Greenwich, and the place of the observed altitude.

Time determined from an observed Altitude of a fixed Star.

The altitude is to be observed as in the former instance: the latitude is supposed to be known from previous observation, and, the star's north polar distance from his *mean* north polar distance (contained in Tables) corrected for the several inequalities of precession, aberration, and nutation; (see Chapters XI, &c.) Then, the computation of the angle ZPS, or of h, will be exactly the same as in the preceding case. That angle will be the star's angular distance from the meridian; therefore, since the star's right ascension is known, the right ascension of a point of an imaginary star, at that time supposed to be on the meridian, is known. But, the right ascension of a star on the meridian being known, the hour of the day is (see pp. 779, &c.)

All stars on the meridian at the same time have the same right ascension; therefore, we may place the imaginary star on the

equator, and then (see p. 748,) its right ascension will be that of the *Mid-Heaven*; consequently we may give the rule for finding the time under the following form:

$$*\text{'s } Ꭿ \pm h = Ꭿ \text{ of mid-heaven,}$$

$$Ꭿ \text{ of mid-heaven} - \odot\text{'s } Ꭿ - \textit{acceleration} = \text{time (see p. 780.)}$$

EXAMPLE.

April 14, 1780. *In Latitude* 48° 56′, *Longitude* W = 66° ($4^h\ 24^m$) *the Altitude of* Aldebaran *in the West was observed* = 22° 20′ 8″. *Required the Time.*

$$\begin{array}{lrrrll}
L = & 48^\circ & 56' & 0'' & \ldots\ldots\ldots\ldots \text{ cos.} = & 9.8175235 \\
p = & 73 & 56 & 59 & \ldots\ldots\ldots\ldots \text{ sin.} & \underline{9.9827322} \\
A = & \underline{22} & \underline{17} & \underline{50} & (\text{refrac.} = 2'\ 18'') & \underline{19.8002557} \\
2) & \underline{145} & \underline{10} & \underline{49} & & 20 \\
\tfrac{1}{2}\text{ sum} = & 72 & 35 & 24 & \ldots\ldots\ldots\ldots \text{ cos.} & 9.4759722 \\
\tfrac{1}{2}\text{ sum} - A = & 50 & 17 & 34 & \ldots\ldots\ldots\ldots \text{ sin.} & \underline{9.8861065} \\
 & & & & & 39.3620787 \\
 & & & & & \underline{19.8002557} \\
 & & & & 2) & \underline{19.5618230}
\end{array}$$

$$\text{log. sin. } \frac{h}{2} = 9.7809115$$

$$[= l \text{ sin. } 37^\circ\ 8'\ 39''.75;$$

$$\begin{array}{lrrrl}
\therefore h = & 74^\circ & 17' & 19''.5 & \\
*\text{'s } Ꭿ = & \underline{65} & \underline{49} & \underline{49.5} & (\text{by Tables}) \\
*\text{'s } Ꭿ + h = & 140 & 7 & 9 & = Ꭿ \text{ of mid-heaven.}
\end{array}$$

$$\begin{array}{lrrrl}
\text{But, April 14, } \odot\text{'s } Ꭿ = & 1^h & 31^m & 1^s & \\
\text{April 15, } \ldots\ldots = & \underline{1} & \underline{34} & \underline{42} & \\
\text{Increase in } 24^h \ldots\ldots = & 0 & 3 & 41 & \therefore \text{prop}^l.\text{inc}^e.\text{in } 4^h\ 24^m = 40^s.
\end{array}$$

$$\begin{array}{lrrr}
\text{Hence, } Ꭿ \text{ of mid-heaven } (140^\circ\ 7'\ 9'') \ldots\ldots = & 9^h & 20^m & 28.6 \\
\odot\text{'s } Ꭿ\ (= 1^h\ 31^m\ 1^s + 40^s) \ldots\ldots\ldots\ldots = & \underline{1} & \underline{31} & \underline{41} \\
 & 7 & 48 & 47.6 \\
\textit{Acceleration} \text{ (see p. 780,)} \ldots\ldots\ldots\ldots\ldots & \underline{0} & \underline{1} & \underline{16.8} \\
\therefore \text{ apparent time} = & \underline{7} & \underline{47} & \underline{30.8}
\end{array}$$

This method, as a practical one, is inferior to the former, partly from the greater length of its computations, and partly from the difficulty of exactly noting the altitude of a star with a sextant. The errors of the Solar Tables affect both methods. In order to lessen the errors of observation, several successive altitudes, distant from each other by nearly equal intervals of time, are noted, and the *mean* altitude deduced corresponding to a *mean* time.

In the sextant there is always some difficulty (and consequently some chance of error) in *reading* off the graduations at the end of each observation. This kind of error is avoided, at least much lessened, in *repeating circles*. Since, with such instruments the *reading off* is not made till after all the observations. The reading off then is the *sum* of all the several altitudes (if they are altitudes which are observed), and the mean altitude is to be had by dividing the above *sum* by the number of observations.

In an Observatory, that has its instruments fixed in the plane of the meridian, the time of apparent noon is easily determined. It may be also ascertained by a sextant, which (see p. 774,) is adapted to measure altitudes: by means of it we can determine when the Sun is at its greatest altitude, or in the meridian. But the altitude of the Sun, when near to the meridian, varying very little, it is difficult to ascertain by a sextant the precise time of the greatest altitude, and consequently, that of apparent noon. Out of the meridian, the variations of altitude are quicker: where they are most quick, then, an error in the altitude (and such there will always be in an observation with a sextant) must be of the least consequence, since it least affects the time; which time would be truly computed by the preceding method, if the altitude were rightly observed.

Since the altitude changes most slowly, when the star is near the meridian, either towards the south or the north, it seems probable, that it would change most rapidly, half way between the north and south; and this is the case, as we shall prove in the solution of a problem, which is usually thus announced.

Given the Error in Altitude; it is required to find where the corresponding Error in Time will be the least.

By p. 795,

$$\cos. h = \frac{\sin. A - \sin. L . \cos. p}{\cos. L . \sin. p},$$

take the differential or fluxion of this equation, and put $dh = \epsilon$, $dA = a$, then

$$-\epsilon \sin. h = a \frac{\cos. A}{\cos. L . \sin. p},$$

but by *Trigonometry*, $\sin. h \times \sin. p = \sin. PZS \times \cos. A$;

$$\therefore \epsilon = \frac{a}{\sin. PZS \times \cos. L};$$

consequently, if L and a, the error in altitude, be given, ϵ is least, when $\sin. PZS$ is the greatest, that is, when $PZS = 90^\circ$, or the azimuth, is 90°, or the body is on the *Prime Vertical:* which is that vertical circle which passes through the east and west points.

The above is the reason of the precept given by Dr. Maskelyne at p. 152, Nautical Almanack, in which he directs the altitude to be observed near the west and east points. To this precept may be added another; that those stars should be selected for observation, which move most quickly; those, therefore, which are situated near the equator.

Besides the error of altitude, there may be an error in the assumed latitude. For, between the observation which determines the latter from the Sun's meridian altitude, and the observation of the altitude, the observer, if on board a ship, may have changed his place, and, if so, has probably changed his latitude. The relation between its error and that of the time may be determined exactly as the relation between ϵ and δ was in p. 789. Instead of making PS to vary, we must make ZS, $(90 - L)$; let λ be the variation of L, then,

$$\epsilon = \lambda \left(\tan. \text{dec.} \times \text{cosec.} \frac{t}{2} - \tan. \text{lat.} \times \cot. \frac{t}{2}\right).$$

There are several methods and instruments used to ascertain, in the interval between observations, the situation of the ship. Dating from a latitude and longitude astronomically determined, navigators *carry* on a latitude and longitude by *account*. This they are enabled to do, by the chronometer, by the *Log* (by which instrument they ascertain the ship's velocity,) and by an instrument of which we shall now give a short account, and called

The Magnetic Compass.

The *Needle* of the Magnetic compass, is a thin bar of steel, made to move about a centre, in a plane nearly horizontal; which needle in different parts of the Earth points to different parts of the horizon. In scarcely any place, is its direction true north and south. The *Magnetic North*, almost always, differs from the true. And the difference is, technically, called the *Variation* of the compass, differing in degree at different places, and not remaining the same at the same place. Navigators are provided with charts of this *Variation*. Therefore, by observing the *variation* they are to form some probable conjecture of the situation of the ship: and if, by independent means, they know the latter condition, they will be able to examine and to correct the charts.

We must now then consider by what astronomical methods the deviation of the *Magnetical* from the *true* north may be ascertained.

The Magnetic north is always known from the direction of the Magnetic needle. The true north may be computed from the Sun's azimuth, at the time of his rising, or from his observed altitude at any other time. The azimuth is the angle PZS; the computation of which is exactly similar to that of the hour angle ZPS (h) in p. 795.

Let the declination and zenith distance of the Sun be d, z, then,

$$\cos. PZS = \frac{\cos. PS - \cos. ZP . \cos. ZS}{\sin. ZP . \sin. ZS} = \frac{\sin. d - \sin. L \cos. z}{\cos. L . \sin. z}$$

when the Sun rises, or is on the horizon, $z = 90^\circ$;

$$\therefore \cos. z = 0, \text{ and } \sin. z = 1,$$

$$\text{and } \cos. PZS, \text{ or } \sin. \textit{amplitude}^* = \frac{\sin. d}{\cos. L}.$$

In other situations, deducing 2 log. sin. $\frac{PZS}{2}$, exactly as 2 log. sin. $\frac{h}{2}$ was, in p. 795, we have

$$2 \log. \sin. \text{azimuth} = 20 + \log. \cos. \tfrac{1}{2}(L + z + d) +$$
$$\log. \sin. \tfrac{1}{2}(L + z - d) - \log. \cos. L - \log. \sin. z.$$

Example to the First Method.

In Lat. 51° 52′ *N. the Sun's Declination being* 23° 28′ *N. Required the amplitude, in the Morning.*

$d = 23^{\circ}\,28'$ .... sin. 9.6001181
$L = 51\;\;52$ .... cos. 9.7906325

9.8094856 = log. sin. 40° 9′ 26′

∴ the Sun's distance from the east point = 40° 9′ 26″.

Or the computed true amplitude is	40°	9′	26″	N. E.
∴ if the amplitude by the compass be....	52	12	28	N. E.
the variation of the compass........	12	3	2	

This operation cannot be a very exact one, since the *computed* amplitude is the amplitude of the Sun when its centre is on the *true* horizon. The observation with the compass can only be made when the Sun is on the *visible* horizon.

Some precautions, therefore, must be taken: and the writers on Nautical Astronomy direct us to take, with the compass, the amplitude of the Sun's centre when the lower limb appears elevated above the horizon by a space somewhat greater than the Sun's semi-diameter. This, however, must needs be an imperfect and rude operation.

---

* The amplitude is frequently appropriated to signify the complement of the azimuth, when the star *rises* or *sets*.

Example to the Second Method.

In Lat. 51° 32′, the Sun's Declination being 23° 28′, and his Altitude corrected for Refraction 46° 20′. Required the Azimuth.

$$
\begin{array}{rll}
L = 51°32' \ldots\ldots \cos. & = 9.7938317 & \\
z = 43\ 40 \ldots\ldots \sin. & = 9.8391396 & \\
\hline
d = 23\ 28 & \ 19.6329713 & (a) \\
\hline
\text{sum} = 118\ 40 & \ 20 & \\
\tfrac{1}{2}\,\text{sum} = 59\ 20 \ldots\ldots \cos. & = 9.7076064 & \\
\tfrac{1}{2}\,\text{sum} - d = 35\ 52 \ldots\ldots \sin. & \ 9.7678242 & \\
\hline
 & \ 39.4754306 & \\
(a) & \ 19.6329713 & \\
\hline
 & 2)19.8424593 & \\
\hline
 & \ 9.9212296 & = \log.\sin.\,56°31'28''
\end{array}
$$

$\therefore$ the Sun's azimuth $= 56°\ 31'\ 28''$.

We will now briefly explain the

Methods of regulating Chronometers.

We have already in pp. 100, &c. explained the method of regulating an Astronomical Clock by means of a fixed transit instrument. But it is necessary, in geodesical operations, for instance, to employ portable instruments and chronometers, and we have now to explain by what means the latter may be regulated, or, rather, their irregularities detected and valued.

The *error* of a chronometer at any time is the difference between the time deduced from astronomical phenomena, and the time its index denotes. The *rate* of a chronometer is the difference between two successive *errors:* it is called the *daily* rate when it is the difference between two errors that happen at the interval of twenty-four hours; or, the *daily* rate may be made to mean the quotient arising from dividing the difference of two more distant errors by the number of intervening days. In order to know, from astronomical phenomena,

the time when we are not possessed of a transit instrument, there is no better method than that of *corresponding altitudes* taken by means of an equal altitude instrument, or sextant. In the Example of p. 793, the mean of sixteen observations gave

$$12^h\ 24^m\ 13^s.792,$$

as the apparent time by the chronometer of the Sun's transit over the meridian. Now on the day of observation (August 28, 1822.) the equation of time was $1^m\ 9^s.3$ additive of apparent time; consequently, the chronometer, if it had been properly adjusted to mean solar time, ought to have denoted

$$12^h\ 1^m\ 9^s.3,$$

as the time of the Sun's transit.

The *error*, therefore, of the chronometer on that day (the difference between $12^h\ 24^m\ 13^s.792$, and $12^h\ 1^m\ 9^s.3$) was $23^m\ 4^s.492$, and hence, as a general rule, *correct the chronometer's time of the Sun's transit (determined as above, or by like methods) by the equation of time with a contrary sign, and the result is the time of mean noon by the chronometer.*

We have been speaking of portable chronometers to be examined or regulated at different stations. Now the *equation of time*, of which we have just spoken, is the *equation* when the Sun is on the meridian of the place of observation, and, consequently, not (except in particular cases,) the *equation* inserted in the Nautical Almanack; which latter *equation* is the correction of the apparent time of the Sun's transit over the meridian of Greenwich. In practice, therefore, it will be, almost always, necessary to compute the equation of time for the noon of the place of observation. This is easily done: for instance, if the place of observation were Cadiz, the longitude of which is $25^m\ 8^s$ west of Greenwich, it would be necessary to compute the *equation of time*, for a time $25^m\ 8^s$ *after* the noon of Greenwich. Suppose the observation made on September 8, 1808: in the Nautical Almanack, p. 98, we have

equation of time subtractive $2^m\ 29^s.4$, difference $20^s.4$,

and, therefore, the difference, corresponding to $25^m\ 8^s$,

$$= 20^s.4 \times \frac{25^m\ 8^s}{24^h} = 0^s.36 \text{ nearly};$$

consequently, the equation of time when the Sun was on the meridian at Cadiz, is equal to

$$2^m\ 29^s.76,$$

$$\text{or nearly, } 2^m\ 29^s.8.$$

This, and the previous explanation are sufficient for the following example, and the mode of solving it.

Example.

In September 1808, at Cadiz (longitude $25^m\ 8^s$, latitude $36^\circ\ 31'$ N.) by means of corresponding altitudes (see p. 786,) the following times of noon were obtained *:

Times of Noon.				Equation of Time for Cadiz.		Times of Mean Noon.			Chronometer too slow.		Differences.
Sept. 8,	11^h	51^m	$48^s.38$	2^m	$29^s.8$	11^h	54^m	$18^s.18$	5^m	$41^s.82$	
											$-12^s.64$
11,		50	59.22	3	31.6		54	30.82	5	29.18	
											-16.11
15,		49	51.83	4	55.1		54	46.93	5	13.07	
											-12.53
18,		49	1.46	5	58		54	59.46	5	0.54	
											-12.51
21,		48	11.27	7	0.7		55	11.97	4	48.03	
											-11.85
24,		47	21.22	8	2.6		55	23.82	4	36.18	
											65.64

Here the sum of differences in 16 days is $65^s.64$, and, accordingly, the mean daily rate, estimated by dividing the sum by the number of days, is $-4^s.1025$.

---

* The column of *equations of time* for Cadiz is formed by adding .4 (nearly the proportional difference, see above) to the equations of time expressed in the Nautical Almanack.

If we estimate the daily rates, by dividing the numbers in the last column, by the numbers of intervening days (3, 4, &c.) we shall have the mean daily rates

from Sept. 8 to 11	....	$-4^{s}.21$
11 to 15	....	-4.03
15 to 18	....	-4.18
18 to 21	....	-4.17
21 to 24	....	-3.95

which differ slightly from the preceding mean daily rate of p. 804.

This is, in effect, the method of determining the *errors and daily rates* of chronometers, by whatever operation or process the time of apparent noon be determined: whether such time be determined by a transit instrument* or be computed (see pp. 795, &c.) from the observed altitude of the Sun or a star, and the latitude of the place of observation.

The present Chapter, unlike the preceding ones, is not confined to the same subject. It contains several methods unconnected as to their nature, and capable of being classed together only because they are useful, or subsidiary to the same astronomical instrument, such as the sextant. We shall soon speak of other uses of that instrument, and of its principal one in determining the longitude of a vessel at sea. That subject, however, claims

* The *rate* of a chronometer may be determined by a telescope even if it should not be fixed in the plane of the meridian. It is only necessary to take care that the wires of the telescope be at right angles to the star's motion. The interval between two successive returns of the same star to one of these wires is a sidereal day, which differs from a mean solar day by the *acceleration*: so that a chronometer, exactly adjusted to mean solar time, ought to note 24^{h} – *acceleration* during two successive transits of the star over the same wire of the telescope. Thus, May 3, α Libræ passed the vertical wire of a fixed telescope at .... $10^{h}\ 44^{m}\ 41^{s}$

acceleration	0	3	55.9
	10	40	45.1
but chronometer at the ✻'s transit on May 3, noted........	10	40	47
∴ rate	+ 0		2.1

a separate Chapter: the present we will conclude with the solution of a few astronomical problems, as they may be called, flowing easily from the Trigonometrical formula, of which, such frequent use has already been made.

If h be the hour angle, z the zenith distance, L the latitude of the place, p the polar distance of the star or Sun, then

$$\cos. h = \frac{\cos. z - \sin. L . \cos. p}{\cos. L . \sin. p}.$$

When the Sun rises or sets, $z = 90^{\circ}$, $\cos. z = 0$;

$$\therefore \cos. h = -\frac{\sin. L . \cos. p}{\cos. L . \sin. p} = -\tan. L . \cot. p,$$

the negative sign indicating that, if p be $< 90^{\circ}$, h is $> 90^{\circ}$, in other words, that, if the Sun have north declination, h will be greater than 6 hours, or that the length of the day will exceed 12 hours.

Again, if $h = 0$,

$$\begin{aligned}\cos. z &= \cos. L \sin. p + \sin. L . \cos. p \\ &= \sin. (p + L) \\ &= \cos. [p - (90^{\circ} - L)].\end{aligned}$$

If P, Z, S, be the places of the pole, zenith, Sun (or star),

$$\cos. ZS = \cos. (PS - ZP),$$

and $ZS = PS - ZP$, the body being on the meridian. In this case, then, ZS the meridional zenith distance, is the least zenith distance, since in every other position of S, there is formed a triangle ZPS, in which $PS - ZP$ is $< ZS$.

Twilight is the light of the Sun, when below the horizon, faintly reflected by the atmosphere; and, by computation, it is found to be just sensible when the Sun is within 18^{0} of the horizon; or, when $z = 118^{0}$. We may find the time, therefore, of twilight's beginning or ending, by substituting in the preceding expression, or in that which is immediately deduced from it, (see p. 795,) instead of $A\ (= 90^{0} - z)$, $-\ 18^{0}$.

The *duration* of twilight is the interval of time due to the Sun's ascending or descending through 18^{0}, it is, therefore, equal

to the difference of the last, and that expression (p. 806, l. 9,) which expresses the time of the Sun's rising or setting.

The boundary of twilight, a small circle, parallel to the horizon and 18^0 from it, is called the *Almaçanter*.

The length of a day, in its common acceptation, is the interval of time between the rising and setting of the Sun; it is, therefore, equal to twice the angle h, estimated from that expression of cos. h, in which $A = 0$, that is, it is equal to $2 \,.\, \tan. L \,.\, \text{co-tan}. p$,

At the equinoxes, p the ☉'s N.P.D. $= 90^0$;

$$\therefore \cot. p = 0; \therefore \cos. h = 0; \therefore h = 90^\circ = \text{(in time) } 6^h;$$
$$\therefore \text{the length of the day} = 12^h.$$

At the solstices, p, either, $= 90^\circ - 23^\circ\,28'$, or $90^0 + 23^0\,28'$; therefore, the lengths of the longest and shortest day at Greenwich are to be computed from this expression,

$$\cos. h = \mp\, 2 \tan. 51^\circ\,28'\,39''.5 \times \tan. 23^\circ\,28',$$

the upper sign $-$, for the longest day, denoting h to be $> 90^0$, and the lower sign $+$, for the shortest, denoting h to be $< 90^\circ$, and equal to the supplement of the former.

If we wish to investigate the latitude in which the Sun's centre, in its greatest depression, just reaches, but does not descend below, the horizon, we must make $h = 180^\circ$,

$$\text{then } \cos. 180^\circ = -1 = -\tan. L \,.\, \cot. p = -\frac{\tan. L}{\tan. p};$$

$$\therefore \tan. L = \tan. p, \text{ and } L = p = 90^\circ - \text{declination},$$

or, the co-latitude of the place equals the Sun's declination.

In a similar way, and still using the expression for cos. h, we may express the relation between the latitude and the Sun's declination, when there is *just* twilight all night; thus, z being the zenith distance, since

$$\cos. h = \frac{\cos. z - \sin. L \,.\, \cos. p}{\cos. L \,.\, \sin. p},$$

$$\cos. 180^0 = \frac{\cos. 118^\circ - \sin. L \,.\, \cos. p}{\cos. L \,.\, \sin. p};$$

$\therefore$ sin. L cos. p − cos. L sin. p, or sin. $(L - p)$, $=$ cos. $118^\circ = -$ sin. 18°;

$$\therefore\ L - p = -18^0, \text{ or } L - (90^0 - \odot\text{'s dec.}) = -18^\circ;$$

$$\therefore\ \odot\text{'s declination} = (90^0 - L) - 18^0.$$

If L therefore be given, search in the Nautical Almanack for that declination, which equals the difference of the co-latitude and 18^0.

Since, $L = p - 18^0$, and the least value of p, is $66^0\ 32'$; therefore the *least* value of L is $48^0\ 32'$; or in latitudes less than $48^0\ 32'$, there never can be twilight all night.

CHAP. XLII.

On Geographical Latitude.

LATITUDE of places at land, (see p. 11, &c.)

1st. *Method by the Altitudes of Circumpolar Stars.*

This method has been already described in pp. 129, &c. Another instance of it is subjoined, in which, the circumpolar star is that particular one, which, for distinction, is called the *Pole Star*, (the *a Polaris* of Astronomical Catalogues.)

By means of an Astronomical Circle, (see Chap. V,) the following zenith distances (*Z. D.*) were observed at Dublin Observatory on August 23, 1808 :

Greatest Z. D.	38°	18′	59″.1
Refraction (barom. 29.97, thermom. 67,) ...	0	0	44.01
Corrected Z. D.	38	19	43.11
Least Z. D. of *a Polaris*	34°	53′	10″.1
Refraction, (barom. 29, 99, thermom. 58,) ..	0	0	39.45
Corrected Z. D	34	53	49.55
	38	19	43.11
2)	73	13	32.66
∴ co-latitude of Observatory, ...	36	36	46.33

and latitude is 53°. 23′ 13″.67.

2dly, *Method by the Zenith distances of Stars near the Zenith.*

This method determines merely the difference of latitude by means of an instrument, (the zenith sector) capable of measuring

small zenith distances with great exactness. We have had already (pp. 12, &c.) specimens of it, and we here subjoin another.

Example.

By observation, at the College of Mazarin, *Mem. Acad.* 1755.)

Z. D. of γ *Draconis reduced* (see p. 380,) to Jan. 1750,	2°	$40'$	$15''$
At Greenwich Z. D. reduced to the same epoch	0	3	4.5
(The star is to the north of both zeniths) diff. lat....	2	37	10.5
Hence, if the latitude of Greenwich be	51	28	39.5
Latitude of Observatory, at College of Mazarin....	48	51	29

It is essential, as it has been fully explained in pp. 306, &c. that, for finding the difference of latitudes, by this operation, the zenith distances of the star observed at different epochs, should be reduced to the same. If, however, we should be possessed of two observations of the same star, made on the same day, of the same year, then, since the corrections of aberration, precession, and nutation, (see Chap. XI, XIII, XIV,) would be the same in each observation, it would be necessary merely to apply the corrections for refraction, before we subtracted or added the zenith distances.

This method of determining the latitude, and capable of great accuracy, was employed in the Trigonometrical Survey of England. See *Phil. Trans.* 1803, pp. 483, &c.

Method of determining the Latitude, by reducing to the Meridian the observed Zenith Distances of the Sun, or a Star when near to the Meridian.

The principle and peculiar processes of this method have already, in substance, been explained in pp. 417, 418, &c. The illustrations there given, were, with observations, made with the *circle* of the Dublin Observatory. It now remains to adapt the method to observations made with small portable instruments: for with such observations and instruments is our present concern.

By observing the zenith distance of a star out of the meridian, and by *reducing it* to the meridian, we obtain a result which is equal to the star's meridional zenith distance. When, therefore, as in the instance of the star Arcturus, (see pp. 422, &c.) we observe four zenith distances, two before, two after, the star's transit over the meridian, we obtain four meridional zenith distances: one-fourth of the sum of which, the *mean* meridional zenith distance, is to be held, according to astronomical usage, and as it probably is, a more true value than any individual zenith distance.

It follows from this, that if we could multiply our observations near to the meridian, we should obtain a truer value of the star's meridional zenith distance. But, with an instrument, such as that of the Dublin *Circle*, there are limits to such multiplication. From the size of the instrument, the *readings off* at the three verniers cannot be very quickly effected: add to this, the instrument must be *adjusted* at each observation: so that, at the distance of ten or twelve minutes of time from the meridian, more than two observations cannot be *conveniently* made; and if we begin to observe the star at greater distances from the meridian, the computations of the *corrections* (see pp. 420, &c.) become more operose and less exact.

With instruments, however, of less magnitude which the observer can adjust and *read off*, without hardly shifting his position, a greater number of observations may be made; and no instrument is so fitted to the rapid multiplication of observations as the *repeating circle*, because, in that, the *readings off* are not made till the termination of the observations.

We shall soon give an instance of a meridional zenith distance, deduced from twenty-six observations made out of the meridian. But the advantage of so many observations, is not solely that of giving, by their number, a more exact *mean* result. It is easy to see, by referring to pp. 420, &c. that the corrections c, c', c'' become less, the nearer the star is to the meridian: it will, therefore, frequently happen (it will always so happen with those stars which are selected for the use of *repeating circles*) that, in computing the reduction, we may confine our computation to that of

the first correction: since the second and third corrections, which must be inconsiderable, except in the extreme observations (those which are most to the east and west) will have scarcely any effect on the mean result.

Thus, if there should be twenty-six results, and the values of the second and third corrections should amount to one-fourth of a second, the mean result could only be affected by them to the amount of $\frac{1}{104}$th of a second.

Let us suppose, however, that we are able, either by computing the three corrections or only one, to determine the star's meridional zenith distance: such distance, if corrected solely on account of refraction, and not on account of the inequalities of precession, aberration and nutation, is an *apparent zenith distance.* If, therefore, the star be to the south of the pole and zenith, the co-latitude (ZP) is to be obtained by subtracting the above *apparent zenith distance,* from the star's *apparent north polar distance.* If the star be south of the pole, but between the pole and zenith, the co-latitude is the sum of the above two apparent zenith distances. If, however, we choose to correct the observed zenith distance by the *equations* due to precession, &c. we must then instead of the above-mentioned *apparent* polar distance, use the *mean* polar distance. The result in each case, as it has been abundantly explained in the preceding pages, must be the same. The formulæ of reduction which we shall use in the succeeding Examples, are those which are given at p. 420, in which A depends on the latitude of the place, and C on A and the star that is observed. In two of the Examples that follow, the places of observation are Dunkirk and Leith: at the former the pole star was observed, at the latter the Sun.

Hence, for these two places, the latitudes of which are respectively $51^{\circ}\ 2'\ 5''$ and $55^{\circ}\ 58'\ 4''$, we have (see pp. 420, 421,) the following computations of log. A,

Dunkirk.		Leith.	
log. sin. $1''$	4.68557		
2 log...15	2.35218	sum	16.73672
ar. com. 2	9.69896	log. cos. $55^{\circ}\ 58'\ 41''$	9.74781
log. cos. $51^{\circ}\ 2'\ 5''$	9.79854		26.48453
	26.53526		

Hence, for Dunkirk, (see p. 421,)

$$\log. C = 6.53526 + \log. \sin. D + \log. \text{cosec}. z - 20 + 2 \log. h'.$$

For Leith,

$$\log. C = 6.48453 + \log. \sin. D + \log. \text{cosec}. z - 20 + 2 \log. h'.$$

We will now, in the case of Dunkirk, farther reduce the value of log. C; for which end it is necessary to take account of the other conditions of the observations.

The observed star was Polaris: the time Dec. 19, 1808;

therefore, since co-lat. .... =	38° 57′ 55″		
and (from Tables) ✱'s N.P.D. =	1 42 18.5	.... sin.	8.47357
$(ZP - PS)$	37 15 36.5	.. cosec.	10.21793
		(from l. 2,)	6.53526
			25.22676

Accordingly,

$$\log. C = 5.22676 + 2 \log. h',$$

which is the formula of computation, from which the correction C is to be computed, when h' the horary angle is given.

Suppose, for instance, a value of h' to be $27^m\ 42^s$,

$$
\begin{aligned}
{}^*\log. 27^m\ 42^s &= 3.22063 \\
&\quad\ \ \underline{2} \\
&\quad\ 6.44126 \\
&\quad\ \underline{5.22676} \\
&\quad\ 1.66803 = \log. 46''.561,
\end{aligned}
$$

and so for other values. The following Table contains the values of h', according to the observations (made in the instance we are quoting) and the corresponding values of the corrections.

* These logarithms may be had very conveniently from Mendoza's Tables, (Tab. XV.)

Values of h'.		Logarithms of C.	Values of C.
27^m	42^s	1.66802	46″.561
26	26	1.62736	42.400
25	38	1.60068	39.875
24	57	1.57720	37.775
24	17	1.55368	35.783
23	39	1.53072	33.940
22	58	1.50526	32.008
15	18	1.15244	14.205
14	34	1.10978	12.877
5	47	0.30742	2.030
2	21	9.52520	0.335
1	45	9.26914	0.185
1	1	8.79742	0.063
4	35	0.10542	1.274
17	15	1.25664	18.057
21	10	1.43436	27.187
21	52	1.46262	29.015
22	28	1.48614	30.630
23	8	1.51154	32.475
23	47	1.53560	32.324
24	19	1.55488	35.883
25	37	1.60010	39.820
26	20	1.62408	42.080
28	3	1.67892	47.744
29	56	1.73538	54.373
35	34	1.88514	76.761
		26)	767.66
		Mean value of C	29.52

The values of h' are thus to be obtained. Note by the chronometer the hour of the passage of *Polaris* over the meridian, using a transit instrument, or, in default thereof, a sextant or *repeating circle*, or any instrument that enables us to take (see pp. 786, &c.) *corresponding altitudes*. Note, also, by the same

chronometer the times of the several observed zenith distances: the differences of the hours of transit, and of the hours of observation are, the chronometer going sidereal time, the hour angles. Thus, in the instance we are considering, the hour of the transit of Polaris was $0^h\ 24^m\ 44^s$, and the times of the first and second observations were, respectively, $23^h\ 57^m\ 2^s$, $23^h\ 58^m\ 18^s$, consequently the two corresponding values of h' are

$$2^m\ 58^s + 24^m\ 44^s$$

and

$$1\ \ 42 + 24\ \ 44,$$

or, respectively, $27^m\ 42^s$, $26^m\ 26^s$, (see the Table of p. 814).

The values of the preceding hour angles depend on the chronometer or clock going exactly sidereal time. This may not be the case. The pendulum may be retarded. The consequence of which would be that the number of beats between each observation, and the star's passage over the meridian, would be too small. The *corrections*, or *reductions*, therefore, which depend on such hour angles would be all too small, and, by consequence, the whole reduction. It will be necessary, therefore, should the retardation be considerable, to apply a corresponding correction. But should the clock be nearly adjusted to sidereal time, the last-mentioned correction will be inconsiderable, since the observations are seldom made at a greater distance of time from the meridian, than 20 minutes.

It may happen that the chronometer of the observer is adjusted to mean solar time. Such chronometer, therefore, may be immediately used in obtaining the values of the horary angles, or the times from noon, when the Sun is the body observed: but should, which usually happens, a star be the observed body, the hour angles, for the reasons just stated, will be all too small. They must, therefore, be all increased in the proportion (see p. 780,) of $24^h\ 3^m\ 56^s.55$ to 24^h, or be corrected for retardation. Since we may consider a clock adjusted to mean solar time as a retarded sidereal clock.

We will now deduce a formula of correction for the retardation (or should it so happen the acceleration) of a pendulum, applicable to any small degrees of retardation.

Formula of Correction for the Retardation of the Pendulum.

If a seconds' pendulum loses, in 24 hours, r seconds, it must beat $86400 - r$ times, instead of 86400.

The true value, therefore, of an hour angle h' noted by such a pendulum is

$$h' \cdot \frac{86400}{86400 - r}, \text{ or } h' + \frac{r h'}{86400 - r};$$

if, therefore, we substitute this true value instead of h', in $\sin.^2 \frac{h'}{2}$, we have

$$\sin.^2 \frac{h'}{2} \text{ equal to } \left(\sin. \frac{h'}{2} + \cos. \frac{h'}{2} \cdot \frac{h'}{2} \cdot \frac{r}{86400 - r}\right)^2,$$

nearly, since h' is a small quantity; but $\frac{h'}{2} = \sin. \frac{h'}{2}$, nearly, and $2 \cos. \frac{h'}{2} \cdot \sin. \frac{h'}{2} = \sin. h' = 2 \sin. \frac{h'}{2}$, nearly.

Hence, the above formula becomes

$$\sin.^2 \frac{h'}{2} \cdot \left(1 + \frac{2r}{86400 - r}\right).$$

If we refer to p. 419, the first term of the expression for δ is

$$2 \sin.^2 \frac{h'}{2} \cdot \frac{\cos. L \cdot \cos. D}{\sin. 1'' \cdot \sin. z} \ldots\ldots (C);$$

which, by increasing h' on account of the retardation of the pendulum, will be increased by

$$2 \sin.^2 \frac{h'}{2} \cdot \frac{\cos. L \cdot \cos. D}{\sin. 1'' \cdot \sin. z} \cdot \frac{2r}{86400 - r},$$

so that C representing the first correction on the supposition that the values of h' are exact, or that the pendulum is accurately adjusted to sidereal time, (supposing a star to be observed) the additional correction for the retardation of the pendulum will be

$$\frac{2Cr}{86400 - r}.$$

What now remains to be done with regard to the instance before us, is the deduction of the numerical value of the latitude,

according to the actual circumstances (the zenith distances, barometer, thermometer, &c.) of the observation

Mean of 26 zenith distances	37°	15′	20″.89
Refraction	0	0	46.41
Apparent mean instrumental zenith distance	37	16	7.3
Reduction, (see p. 814,).................		−	29.52
Retardation [the daily rate (r) of clock being 69^s.5]		−	.05
	37	15	37.73
North Polar Distance, (p. 813,)	1	42	18.5
Co-latitude..........................	38	57	56.23
Latitude of Dunkirk	51	2	3.77

This is the value of the latitude of Dunkirk from 26 observations, or, from one series of that number, made with a repeating circle. It differs, however, considerably (by several seconds) from the mean value deduced by Mechain and Delambre, from several hundreds of observations, and which are detailed in the second Volume of the *Base du Systême Metrique*, p. 273 to p. 293. The latitude of Dunkirk from the mean of these observations is concluded to be about 51° 2′ 8″.7, *using a certain formula of refractions:* for, as we have shewn in pp. 220, &c., the latitude of a place is no absolute value (we speak of our means of determining values) but depends on the assumed law of refraction, (see also on this subject, tom. II, *du Systême Metrique*, pp. 640, &c.)

We subjoin as a second Example, one taken from the abovementioned Work (*Base du Systême Metrique*).

Example II.

Paris, Rue de Paradis, 17 Dec^r. 1798.

Approximate latitude	48° 51′ 38″ .. cos. =	9.81815
N.P.D. of Polaris (the star observed) ..	1 45 40.16 .. sin. =	8.48760
z $(ZP - PS)$	cosec.	10.19767
const. log. or sum of log. sin. 1″, 2 log. 15, arith. comp. 2 ..		6.73671
		5.24007

therefore, see p. 421, the formula of computation for Paris with the pole star, at the time of observation, is

$$\log. C = 5.24007 + 2 \log. h'.$$

$52^m\ 4^s$ sidereal time of pole star on the meridian.
$\quad\ \ 42$ clock too slow,

51 22 hour of ∗'s passage by the clock,

	Values of h'.	Values of C.
$24^m\ 37^s$	$26^m\ 45^s$	$44''.77$
26 51	24 31	37 . 62
28 3	23 19	34 . 10
29 20	22 2	31 . 21
30 58	20 24	26 . 04
32 0	19 22	23 . 47
33 3	18 19	20 . 99
33 55	17 27	19 . 05
35 12	16 10	16 . 35
36 24	14 58	14 . 02
37 55	13 27	11 . 32
39 39	11 43	8 . 59
		12) 287 . 53
		23 . 96

Mean of 12 zenith distances..........	$39^\circ\ 22'\ 18''.93$
Meridional Z. D..................	39 21 54 . 97
Refraction......................	0 0 46 . 42
True Z. D......	39 22 41 . 39
∗'s N. P. D.....................	1 45 40 . 16
Height of equator..................	41 8 21 . 55
Latitude.......................	58 51 38 . 45

The numbers in the first column are the times of observation by the clock; the numbers in the second are formed by deducting the former numbers from $51^m\ 22^s$, the star's time of transit. The numbers representing the values of C in the third column, do not exactly agree with those in the *Base du Systême*, &c. p. 311, &c. which latter were taken from a Table (p. 250,) constructed for the latitude of Dunkirk and the pole star. The sum of the corrections instead of being, as we obtained it, $287''.53$, is stated to be $288''.14$.

The corrections of third column, p. 818, are merely the first corrections computed, as we have shewn, from

$$\log. C = 5.24007 + 2 \log. h',$$

the formulæ for computing the other two corrections are (see pp. 421, &c.)

$$\log. C' = 7.19899 + 4 \log. h',$$
$$\log. C'' = 4.95046 + 4 \log. h',$$

the greatest value of log. C', therefore, in the preceding instance, when $h' = 26^m\ 45^s$, is

$$4 \log. 26^m\ 45^s + 7.19899 = 0.02091,$$

and, accordingly, $C' = 1''.05$.

In the following observations which were made at Barcelona, and for the purpose of determining its latitude, the clock was adjusted to mean Solar time, and consequently, according to what was said in p. 815, in computing the reduction it is necessary either to increase the hour angles marked by the clock, or to correct the reduction computed on the supposition of the hour angles expressing sidereal time.

Example III.

Barcelona the place of observation, Capella the Star observed, the Time, March 16, 1794.

Approximate latitude....41° 22′ 43″......cos. =	9.87527
∗'s N. P. D..........44 13 50 ....sin.	9.84344
z.................. 4 23 27 .....cosec.	11.11600
Sum of log. sin. 1″, 2 log. 15, arith. comp. 2....	6.73671
(See p. 421,) constant logarithm in log. C........	7.57142
(See p. 421,) sum of 2 log. sin. 1″ 2 log. 15, arith. comp. 12.....	0.64413
Constant logarithm in log. C'....................	8.21555
Again, (see p. 421,).........................	4.38454
2 constant logarithm in log. C................	5.14284
Log. cot. z..................................	11.11472
Constant logarithm in log. C''.................	0.64210

Hence, the three formulæ of computation are

$$\log. C = 7.57142 + 2 \log. h',$$
$$\log. C' = 8.21555 + 4 \log. h',$$
$$\log. C'' = 0.64210 + 4 \log. h'.$$

The three formulæ are given, since Capella passing near to the zenith of Barcelona, renders the third correction of some moment, when the star is observed at more than five minutes of time from the meridian.

*'s Æ........	5^h	23^m	$32^s.1$
clock too slow ..	0	12	36.1
time of *'s transit	5	10	56

	Values of h'.	Values of C.
5^h 7^m 45^s	3^m 11^s	$135''.98$
8 55	2 1	54.57
10 27	0 29	3.13
11 20	0 24	2.15
12 42	1 46	41.88
13 51	2 55	114.15
		6) 351.86
	mean reduction........	58.64

Now log.5 8.64........... 1.76823

$r = 3^m\ 55^s.9$, and log. $\dfrac{471.8}{86164.1}$.......... 7.73843

(log. 321)............ 9.50666

Hence, allowing for the *retardation* of the clock on sidereal time, (see p. 815,) the value of C, the first of the corrections, is

$$58''.64 + 0''.32, \text{ that is, } 58''.96.$$

If we compute C', C'', from the formulæ of p. 819, we have

Horary angle.	Values of C'.	Values of C''.
3^m 11^s	.002	.584
2 1		.094
0 29		
0 24		
1 46		.055
2 55		.410
		1.143
		.002
		6) 1.145
		.19

The values corresponding to the horary angles 29^s, 24^s, &c. are too inconsiderable to be made account of. But, as it appears, the reduction obtained solely from C, is affected by the values of C', C'', only to the amount of $0''.19$.

We have now given examples of different stars, and different rates of the chronometer. In the fourth Example, which is subjoined, the zenith distances of the Sun's upper limb are observed, and the times of observation noted by a chronometer adjusted to mean solar time.

Example IV.

(*From the Philosophical Transactions,* 1819.) *Leith Fort. Approximate Latitude* $55^{\circ}\ 58'\ 41''$. *Longitude* $12^m\ 46^s.7$ *West. Sept.* 17, 1818. *Barometer* 30.05 *Inches. Thermometer* 66°.

Time of Apparent Noon by the Chronometer.
*$0^h\ 3^m\ 15^s$

	Times from Apparent Noon.	Values of C.
$23^h\ 52^m\ 28^s$	$10^m\ 47^s$	$2'\ 38''.6$
23 54 21	8 54	1 48.05
0 10 6	6 51	1 4
0 11 26	8 11	1 31.34
0 13 6	9 51	2 12.34
0 14 19	11 4	2 47.02
		6) 12 1.35
		2 0.22

* Chronometer $8^m\ 42^s.18$ too fast.

For, equation of time at Greenwich (subtractive)......	0^h	5^m	27^s	diff. $2''.2$
Proportional difference for $12^m\ 46^s$ (longitude).....	0	0	0.2	
∴ equation at Leith	0	5	27.2	
Or, time of apparent noon....................	23	54	32.8	
Add..	0	8	42.2	nearly.
Time by chronometer.........................	24	3	15	

From preceding page	2 0.22
$\frac{1}{6}$ Sum (319° 57′ 38″.4) of the corrected angles read off on the repeating circle	53ʰ 19ᵐ 36ˢ.4
	53 17 36.18
Refraction 1′ 15″.85, parallax 7″.03, difference....	0 1 8.82
☉'s semi-diameter	0 15 57.26
	53 34 42.26
Change of declination, (see pp. 442, &c.)..........	0 0 2.62
Z. D. ☉'s centre	53 34 39.64
* ☉'s apparent declination on the meridian at Leith..	2 24 1.6
Latitude at Leith	55 58 41.24

In the above case, the chronometer was not *exactly* regulated to sidereal time. Its retardation, however, was too small to affect the preceding results.

For a like cause, that of minuteness, the corrections C', C'', are not taken account of in the above computation†.

---

* September 17, ☉'s declination by N. A...2° 24′ 14″

18	2 1 0	
	0 23 14	in 24ʰ
∴	0 0 11.62	in 12ᵐ
	0 0 0.74	in 46ˢ
	0 0 12.36	in 12ᵐ 46ˢ

∴ ☉'s declination on the meridian at Leith.... 2 24 1.64.

† C', C'', computed from their formulæ, are as follows

	10ᵐ 47ˢ	8ᵐ 54ˢ	6ᵐ 51ˢ	8ᵐ 11ˢ	9ᵐ 51ˢ	11ᵐ 4ˢ
C' =	.045	.0136	.00476	.0079	.0203	.0324
C'' =	.029	.0209	.00732	.0149	.0312	.0498

the whole value, therefore, of the corrections, or their sum computed from the formula, (see pp. 420, &c.)

$$C - C' - C'',$$

will be 12′ 0″.972, one-sixth of which is 2′ 0″.162, instead of 2′ 0″.22, as was deduced in page 821. The difference, then, in the two results is only 0″.06.

A great part of the Second Volume of the *Base du Systême Metrique*, is occupied with computations, like the preceding, for determining the latitudes of Dunkirk, Barcelona, Paris, &c. The Observer's instruments were, as it has been already mentioned, small repeating circles, their chief star of observation, Polaris; but, besides, other stars, Capella, β Ursæ Minoris, ζ Ursæ Majoris, β Pollucis, β Tauri, &c. were observed, and as, with each of these stars, a vast number of observations were made, it was found to be most commodious to construct separate Tables of *reduction*, (see pp. 302, &c. *Base Metrique*,) for each star and place: for, it is evident from the formulæ of computation given in pp. 421, 819, that the *reduction* depends on the star, the time of its observation, and the latitude of the place.

The preceding methods cannot be practised at sea, where the motion of the vessel renders the level and plumb line useless. In order, then, to determine the latitude of a ship at sea, recourse must be had to the sextant. By means of that the necessary observations are to be made. The results obtained from them, with the aid of Solar and other Tables, give (under skilful management) the latitude to within half a mile: an accuracy sufficient for the navigator, but quite inferior to that which may be obtained from the repeating circle, and its appropriate methods.

LATITUDE OF A VESSEL AT SEA.

Method by the Meridional Altitude of the Sun.

If the latitude and the declination be of the same denomination, that is, either both north, or both south, then, the latitude $= \text{Z. D.} \odot + \text{decl.} \odot$

$$\text{or} = \text{decl.} \odot - \text{Z. D.} \odot, \text{ if decl.} > \text{lat.}$$

If the latitude and declination be of different denominations then, the latitude $= \text{Z. D.} \odot - \text{decl.} \odot$.

EXAMPLE.

July 24, 1783. *Longitude* 54° (3^h 36^m) *West of Greenwich, the Altitude of the Sun's Lower Limb was observed by the Sextant to be* 59° 16′. *Required the Latitude.*

Altitude of the Sun's lower limb	59°	16′	0″
Refraction (Chap. X.)	0	−	34
Parallax (Chap. XII.)	0	+	4
Sun's semi-diameter	0	15	48
True altitude of Sun's centre	59	31	18
∴ Z. D.	30	28	42
Sun's decl. (found as in p. 822,)	19	51	0
∴ latitude (*N*)	50	19	42

By the Meridional Altitude of a fixed Star.

March 29, 1783. *South Latitude, the Meridional Altitude of* Procyon *was* 77° 27′ 15″: *the Height of the Observer's Eye,* 22 *Feet above the Surface of the Sea. Required the Latitude.*

Meridional alt. of *Procyon*	77°	27′	15″	
Refraction	0	0−	13	
Dip of the horizon	0−	4	28	
True alt. of ✳	77	22	34	
∴ true zen. dist.	12	37	26	S.
Decl. of *Procyon* (from Tables)	5	46	17	N.
∴ latitude	6	51	9	S.

In this Example, a correction called the *Dip*, and not before mentioned, is made. That correction arises from the increase of the apparent altitude occasioned by the elevation of the observer above the surface of the sea*.

* See Tables for computing the Dip: Mendoza's Tables I, II. Lax's Tables VIII, IX.

By the Meridional Altitude of the Moon.

March 26, 1810. *Longitude* 40° 47′ *West of Greenwich, the Altitude of the Moon's Upper Limb was observed to be* 46° 14′ 19″. *Required the Latitude.*

Alt. of Moon's upper limb		46°	14′	19″
Horiz. $\frac{1}{2}$ diam. .... 16′ 6″ ; Augmentation (see p. 656,) 0 12	semi-diameter	0	16	18
Apparent alt. of Moon's centre		45	58	1
(Horiz. Parallax 59′ 7″) Parallax (p. 323,)		0	41	0
Refraction		0	—	55
True alt. Moon's centre		46	38	6
Declination S.		17	42	0
Alt. of equator, or co-latitude		64	20	6
∴ latitude		25	39	54 N.

The difference of the parallax and refraction is given as *one result* in Astronomical Tables, (See Tab. VIII. of the Requisite Tables: also Tab. VIII. of Mr. Mendoza's.)

Of these three methods, the first, in which the altitude of the Sun is observed, is most commonly used: the second, very rarely, by reason of the difficulty of observing the star's altitude with a sextant: the third, as it is plain, can only be used in certain parts of the month; and, since in all the observed body must be on the meridian, clouds may prevent any of the three from being used. A subsidiary method, therefore, is provided, in which the latitude may be computed from two observed altitudes of the Sun, and the interval of time between the observations.

Method of finding the Latitude by two Altitudes of the Sun and the Time between.

We have already used a triangle ZPS, and we will now introduce another, ZPs, exactly similar to it: in which s is a position of the Sun, separated from that of S, by the angle SPs, and, in time, by the interval t. Conceive the places S, s (S being nearest

to the meridian) to be joined by the arc Ss of a great circle; then we have given

$$ZS,\ Zs\ (90 - a,\ 90 - a')\ \text{the observed zenith distances,}$$

$$PS,\ Ps\ (p,\ p_{,})\ \text{equal N. P. D. of the Sun,}$$

and $\angle\, SPs$ (t) measuring the interval between the observations. Now the investigation will consist of several steps, which all tend to the finding of the angle ZsP; for, that being found, we have given Zs, Ps, and the included angle ZsP, to find ZP the co-latitude. The steps for finding ZsP are according to the following order. First,

$$Ss \text{ is found; then } \angle\, PsS\text{; next } \angle\, ZsS, \text{ and last,}$$

$$\angle\, ZsP = \angle\, PsS - \angle\, ZsS.$$

Ss *found.*

$$\text{Cos. } Ss = \text{cos. } SPs \,.\, \text{sin. } SP \,.\, \text{sin. } sP + \text{cos. } SP \,.\, \text{cos. } sP$$

(*Trigonometry*, p. 139.)

$$\therefore\ 1 - \text{cos. } Ss, \text{ or, } 2\,\text{sin.}^2 \frac{Ss}{2} = 1 - \text{cos.}^2\, p - \text{cos. } t\, \text{sin.}^2\, p$$

$$= \text{sin.}^2\, p \,.\, 2 \,.\, \text{sin.}^2 \frac{t}{2}; \text{ and in log}^s.$$

$$\text{log. sin. } \frac{Ss}{2} = \text{log. sin. } p + \text{log. sin. } \frac{t}{2} - 10.$$

Angle SsP *found.*

$$\text{Sin. } SsP = \frac{\text{sin. } p \,.\, \text{sin. } t}{\text{sin. } Ss}\ ^{*},$$

$$\text{cos. } SsP = \frac{\text{cos. } p\,(1 - \text{cos. } Ss)}{\text{sin. } p \,.\, \text{sin. } Ss};$$

---

* The angle might be deduced from this expression; but the last in practice, is more convenient, since, by taking out the log. sin. $\frac{t}{2}$, we can, without turning over the leaves, take out the log. cot. $\frac{t}{2}$.

$$\therefore \tan. SsP = \frac{\sin. t \,.\, \sin.^2 p}{\cos. p\,(1 - \cos. Ss)} = \frac{\sin. t \,.\, \sin.^2 p}{2 \,.\, \cos. p \,.\, \sin.^2 p \,.\, \sin.^2 \frac{t}{2}},$$

$$= \frac{\cot. \frac{t}{2}}{\cos. p}.$$

In logarithms,

$$\log. \tan. SsP = 10 + \log. \cot. \frac{t}{2} - \log. \cos. p.$$

Angle ZsS *found.*

$$\cos. ZsS = \frac{\cos. ZS - \cos. Ss \,.\, \cos. Zs}{\sin. Ss \,.\, \sin. Zs} = \frac{\sin. a - \sin. a' \,.\cos. Ss}{\cos. a' \,.\, \sin. Ss};$$

$$\therefore\ 1 + \cos. ZsS, \text{ or } 2 \cos.^2 \frac{ZsS}{2}$$

$$= 2 \,.\, \sin. \left(\frac{Ss + a + a'}{2} - a'\right) \cos. \left(\frac{Ss + a + a'}{2} - Ss\right)$$

$$\times \operatorname{cosec}. Ss \,.\, \sec. a'.$$

In logarithms,

$$2 \log. \cos. \frac{ZsS}{2} = \log. \sin. \left(\frac{Ss + a + a'}{2} - a'\right)$$

$$+ \log. \cos. \left(\frac{Ss + a + a'}{2} - Ss\right)$$

$$+ \log. \operatorname{cosec}. Ss + \log. \sec. a' - 20.$$

Now $\angle ZsS$ being found, $\angle ZsP = \angle SsP - \angle ZsS$ is known.

ZP *the Co-latitude found.*

In the triangle ZsP we have Zs, Ps and the angle ZsP given, and the side ZP is required. This side will be found by the formula of p. 171, *Trigonometry*.

Thus,

$$\log. \sin. M = \tfrac{1}{2}\left(2 \log. \cos. \frac{ZsP}{2} + \log. \sin. p + \log. \cos. a' - 20\right)$$

$$\text{and log. sin. } \frac{ZP}{2} = \tfrac{1}{2}\left\{\begin{array}{l}\text{log. sin. } \left(\frac{p}{2} + \frac{90 - a'}{2} + M\right) \\ + \text{ log. sin. } \left(\frac{p}{2} + \frac{90 - a'}{2} - M\right)\end{array}\right\}.$$

This method, although it may be called a *direct* one, cannot give an exact result, because, in the first operation (see p. 826,) the Sun's declination is supposed not to alter during the observations. It will be necessary, therefore, to introduce a correction dependent on the change of declination.

Example.

$a = 42^\circ\ 14'\ 0''$, $\quad p = 81^\circ\ 43'\ 30''$

$a' = 16\ \ 5\ \ 47$ $\quad p' = 81\ \ 45\ \ 0$

$\frac{p + p'}{2}$ (mean N. P. D.) $81\ \ 44\ \ 15$,

t, the interval between the observations, 3^h, or in space 45°.

Ss		$\angle SsP$	
	10 10		10 10
sin. 81° 44′ 15″ . .	9.9954800	cot. 22° 30′ 0″ . .	10.3827757
sin. 22 30	9.5828397	cos. 81 44 15 . . .	9.1574825
(sin. 22° 15′ 16″)	9.5783197	(tan. 86° 35′ 36″.3)	11.2252932
∴ Ss = 44° 30′ 32″		∴ SsP = 86° 35′ 36″.3	

$ZsS.$

			−20 = −20
a =	42° 14′ 0″		
a' =	16 5 47	. . . . sec. =	10.0173684
Ss =	44 30 32	. . . . . . cosec. =	10.1542695
sum . .	102 50 19		
½ sum . .	51 25 9.5		
½ sum − Ss	6 54 37.5	. . . . cos. =	9.9968337
½ sum − a'	35 19 22.5	. . . . sin. =	9.7620664
		2)	19.9305380
		(cos. 22° 36′ 36″)	9.9652690

$$\therefore\ ZsS = 45^{0}\ 13'\ 12''$$
$$\text{but } SsP = 86\ \ 35\ \ 36.3$$
$$\therefore\ ZsP = 41\ \ 22\ \ 24.3$$

ZP.

$$\begin{array}{lr}
2 \text{ log. cos. } 20^{0}\ 41'\ 12'' \ldots\ldots & 19.9421120 \\
\text{log. sin. } 81\ \ 44\ \ 15 \ldots\ldots\ldots & 9.9954800 \\
\text{log. cos. } 16\ \ \ 5\ \ 47 \ldots\ldots\ldots & 9.9826315 \\
2)\ & 19.9202235 \\
 & 9.9601117 ;\ \therefore M = 65^{\circ}\ 49'\ 3''.
\end{array}$$

Again,

$$\begin{array}{rl}
p = & 81^{0}\ 44'\ 15'' \\
90 - a' = & 73\ \ 54\ \ 13 \\
2) & 155\ \ 38\ \ 28 \\
\frac{1}{2}\text{ sum} & 77\ \ 49\ \ 14 \\
M & 65\ \ 49\ \ \ 3
\end{array}$$

$$\begin{array}{lllr}
\frac{1}{2}\text{ sum} + M & 143\ \ 38\ \ 17 \ldots\ldots\ldots\ldots & \text{sin.} = & 9.7729698 \\
\frac{1}{2}\text{ sum} - M & 12\ \ \ 0\ \ 11 \ldots\ldots\ldots\ldots & \text{sin.} & 9.3179879 \\
 & & 2) & 19.0909577 \\
 & \text{sin. } (20^{0}\ 33'\ 25'') & & 9.5454788
\end{array}$$

$$\therefore\ ZP = 41^{\circ}\ \ 6'\ 50''$$
$$\text{latitude} = 48\ \ 53\ \ 10.$$

The formula of correction, for a change in the Sun's declination, which happens between the two observations, is

$$\pm (D - d)\,\frac{\cos.\dfrac{a + a'}{2}\,.\,\sin.\dfrac{a - a'}{2}}{\cos. D\,.\cos. L\,.\sin.^{2}\dfrac{t}{2}}.$$

D being the Sun's declination, at the mean time between the observations, and *d* being the less declination.

Now if the whole change of declination be 1′ 30″,

$D - d$	$= \frac{1}{2}(1' \; 30'') = 0''.75$			.. log. =		9.8751
$\dfrac{a+a'}{2}$	= 29	9	53	....	cos.	9.9411
$\dfrac{a-a'}{2}$	= 13	4	7	....	sin.	9.3548
D	= 8	15	45	....	sec.	10.0045
L	= 48	53	10	....	sec.	10.1820
$\dfrac{t}{2}$	= 22	30	0	......	2 cosec.	20.8343
					(60 taken away)	.1918 (log. 1′.55.)

Hence, the correction is + 1′.55, or + 1′ 33″,

and since the value of L is 48° 53′ 10″,

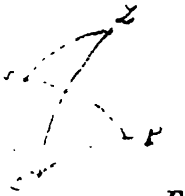
the corrected latitude is 48 54 43.

This method founded on the false supposition of the constancy of the Sun's declination during the observations, with the subsequent correction for the change of declination, form a process as long as that would have been in which no change should have been supposed. It is scarcely worth the while to set down all the logarithmic operations in the latter method, but we subjoin the formulæ and their several arithmetical results.

In the triangle PSs, S belongs to the greater altitude, and Ps is the greater N. P. D, and we have to determine, from the two sides and the included angle, the third side and the other angles.

Given Quantities.

$$Ps = 81^\circ \; 45' \; 0''$$
$$PS = 81 \;\; 43 \;\; 30$$
$$\angle \, SPs = 45 \;\; 0 \;\; 0.$$



Formulæ, (Trig. p. 167.)

$$\tan.\frac{PSs+PsS}{2}=\cot.\frac{SPs}{2}.\cos.\frac{Ps-PS}{2}.\sec.\frac{Ps+PS}{2},$$

$$\tan.\frac{PSs-PsS}{2}=\cot.\frac{SPs}{2}.\sin.\frac{Ps-PS}{2}.\operatorname{cosec}.\frac{Ps+PS}{2},$$

$$\sin. Ss=\sin. Ps.\frac{\sin. SPs}{\sin. PSs}.$$

Results.

$$\begin{aligned}PSs&=86^\circ\ 37'\ 26''\\ PsS&=86\ \ 33\ \ 46.5\\ Ss&=44\ \ 30\ \ 28.\end{aligned}$$

ZsS is to be determined from the formula of 828, by substituting the present values of Ss, instead of the value therein used: if this be done,

$$ZsS=45^\circ\ 13'\ 10'',\quad ZSs=112^\circ\ 54'\ 54'',$$

$$\text{but, } PsS=86\ \ 33\ \ 46.5$$

$$\therefore\ ZsP=41\ \ 20\ \ 36.5$$

In order to determine ZP, we must also use the same formulæ as were used in p. 829. The results of those formulæ (substituting instead of their former values, the new values of ZsP and Ps, namely, $41^\circ\ 20'\ 36''.5$, and $81^\circ\ 45'$,) will be

$$\begin{aligned}M&=65^\circ\ 49'\ 49''.7\\ \tfrac{1}{2}ZP&=20\ \ 32\ \ 46.25\end{aligned}$$

$$\text{and therefore latitude}=48\ \ 54\ \ 27.5$$

differing from the former result by 15.5 seconds.

We may derive from this method the following mode of correcting the approximate latitude obtained by the first process of pp. 826, &c., and dispense with the correction of page 829. Thus, the value of PsS, deduced in this page, is an exact value: so is Ss; therefore, ZSs, deduced from ZS, Zs (given quantities,) and Ss, is also an exact value. Compute, then, the angle ZSP, from ZS, PS, and that value of ZP which results from the

approximate method of pp. 826, &c. If such be a true value of ZP, $ZSs - ZSP$ ought to equal PSs: or, not being equal, their difference will indicate how much, and which way, the value of ZP ought to be changed, in order to procure a more exact agreement. For instance, from

$L = 48°\ 53'\ 10''$ first approximate value, p. 829.
$p = 81\ \ 43\ \ 30$ least N.P.D. corresponding to greatest altitude,
$a = 42\ \ 14\ \ \ 0$ greatest altitude,

and this formula, to wit

$$\cos.^2 \frac{ZSP}{2} = \sin.\left(\frac{L+p-a}{2}\right)\cos.\left(\frac{L+a-p}{2}\right)\sec. a . \text{cosec}. p,$$

may be derived

$$\frac{ZSP}{2} = 13°\ 10'\ 2'', \text{ and } ZSP = 26°\ 20'\ 4''$$

but (see p. 831,) $ZSs =$	112	54	54
$\therefore\ PSs =$	86	34	50
but the true value (see p. 831,) of $PSs =$	86	37	26
difference	0	2	36

consequently, since ZSs is an exact value, this difference can only arise from ZSP being too large. In order to discover how much we must either augment or diminish the latitude, for the purpose of properly diminishing ZSP, we have this equation,

$$\cos. ZSP = \frac{\sin. L - \sin. a . \cos. p}{\cos. a . \sin. p},$$

$$\text{whence } - d\,(ZSP) . \sin. ZSP = dL . \frac{\cos. L}{\cos. a . \sin. p},$$

we must, therefore, in order to diminish ZSP, augment the latitude, and by the result from the preceding differential formula; thus,

log. $2'\ 36''$, or log. $2''.6$ =	0.41497
log. cos. a . . . . =	9.86947
log. sin. p . . . . =	9.99550
log. cos. ZSP . . . . =	9.64700
log. sec. L . . . . =	10.18190
	0.10884 = log. $1''.284$.

$$dL \therefore = 0^{\circ}\ 1'\ 17''$$
$$\text{and, since } L = 48\ 53\ 10$$
$$\text{corrected latitude} = 48\ 54\ 27.$$

These latter observations and processes have been introduced because they fully explain the method which Dr. Brinkley has given in the Nautical Almanack of 1825, for finding the latitude from the observed altitudes of two known stars. Instead of S, s being two different positions of the Sun, suppose those points to denote two different stars: then the angle SPs will be the difference of the right ascensions of the two stars, and since Ps, PS, the north polar distances of the two stars, and SPs the difference of their right ascensions is known, their distance Ss, and the angle PsS can be computed: which latter quantities, for certain pairs of stars are, in the Nautical Almanack, already computed for the use of the Observer. For instance, the first pair of stars in Table I. (see *Nautical Almanack* 1825, p. 5, of Appendix,) are Capella and Sirius. Now, for 1822, taking

N. P. D. of Capella =	$44^{\circ}\ 11'\ 42''$ ……	Æ	$5^{h}\ 3^{m}\ 33^{s}$
of Sirius	106 28 40 …….	Æ	6 37 18
		difference	1 33 45

we may, as in page 831, and by the same formula, find Ss (D) and the angles PSs, PsS, one of which like PSs is the *angle of comparison* (C) and answers the same end. Their values will be, according to the above data,

$$Ss\,(D =)\ 65^{\circ}\ 47'\ 48''$$
$$PSs\,(C) = 17\ 41\ 50$$
$$PsS = 155\ 16\ 51,$$

and these values (very nearly the same) are expressed in Tab. I. to save the Observer, as we have said, the trouble and difficulty of computation. The parts of the Rule for finding the latitude are, in substance, precisely the same as those we have already used in pages 831, 832, for finding the latitude from two altitudes of the Sun, and the time between. Dr. Brinkley, indeed, instead of a process wholly logarithmic, uses one partly so, and partly constructed by the aid of natural cosines.

The latitude in the first method (see p. 827,) before correction, was supposed to be approximately found, on the supposition of the Sun's declination remaining constant. But we may suppose it approximately known by *account*, as Dr. Brinkley supposes it in his method of two stars, and correct as before.

These methods, whether the Sun be twice observed after a short interval, or two stars be observed at the same time, have been invented for the use of the mariner; and when they are practised whilst the vessel is in motion, the latter has, in one respect, a considerable advantage over the former: which is, that it is not necessary to make in it any allowance for a change of latitude, which it is almost always necessary to do in the other method *.

Instead of the direct method (if such it may be called) of finding the latitude from two altitudes, and the intervening time, several indirect and approximate methods, and made easy by proper Tables, have been invented (see *Nautical Almanack* 1797, 1798, 1799, 1800, 1822: Mendoza's and Lax's Tables on *Nautical Astronomy*. Delambre, tom. III. pp. 641, &c. *Phil. Mag.* 1821, pp. 81, &c.)

It is evident, the preceding methods (pp. 823, &c.) which are the only ones that can be practised at sea, may be practised at land, when the sextant is used with an artificial horizon, (see p. 774.). But then, they are to be used only when no great accuracy is required, and in default of better instruments. The errors of observation with the sextant, and those of the Solar Tables, must always be presumed to be of some magnitude; and, of both of these errors, the above-mentioned methods necessarily partake.

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* The inconvenience of the *latter* method is the difficulty of observing, with accuracy, the altitudes of stars.

CHAP. XLIII.

On Geographical Longitude.

THE Earth revolves round its axis in $23^h\ 56^m\ 4^s.091$ of mean solar time; but, a meridian passing through the Sun returns to it after the lapse of a greater time, viz. 24^h, and consequently, after describing a greater angle than 360^0. This arises from the increase of the Sun's right ascension in the time of the Earth's rotation; the mean value of which increase is $59'\ 8''.3$: consequently, the angle, through which a meridian revolves in a mean solar day of 24 hours, is $360^0\ 59'\ 8''.3$.

If we suppose a number of meridians to be drawn at equal intervals, that is, to form successively with each other, equal angles at the poles, then, in the course of 24 hours, each of these meridians (supposing their planes produced) will pass through the Sun and, since both the Earth's rotation, and the Sun's mean motion in right ascension, are supposed to be uniform, at equal intervals of time. If the meridian of a given place passed through the Sun at the beginning of the 24 hours, it would again pass through it at the end; 24 hours then of mean solar time would correspond to 360 degrees of longitude; for, the whole scale of longitude must be comprehended between the eastern and western sides of the meridian of the same place. At places situated on the meridian opposite that on which the Sun was at 0^h, or, in civil reckoning, at 12 at noon, the time would be 12^h, or 12 at night; and 12^h would correspond to 180 degrees of longitude. At places situated on the meridian, at right angles to the former, the time would be 6^h or 18^h; or, in civil reckoning, 6 in the morning, or 6 in the evening; and accordingly, 6 and 18 hours of mean solar time, would correspond to 90^0, or 270^0 of longitude; and similarly for intermediate meridians.

The selection of a meridian, from which the longitudes of all other places are to be reckoned, is entirely arbitrary. The English have selected that which passes through the Royal Observatory at Greenwich: it is called the *First Meridian*, and its longitude is called 0^h. The French use a different one: their *Premier Meridien* passes through the Observatory at Paris, and is $9^m\ 21^s$ east of the former.

If then at Greenwich, (and consequently at all places through which its meridian passes) the Sun were $7^\circ\ 30'$ to the west of the meridian, or the time were $0^h\ 30^m$, at other places, the meridians of which should be 15^0, 30^0, 45^0, &c. distant from that of Greenwich and to the east, or which should have, respectively, 15°, 30°, 45^0, &c. of *east longitude*, the times, or the reckoned hours of the day, would be, respectively, $1^h\ 30^m$, $2^h\ 30^m$, $3^h\ 30^m$, &c. At places, 10°, 20°, 30^0, &c. of *west longitude*, the times would be respectively, $23^h\ 50^m$, $23^h\ 10^m$, $22^h\ 30^m$, &c. or in civil reckoning, $11^h\ 50^m$, $11^h\ 10^m$, $10^h\ 30^m$, &c. in the morning. Now, some of the methods of determining the longitude, depend solely on the reverse of this; that is, they find the differences between the reckoned time at a given place and at Greenwich, and thence deduce the difference of longitude, or, (since that of Greenwich is 0), the real longitude, converting the time into degrees at the rate of 15 for each hour.

The methods that depend solely on the difference of the reckoned times, are those which are connected with phenomena that happen and are observed at the same point of *absolute time*. Such phenomena are the eclipses of the Moon and of the satellites of *Jupiter*. There are other methods, however, which depend partly on the difference of the reckoned, and partly on that of the absolute times. Such are founded on the phenomena of solar eclipses, of occultations, and of transits, which are not observed, at the same point of absolute time, at all parts of the Earth's surface. (See p. 738.)

This may be illustrated by an instance. Berlin is $44^m\ 10^s$ east of Paris; therefore, if an eclipse of one of *Jupiter's* satellites were observed to happen at the latter place at $13^h\ 1^m\ 20^s$, it would be reckoned to happen at the former at $13^h\ 45^m\ 30^s$: for, since the

phenomenon takes place by the actual falling of the shadow on the satellite, the observer at Berlin must see it at the same point of absolute time, as the observer at Paris. But, the occultation of *Antares* by the Moon, (see p. 748,) was observed at Paris at $13^h\ 1^m\ 20^s$, and at Berlin at $14^h\ 6^m\ 19^s$. The difference ($1^h\ 4^m\ 59^s$) of the *reckoned* times, then, is not entirely due to the *difference of meridians* ($44^m\ 10^s$), but partly to that, and partly to the difference in the absolute times of the observations of the phenomena: which latter difference, equal to $20^m\ 49^s$, is entirely the effect of parallax. In the former case, the satellite was *obscured* by the shadow of *Jupiter*, in this latter, the star is *concealed* by the interposition of the Moon.

The methods of finding the longitude, then, naturally arrange themselves into two classes: one belonging to phenomena of the first description, the other, to phenomena of the second. The methods of the former being very simple in their application, but not very accurate in their results; the latter requiring tedious computations, but capable of great exactness. We will, however, first shew how to determine

The Longitude by a Chronometer or Time-keeper.

From the *error* of a chronometer at the beginning of a period and its daily *rate*, we can, supposing the latter constant, determine the error at the end of the period. If the chronometer on June 1, be $2^m\ 13^s$ too slow, and its daily rate be $-0^s.5$, on June 10, its error will be $2^m\ 18^s$. This is an arithmetical operation: but we can also determine the *error* from astronomical phenomena: by means of the Sun's transit observed by a transit instrument, by equal altitudes, or by calculations from absolute altitudes, (see pp. 104, 786, 796.) Should the two errors, thus differently found, not agree, the inference would be that the *rate* of the chronometer had, during the interval, varied.

In this we suppose the observer to have remained at the same station, at Greenwich, for instance. But should he, in the interval of the two observations, have journeyed to a station west of Greenwich, to Edinburgh, for instance, he would have to

account for the difference of the longitudes of the two stations, before he could rightly estimate the equability of the chronometer's rate.

We may illustrate this point by an instance taken from the *Philosophical Transactions*, 1819. Part III, p. 384.

Thus, June 15, 1818, the *equation of time* at Greenwich being $-5^s.6$, the Sun's centre was on the meridian at $11^h\ 59^m\ 54^s.4$ of mean time, but the chronometer noted $11^h\ 58^m\ 38^s.6$, it was, therefore, slow by $1^m\ 15^s.8$, and its daily rate being $-0^s.2$, on Sept. 17, it ought, on the supposed constancy of the daily rate, to have been slow by $1^m\ 34^s.6$: in other words, it ought to have noted the time of noon by $11^h\ 52^m\ 58^s.4$, since $-5^m\ 27^s$ being the equation of time at Greenwich, the mean time of apparent noon was $11^h\ 54^m\ 33^s$. Now the chronometer was carried to Edinburgh, and there examined on Sept. 17, by one of the methods mentioned in pp. 786, 802. The longitude of Edinburgh, known by previous methods, is $12^m\ 46^s.7$ west, and the equation of time for that place on the noon of September 17, being $-5^m\ 27^s.2$, the time of apparent noon was $11^h\ 54^m\ 32^s.8$, but the chronometer denoted $12^h\ 3^m\ 14^s.4$; it was, therefore, too fast by $8^m\ 41^s.6$, but if $-0^s.2$ had been its rate, it ought to have been fast by $12^m\ 46^s.7 - 1^m\ 34^s.6$, or $11^m\ 12^s.1$: instead then of having lost in 94 days $18^s.8$, the chronometer had really lost $11^m\ 12^s.1 - 8^m\ 41^s.6$, or $2^m\ 50^s.5$, and its daily rate instead of $-0^s.2$, appeared to be $-1^s.8$.

By methods, then, like this it is ascertained that chronometers by being transported from one place to another change their daily rate, or, widely depart from that mean rate, which, if their construction be good, they preserve at a fixed station. A chronometer, therefore, cannot be relied on for determining the longitudes of places, especially if it be conveyed over land. Their rates are less subject to variation at sea, from the less jolting mode of transport. But the uncertainty attendant on one chronometer is almost entirely got rid of, by the use of several. In the present year, the longitude of Funchal in the island of Madeira has been so determined. Ten or twelve chronometers

were taken from Greenwich to Falmouth, and their errors and rates examined at that latter place, by the method of corresponding altitudes. They were then taken to Madeira, and subjected to a like examination, and the longitude determined by a mean of results.

Longitude by an Eclipse of the Moon.

By means of a perfect chronometer we could always, and in all places, determine the longitude. By lunar eclipses which are rare, we can determine the longitude, only occasionally and at particular conjunctures; but, when such occur, by the following method. The times at which eclipses happen, at the place of observation, are to be computed, by one of the methods given in pp. 396, &c., or, which is commonly the case, may be known by a chronometer previously regulated by observation. The times at Greenwich, previously computed, are inserted in the Nautical Almanack, or may be computed by the observer from the Lunar Tables. The difference of these times is the longitude.

Since the Lunar Tables are not exact, the comparison of the same eclipse, actually observed at two different places, will give the difference of their longitudes much more accurately than the comparison of the eclipse observed at one place, and *computed for another.*

EXAMPLE.

1729, *Aug.* 28. *By observations of Cassini at Paris* (Mem. Acad. 1779.) *and of Mr. Stevenson at Barbados* (Phil. Trans. N°. 416. p. 441.)

At Paris, Imm. ☽	12^h	19^m	13^s	Emer. ☽	13^h	59^m
At Barbados, Imm.....	8	11	0	Emer.........	9	51
	4	8	13		4	8

By the mean of the two, the difference of longitude is, 4^h 8^m $6^s.5$ or 62^0 $1'$ $30''$: that is, Barbados is 62^0 $1'$ $30''$ west of Paris.

This method of determining the longitude is rarely used, since, by reason of a penumbra, it is difficult to ascertain the

exact time of contact of the Earth's shadow with the Moon's limb. The time is uncertain, to the extent of 2^m, or 30′. It has been proposed to amend the method, by observing the contact of the Earth's shadow with some remarkable spots in the Moon's disk. (See *Phil. Trans.* 1786, pp. 415, &c.)

Longitude by the Eclipses of Jupiter's Satellites.

This method, although an inexact one, is yet better than the preceding, and for two reasons; the first is, the more frequent recurrence of the eclipses of Jupiter's satellites than of lunar eclipses. The first satellite, for instance, is regularly eclipsed at intervals of forty-two hours. The second reason is, that the times of the immersion and emersion of the satellites, can be more exactly noted than the times of the contacts of the Earth's shadow with the Moon's limb.

This is, however, only a relative excellence. In noting the eclipses of the first satellite, the time must be considered as uncertain to the amount of 20 or 30 seconds. Two observers, in the same room, observing with different telescopes, the same eclipse, will frequently disagree in noting its time, to the amount of 15 or 20 seconds; and the difference will not be always the same way; that is, the telescope by which an emersion is the soonest seen on one occasion, will not always maintain its superiority. As a general fact, however, the telescope of the greatest power will cause immersions to appear later, and emersions sooner: and this is the reason why observers are directed in the Nautical Almanack, (p. 151,) to use telescopes of a certain power.

The eclipses of the first satellite cannot, as it has been remarked, be observed very exactly. But there is much greater uncertainty in noting the times of the eclipses of the other satellites. M. Delambre thinks that the time of an eclipse of the fourth satellite, may be doubtful to the amount of 4′. Still the method of determining the longitude by the eclipses is much practised, because it can be frequently and conveniently practised. A good telescope, an adjusted chronometer, and the Nautical Almanack, are all the apparatus wanted. We subjoin an Example.

Example.

At the Cape of Good Hope, May 9, 1769,

Emer. 1st Satellite.	$10^h\ 46^m\ 45^s$
At Greenwich, by computation (Naut. Alm.).	9 33 12
Difference of meridians	1 13 33

or the Cape is $18^\circ\ 23'\ 15''$ to the east of Greenwich. The remark which was applied to the former case, applies to this. If we use the emersion *observed* at Greenwich, instead of the emersion *computed* for Greenwich, we shall avoid the errors of the Tables of Jupiter's satellites, and obtain a more exact value of the longitude.

We now proceed to the methods of determining the longitude by means of phenomena of the second class; those, which are not seen by all spectators at the same point of absolute time.

The Longitude determined by an occultation of a fixed Star by the Moon.

In pp. 748, &c. the apparent distance of *Antares*, from the Moon was computed, for the instant previous to its occultation, and found equal to $15'\ 51''.38$. The place of observation was Paris: the hour or apparent time $13^h\ 1^m\ 20^s$ (the mean time $13^h\ 3^m\ 32^s.8$): and the formula for the computation of the distance, was

$$D^2 = (l-l')^2 + (k-k')^2.\cos.^2 l \quad (a).$$

In this formula, l, k, are the *apparent* latitude and longitude of the Moon, obtained, by adding to the true, (see p. 744,) the computed parallaxes in longitude and latitude.

The true longitude and latitude of the Moon were taken, from Lunar Tables *computed for the meridian of Paris*, and for $13^h\ 3^m\ 32^s.8$ *mean solar time at Paris*: and were found, respectively, equal to $9^s\ 5^\circ\ 31'\ 42''.4$ and $3^\circ\ 47'\ 58''.7$. (See p. 749.)

If then the Lunar Tables be correct, D would result from the preceding formula (a) exactly of its proper value, such as the

Tables would assign, or (since D is, in this case, the Moon's semi-diameter) such as might easily be ascertained by observation. But, if D computed from the formula (a) should differ from the value of the Moon's semi-diameter assigned by the Tables, such circumstance would be a proof of the existence of errors in the Tables. And, the difference between the two values of D, would enable us to deduce an equation between the corresponding errors in the Moon's latitude and longitude. In this case, an occultation would serve to *correct the errors of the Lunar Tables.*

But, as it has been already explained in Chap. XXXIV, there is another method of correcting the Lunar Tables. On the day of observation, the Moon's declination and right ascension are observed, and thence, her latitude and longitude are computed. The respective differences between these, and her latitude and longitude computed from the Lunar Tables, will give, for that day, their errors.

Since we have the means then of ascertaining the errors, we will *suppose* the Lunar Tables to be perfectly correct. Let us now see, by what means, D is to be computed, in a place of observation, for the *Meridian of which, there are no Tables constructed.*

In such a place, the observer must use Tables computed for another meridian: either, for the meridian of Greenwich, or for that of Paris: either the Nautical Almanack, or the *Connoissance des Tems**. By these, he must compute l, and k, and accordingly, previously must compute the Moon's true latitude and longitude, that is, the latitude and longitude that belong to the centre of the Earth. The values of these latter depend on the time for which they are computed, and, *on the time as it is reckoned either at Greenwich or Paris.* Now, although (see

* These Ephemerides may be considered a species of lunar and solar Tables, in which certain results, most commonly wanted in practice, and computed from the *general* Tables, are inserted. Such results are the Moon's right ascension, declination, longitude, latitude, parallax, and semi-diameter, for noon and midnight.

Chap. XLI.) the time, at the place of observation, can be exactly known, that, *at the place for which the Tables are computed*, cannot, except by a knowledge of the *longitude* of the former place.

This is easily illustrated: the occultation of *Antares* was observed at Berlin at $14^h\ 7^m\ 31^s$, mean solar time. The Observer at that place in order to compute, by the *French Tables*, the Moon's true longitude, must know the corresponding time at Paris. If he *assume* Berlin to be 44^m east of Paris, the corresponding mean time, at the latter place would be, $13^h\ 23^m\ 31^s$: and the Moon's true longitude computed for $13^h\ 23^m\ 31^s$, would be $8^s\ 5^0\ 43'\ 16''$. But, if he *assume* the difference of longitude to be $39^m\ 49^s$, the corresponding time at Paris will be $14^h\ 27^m\ 42^s$: and the Moon's true longitude computed for $14^h\ 27^m\ 42^s$, will be $8^s\ 5^0\ 45'\ 35''$. The computations for the Moon's true latitude will be similarly affected by a change in the hypothesis of the longitude of Berlin.

A small error in that hypothesis will very little affect the computation * of the parallaxes in longitude and latitude, which depend chiefly on the hour angle; consequently, since the apparent differ from the true longitudes and latitudes, solely by the parallaxes, the change, or error in the hypothesis of the difference of meridians, will produce the same difference in the apparent, as in the true longitudes and latitudes of the Moon.

Hence it follows, that an error in the assumed longitude of Berlin (that being still the place used for illustration) will produce errors in the computation of l, k; and consequently, in the computation of D from,

$$D^2 = (l - l') + (k - k')^2 \cos.^2 l \quad (a),$$

there must be an error in the resulting value of D.

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* If we examine the formulæ of computation, (1), (2), (3), &c. in p. 748, &c. we shall perceive that the parallaxes depend principally on the hour angle which is not changed by altering the hypothesis of the longitude.

Now, the principle of finding the longitude of Berlin, consists in *correcting* the assumed longitude, by means of the error in D. The correction is thus made.

The Moon's latitude and longitude (l, k,) being supposed to be erroneous, let their true value be $l + nt$, $k + mt$, n, m being the Moon's horary motions in latitude and longitude, and t, as an unknown quantity, representing the time, or the error of the hypothesis of the *difference of the meridians*; then, if Δ be the Moon's true semi-diameter, we have

$$\Delta^2 = (l + nt - l')^2 + (k + mt - k')^2 . \cos.^2 l \quad (b),$$

and from this and the preceding equation (a), t is to be determined.

If we suppose, what will always be the case in practice, the longitude of the place of observation to be *nearly* known, and, consequently, the hypothesis of its value to differ but little from the true value, t will be a small quantity; and, if we neglect its square in the expansion of (b), we shall have

$$\Delta^2 = (l - l')^2 + 2nt.(l - l') + [(k - k')^2 + 2mt(k - k')] \cos.^2 l.$$

Subtracting (a) from this,

$$\Delta^2 - D^2 = 2t[n(l - l') + m.(k - k') \cos.^2 l]$$

and, consequently,

$$t = \frac{\Delta^2 - D^2}{2[n(l - l') + m(k - k').\cos.^2 l]} \ldots (c).$$

This value of t, (an approximate one) is the correction to the assumed longitude: suppose, the longitude $= T$, then its corrected value is $T \pm t$; and, if a still more correct value be required, compute again by means of this *corrected hypothesis* of the difference of the meridians ($T \pm t$), the true latitudes and longitudes of the Moon; thence deduce correcter values of l, k, and find a new approximation (t') from the expression (c). The longitude, after this second correction, will be $T \pm t \pm t'$.

This method, from an assumed approximate value, is capable of determining the true value of the longitude, to the greatest exactness. And, we need not be solicitous concerning the *nearness* of the first approximation to the truth. An eclipse of one of

Jupiter's satellites, which is easily observed, will afford us a first value of the longitude, we might almost say, more than sufficiently near. For, we may even take as a first value, the difference of the *reckoned times* of the occultation at the two places which in the preceding illustration was $1^h\ 5^m$, and which (see pp. 837, &c.) is considerably different from the true value.

We have already illustrated the method, by supposing the occultation to have been observed at Berlin, and the Moon's longitude and latitude to have been computed by Paris Tables. We will now attempt to exemplify the mode of computing the *correction* (t), by supposing the occultation to have been *observed at Paris*, and the Moon's longitude and latitude to be computed by Tables adapted to the *Meridian of Greenwich*.

The immersion (see p. 748,) was observed at Paris at $13^h\ 1^m\ 20^s$. In order to find the corresponding time at Greenwich, suppose the latter place to be 9^m west of the former; then, the reckoned time would be $13^h\ 1^m\ 20^s - 9^m$, or $12^h\ 52^m\ 20^s$; for this time, compute the Moon's longitude; the simplest mode of effecting which, now, would be, to take from the Nautical Almanack the Moon's longitudes on April 6th at midnight, and April 7th at noon; to find their difference, and then to add to the former that part of the difference which is proportional to $52^m\ 20^s$. The result would be the Moon's true longitude at $12^h\ 52^m\ 20^s$. (See pp. 784, &c.) Compute in the same way the Moon's latitude: suppose the above quantities to be exactly of those values which are assigned to them in the Example of pp. 748, &c.; then, the parallaxes, &c., being computed exactly as in that Example, the Moon's semi-diameter will be found (see p. 751,) equal to $15'\ 51''.3$. If the Tables be perfectly correct, and the longitude be rightly assumed, such computed value of the semi-diameter ought to be equal to the semi-diameter assigned by the same Tables. But, the latter is found to be $15'\ 37''.7$. The difference or error $13''.6$, *assuming* the Tables to be correct, must arise then solely from an error in the hypothesis of the longitude: computing that error from

$$t = \frac{\Delta^2 - D^2}{2\left[n(l - l') + m(k - k')\cos.^2 l\right]},$$

in which $\Delta = 15' \; 37''.7$, ... $l - l' = \; 4' \; 3''.2$

$D = 15 \; 51 . 3$, ... $k - k' = 15 \; 22 . 7$

$l = 4^{0} \; 36'$, and n and m are the hourly motions*; t will be found nearly $= 25^{s}$. The corrected longitude of Paris then is $9^{m} \; 25^{s}$, and a repetition of the process will give a value still more correct.

Since the illustration of the method of correcting the assumed longitude was our chief object, we have supposed the Lunar Tables to be correct. But, in practice, their errors, which are frequently considerable, must be always attended to.

If the occultation be observed under a known meridian, such as that of Greenwich or of Paris, then, it may be made subservient to the correction of the Lunar Tables. For such an end, Mayer has employed the immersion and emersion of *Aldebaran* †. And, it is easy to see, since the errors in the computation of the Moon's distance from the star, can be only three ‡ (those of the lunar longitude and latitude and of the assumed longitude of the place of observation,) that three observations, to wit, of an immersion, at a place of an ascertained longitude, and of an immersion and an emersion at a place whose longitude is required, will furnish three equations sufficient to correct the three errors abovementioned. (See Cagnoli, *Trig.* pp. 470, &c.)

In page 753, allusion was made to a method, of deducing the longitude from an occultation, in some respects the reverse of the preceding. In the method alluded to, the true latitude and longitude of the *point* of occultation are deduced by correcting the apparent latitude and longitude of the star on account

* To obtain n, m, the hourly motions, compute the Moon's apparent latitudes and longitudes, for $12^{h} \; 51^{m} \; 40^{s}$, and for $13^{h} \; 51^{m} \; 40^{s}$: and the respective differences of these quantities will be the hourly motions in latitude and longitude. In the computation they were assumed to be $1' \; 54''$ and $36' \; 31''$; which are not, however, their exact values.

† Mayer's Lunar Tables, 1770, pp. 39, 40.

‡ The Moon's semi-diameter, on the day of the occultation, may be measured or computed by means of an observation, and accordingly, any error, in it's value assigned by the Tables, corrected.

of parallax. The true latitude of the Moon is taken from the Nautical Almanack. The true distance D, or the semi-diameter of the Moon may be taken from the same source, or may be determined by observation: and thence may the Moon's longitude be determined: for, supposing in the equation (p. 747,)

$$D^2 = (l - l')^2 + (k - k')^2 \cdot \cos.^2 l,$$

that, l, k, &c. represent the true latitudes and longitudes: if D, l, l', are known, $k - k'$ may be determined; and, since k', or the true longitude of the point of occultation is known, k the longitude of the Moon's centre is.

Suppose, then, that by these means, and separate calculations, we obtained, from an occultation, at two different places, the following results:

Greenwich, long. ☽'s centre	67°	$22'$	$26''.1$	hour =	8^h	37^m	$36^s.8$
Dublin	67	18	43.3		8	4	51.5
	0	3	42.8		0	32	45.3

then, $3'\ 42''.8$, is the difference between the Moon's true longitudes at the *absolute* times of the observed occultation: and if the Moon's horary motion be $30'\ 9''.2$, the difference would correspond to $7^m\ 23^s.3$, in time. The occultation therefore at Greenwich *really* happened later than the occultation at Dublin by $7^m\ 23^s.3$: but, it is *reckoned* to happen later at the former by $32^m\ 45^s.3$: consequently part of this, or that part which remains after $7^m\ 23^s.3$ is subducted, is solely due to the difference of the longitudes of the two places: Dublin therefore is *east* of Greenwich, $25^m\ 22^s$.

The Longitude determined by means of a Solar Eclipse.

This method, in all its parts, is like the preceding. The distance (D) which is to be computed, instead of being the Moon's semi-diameter, will be the sum of the semi-diameters of the Sun and Moon. The immersion of the star will correspond to the first exterior contact of the limbs of the Sun and Moon, the emersion to the last. Thence will result, two equations for correcting, if the Lunar and Solar Tables be correct, the hypo-

thesis (see p. 846,) of the assumed longitude. But, since we can also observe other *Phases* of the eclipse, that, for instance, of the nearest approach of the centres (see pp. 724, 732,) we may deduce equations sufficient to correct both the errors of the Tables and the error of the assumed longitude of the place of observation.

We will now proceed to the description of an excellent method of finding the longitude, which cannot be ranged under either of the two preceding classes.

Method of determining the Longitude by means of the Passage of the Moon over the Meridian.

Let us suppose the meridian of a given place, produced to the heavens, to pass through the Moon, the Sun, and a fixed star. In the next instant, the Sun by its motion in right ascension will separate itself from the star; the Moon, by her greater motion in right ascension, both from the star and Sun, and the meridian, by the rotation of the Earth, from the star, Sun and Moon. In other words, in the instant of time (whatever be its magnitude) after that on which the three bodies were on the meridian, the star will be most to the west of the meridian, the Moon least, and the Sun will be in an intermediate position.

The meridian after quitting these bodies, will approach towards them with different degrees of velocity, and will reach them after different intervals of time. It will again pass through the star, after describing 360^0, in $23^h\ 56^m\ 4^s.09$; through the Sun, after describing $360^0\ 59'\ 8''.3$, in 24^h; and, through the Moon, after describing an angle equal to the sum of 360^0 and the increase of the Moon's right ascension in 24^h, and in a time equal to the sum of 24 hours, and of the Moon's *retardation* (see p. 783,) in 24 hours.

This takes place in the interval between two successive transits of the Moon over the same meridian. A spectator on a *different* meridian must note similar effects; but less in degree, and less, proportionally to the distance of his, from the first, meridian. He will note an increase in the Sun's right ascension, (or a

separation of the Sun from the fixed star) but less than 59′ 8″.3: an increase in the Moon's right ascension (or a separation of the Moon from the star), but less than its increase between two successive transits: and, consequently, an excess of the increase of the Moon's right ascension above that of the Sun's, but less than the excess that takes place between two successive transits of the Moon over the first meridian.

Hence, if the spectator, on this second meridian, knows, or is able to compute, the respective increases in right ascension of the Moon and Sun that take place between two successive passages of the Moon over the first meridian, then, since he is able, by actual observation, to ascertain, at the times of their passages, the right ascension of the Sun and Moon, he may, by simple proportion, determine his longitude; and in fact, he has three ways of effecting it: either with the Sun and star; or with the Moon and star; or with the Moon and Sun. Since, however, the first method by reason of the slow motion of the Sun, is not convenient and *practically* useful, we shall not notice it, but consider only the two latter.

Let E be the increase of the Moon's right ascension during two successive transits over the first meridian, e the difference between the Moon's right ascension at the Moon's first passage over the first meridian, and her right ascension at the passage over the second meridian, then,

$$E : e :: 360^{\circ} : 360 \times \frac{e}{E} = \text{difference of the meridians.}$$

This is the case with the Moon and star: and, with the Moon and Sun, there is this only difference, that E (E') must denote the *excess* of the increase of the Moon's right ascension above that of the Sun between two successive transits of the Moon; and e (e') the difference between the hours of Moon's passages over the second and first meridian: for, the hour of the Moon's passage is proportional to the angular distance which then exists between the Sun and Moon.

We must now endeavour to render the above formula more convenient for computation, so that (which ought in practical

Astronomy to be our constant aim) we may avail ourselves of the facilities of the Nautical Almanack.

E is the increase of right ascension between two successive transits of the Moon over the first meridian; it is, therefore, equal to the increase of right ascension in twenty-four hours, plus the increase of right ascension due or proportional to, the Moon's *retardation* (see p. 783,) in twenty-four hours. We have therefore this rule in the case of the Moon and star:

Find from the Nautical Almanack, (see p. 786,) the increase of the Moon's right ascension in twenty-four hours.

Compute also by the rule in p. 155 of the Nautical Almanack, (or from the expression in this Treatise, p. 782,) the Moon's retardation in twenty-four hours.

To the increase (A) of the Moon's right ascension in 24^h add the increase proportional to the retardation: call the sum E.

Then, substituting in p. 849, l. 25, 24^h instead of 360^0, we have

$$\log.\ \text{longitude} = \log.\ 24 + \log.\ e - \log.\ E.$$

In the case of the Moon and Sun, the rule is somewhat more simple: for E' converted into time in the case of the Moon, is the Moon's retardation, and e' is the proportional retardation between the transits at the first and second meridian. The third step, therefore, in the preceding rule, in this case, need not be made.

The above rule is adapted to the Nautical Almanack. But, it is easy to substitute, instead of it, a general formula of computation expressed in symbols. Thus, let A, a, be the respective increases of the right ascensions of the Moon and Sun in twenty-four hours; then, since the interval between two successive passages of the Moon over the meridian is

$$24^h + 24 \times \frac{A - a}{24} + 24\left(\frac{A - a}{24}\right)^2 + 24\left(\frac{A - a}{24}\right)^3 + \&c.$$

(since in this case $t = 24^h$, see p. 782,) the *retardation* in 24^h must equal

$$A - a + \frac{(A - a)^2}{24} + \frac{(A - a)^3}{(24)^2} + \&c.$$

and the increase of A, due to the retardation, must equal

$$\frac{A}{24}\left(A - a + \frac{(A-a)^2}{24} + \frac{(A-a)^3}{(24)^2} + \&c.\right)$$

and consequently, (see p. 783,)

$$E = A + A\left\{\frac{A-a}{24} + \left(\frac{A-a}{24}\right)^2 + \left(\frac{A-a}{24}\right)^3 + \&c.\right\}$$

and the longitude =

$$\frac{24 \times e}{A\left\{1 + \frac{A-a}{24} + \left(\frac{A-a}{24}\right)^2 + \left(\frac{A-a}{24}\right)^3 + \&c.\right\}} \quad ..(1).$$

In the case of the Sun,

$$E' = A - a + \frac{(A-a)^2}{24} + \frac{(A-a)^3}{24^2} + \&c.$$

and $e' = e - \epsilon$, where ϵ expresses the star's *acceleration*, (see p. 780,) proportional to the time corresponding to the difference of meridians. Hence, the longitude =

$$\frac{24 \times (e - \epsilon)}{(A-a)\left\{1 + \frac{A-a}{24} + \left(\frac{A-a}{24}\right)^2 + \left(\frac{A-a}{24}\right)^3 + \&c.\right\}} \quad ..(2).$$

Since $e - \epsilon : A - a :: e : A$, it is plain, the two expressions are, as they ought to be, equal.

The Moon's right ascension is expressed in the Nautical Almanack for every 12^h. Instead therefore of the difference of the increases of right ascension $(A-a)$ in 24 hours, we may employ the difference $\left(\frac{A-a}{2}\right)$ in 12 hours: and accordingly in the Rule, (p. 850, l. 9, &c.) and in the two expressions (1), (2), we must use 12^h instead of 24^h.

The denominators of the expressions, (1), (2), are, strictly speaking, infinite series; but, in practice it will be sufficiently accurate to take the sums of three of their terms.

The application of the Rule of p. 850, to Examples will now be much more easy than it was some years ago: since the Nautical Almanack, will, in future, express the Moon's right ascensions for noon and midnight in degrees, minutes, and *seconds*. We may, therefore, either compute the retardation by the formula of p. 783, or by the Rule given by Dr. Maskelyne in the Nautical Almanack, his explanation of its use, &c.: or by computing by the method given in pp. 698, &c. the time of the Moon's passage over the meridian: since the difference of two successive passages will immediately give us the Moon's retardation in 24 hours. If the passages of the Moon over the meridian of Greenwich were expressed as far as seconds of sidereal, or other, time, the application of the Rule would be still more simple.

EXAMPLE.

April 8, 1800.

Æ of Moon's centre observed at Greenwich....	12^h	36^m	$26^s.6$
On a meridian to the west, ...	12	47	56.7
$e =$	0	11	30.1

By computation from Nautical Almanack

Increase of ☽'s right ascension in 24^h, or A	52^m	6^s
of ☉'s, ... or a	3	39.3
$A - a =$	48	26.7
Moon's retardation in 24^h, or time proportional to the description of $A - a$ (see p. 782,) also Nautical Almanack, Explanation of Rules }	50	7.8
Proportional increase of 52^m 6^s, in 50^m $7^s.8$.........	1	48.8
$\therefore E\,(= 52^m\ 6^s + 1^m\ 48^s.8)$..................	53	54.8

Hence, by the Rule, p. 850,

log. 24 .... 1.3802112
log. 11^m $30^s.1$ 2.8389120

4.2191232

log. 53 54.8 .... 3.5098474

0.7092758 = log. 5.12007;

therefore the longitude $= 5^h.12007 = 5^h\ 7^m\ 12^s.25$.

We will now solve the same Example, by the second method, which is founded on the difference between the hours of the Moon's passages over the meridian, instead of the difference of her right ascensions at those passages. We will also use 12 instead of 24 hours (see p. 852.)

EXAMPLE.

Moon's passage at Greenwich	11^h	26^m	$47^s.82$
—— at the place of observation	11	37	29 . 5
e' or, $e - \epsilon =$	0	10	41.68
Moon's retardation, or E		25	3 . 9

Hence, log. 12 .... 1.0791812
log. $10^m\ 41^s.68$ 2.8073185

3.8864997

log. 25 3 . 9 ...... 3.1772190

.7092807 = log. 5.1201

$\therefore$ longitude $= 5^h.1201 = 5^h\ 7^m\ 12^s.36$.

The results are expressed as far as decimals of a second, merely for *arithmetical* exactness, and with no view of signifying that, in practice, any such exactness is attainable. The method is an excellent one, if it will determine the longitude within 10 seconds: and its original author Mr. Pigott, does not think it capable of a greater degree of accuracy. (See *Phil. Trans.* 1786, p. 419.)

The method, indeed, in a point of view strictly theoretical, cannot be minutely accurate. For the Moon's motion is continually variable, and the increase of its right ascension in 24 hours, will not be 24 times the increase in one hour. But if, from the strict laws of the lunar motions, we corrected the method, we should probably obtain an exactness of no practical value; since, we might only get rid of errors much less than the almost unavoidable errors of observation.

Any means, however, of rendering the method more accurate and simple, are not to be neglected. And, on the ground of accuracy, we shall probably gain something, by employing, instead of the sidereal clock, one of the stars that regulate it: and, *that star*, which shall happen to be nearest the Moon in right ascension and declination. Let both Observers note the right ascensions of this star and of the Moon, at the times of their transits over their meridians; then since, in a short interval, the clocks will not err much, the *difference of the differences in right ascension*, on which the method depends, will be given with sufficient accuracy for its successful application.

Again, the method will be rendered more simple, if instead of computing the transit of the Moon's centre, we are content to note merely the *transit of one of her limbs*. This we may do, with little error, if the required longitude be not great. For, the error, if there be any, can arise, solely from a change in the Moon's semi-diameter during the interval between the transits over the two meridians.

EXAMPLE. (See Vince's *Astronomy*, p. 533.)

June 13, 1791. At Greenwich, difference of Æ of ☽'s first limb, and of *a Serpentis* ...	28^m	$31^s.13$
Difference, at Dublin ...	27	24.74
	1	$6.44 = e$
By Nautical Almanack, $\frac{A}{2}$ ...	30	30
$\frac{a}{2}$ ...	2	4.4
$\frac{A-a}{2}$ ...	28	25.6
Retardation, (see p. 782.) ...	29	35.2
Increase of $\frac{A}{2}$ proportional to retardation ...	1	15.2
$\therefore E\,(=30^m\ 30^s + 1^m\ 15^s.2)$ ...	31	45.2

$$\begin{array}{lr} \text{Hence, log. } 12 \ldots\ldots\ldots\ldots & 1.0791812 \\ \text{log. } 1^{m}\ 6^{s}.44 \ldots\ldots\ldots & 1.8224296 \\ \hline & 2.9016108 \\ \text{log. } 31^{m}\ 45^{s}.2 \ldots\ldots\ldots & 2.2799406 \\ \hline & 0.6216702 = \text{log. } .418475 \end{array}$$

$\therefore$ the longitude $= 25^{m}\ 6^{s}.5$*.

The method of finding the longitude, by an occultation and the eclipses of the Sun and Moon, would, even if they could be practised, be of no use at sea, by reason of the rare occurrence of the phenomena on which they depend. A voyage might be completed before any eclipse happened. The mariner, who continually changes his place, requires a constant method of determining the change of longitude; a method, accordingly, depending on phenomena, continually occurring. Now, the passages of the Moon over the meridian, and the eclipses of *Jupiter's* Satellites, are phenomena of such character. But, of neither of these can he avail himself: for, the method founded on the former requires a nice observation with a telescope adjusted to move in the plane of the meridian: which is an operation evidently impracticable on board a ship. And the other method, on trial, has been found to be equally impracticable. Yet all that is wanted, for its success is, a contrivance that shall enable the Observer to direct, with steadiness, a telescope of sufficient power, towards *Jupiter*. (See Naut. Alm. p. 151.)

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* The principle of the preceding method is to be found, in a letter from Mr. Pigott, to Dr. Maskelyne, inserted in the *Philosophical Transactions* for 1786, pp. 417, &c.; and the method was used by the former in determining the longitude of York. The rule, however, p. 417, given by its author, is inaccurate: immaterially so, with regard to a place of so small a longitude as York, but to the extent, nearly, of 3 degrees, if we should seek to determine, by it, the longitude of a place that exceeds 5^{h}. This inaccuracy, as well as those of other authors, (see Vince's *Practical Astronomy*, p. 91. Wollaston's *Fasciculus*, Appendix, p. 76) who have adopted Mr. Pigott's method, was first pointed out in the *Phil. Mag.* vol. XV.

From the defect, however, of the preceding methods, has arisen one of singular simplicity and ingenuity, in which the sole instrument employed is the *Sextant*. This we shall now proceed to describe and illustrate.

Method of determining the Longitude by the Distance of the Moon from a fixed Star, or from the Sun.

1. By means of the sextant (see Chap. XL.) observe the distance between a star and one of the limbs of the Moon; or between the limbs of the Sun and Moon; then, by adding or subtracting, in the former case, the Moon's semi-diameter, and in the latter, the sum of the semi-diameters of the Sun and Moon, there will result either the distance between the Moon's centre and the star, or between the centres of the Sun and Moon.

2. If there be two Observers besides the one, who takes the above distance, let them, at the instant that distance is taken, observe the altitudes of the Moon and Star, or of the Moon and Sun. If there be only one Observer, he must take the altitudes immediately before and after the observation of the distance, and endeavour to allow for the changes of altitude, that may have taken place in the intervals between their observations and that of the distance.

3. These observations being made, the true altitudes must be deduced from the apparent and observed, by correcting the latter for parallax and refraction, (see Chap. XI, XII.). Which correction, in practice, is effected by means of Tables.

4. The observed distance being an apparent one, must be reduced to a true distance, or, (as it is technically expressed,) must be *cleaned* of the effects of parallax and refraction. This must be effected in every case, by a distinct computation from a proper formula.

5. The true distance being obtained, find the hour, minute, &c. of *Greenwich time* corresponding to it. This is effected by appropriate Tables, previously computed and inserted in the Nautical Almanack. In these Tables the Moon's distances from

certain stars are inserted for every 3^h: and thence, by an easy calculation, the time corresponding to an intermediate distance may be found.

6. Compute the time at the place of observation from the corrected altitude of the Sun or star, the Sun's or star's north polar distance (furnished by Tables), and the latitude.

7. The difference between this latter time and the time at Greenwich, is the *longitude*.

The first thing in the preceding statement that requires our attention, is the

Formula for deducing the True from the observed Distance.

Conceive S, M to be the true places of the star and Moon in two vertical circles SZ, MZ, forming at the zenith Z, the angle MZS; then, since (see Chap. XI, XII.) both parallax and refraction take place entirely in the directions of vertical circles, some point s above S, in the circle ZS, will be the apparent place of the star, and m below M, (since, in the case of the Moon, the *depression* by parallax is greater than the *elevation* by refraction) will be the apparent place of the Moon: let

D (SM) be the true, d (sm) the apparent distance,

A, a $(90^0 - ZM,\ 90^0 - ZS)$ the true altitudes,

H, h $(90^0 - Zm,\ 90^0 - Zs)$ the apparent altitudes;

then,

$$\text{in } \triangle\ SZM,\ \cos.\ SZM = \frac{\cos.\ D - \sin.\ A\ .\ \sin.\ a}{\cos.\ A\ .\ \cos.\ a},$$

$$\text{in } \triangle\ sZm,\ \cos.\ sZm\,(= SZM) = \frac{\cos.\ d - \sin.\ H\ .\ \sin.\ h}{\cos.\ H\ .\ \cos.\ h},$$

and D is to be deduced by equating these two expressions.

Hence,

$$\cos.\ D = (\cos.\ d - \sin.\ H\ .\ \sin.\ h)\frac{\cos.\ A\ .\ \cos.\ a}{\cos.\ H\ .\ \cos.\ h} + \sin.\ A\ .\sin.\ a,$$

$$= [\cos. d + \cos.(H+h) - \cos. H . \cos. h] \frac{\cos. A . \cos. a}{\cos. H . \cos. h} + \sin. A . \sin. a$$

$$= 2 . \cos. \tfrac{1}{2}(H + h + d) . \cos. \tfrac{1}{2}(H + h - d)^{*} \frac{\cos. A . \cos. a}{\cos. H . \cos. h},$$

$$-(\cos. A \cos. a - \sin. A \sin. a.)$$

But the last term $= \cos. (A + a)$; subtract both sides of the equation from 1; then, since

$$1 - \cos. D = 2 . \sin.^2 \frac{D}{2}, \text{ and } 1 + \cos. (A + a) = 2 . \cos.^2 \frac{A+a}{2},$$

we have, dividing by 2, and making F to represent $\dfrac{\cos. A . \cos. a}{\cos. H . \cos. h}$,

$$\sin.^2 \frac{D}{2} = \cos.^2 \tfrac{1}{2}(A+a) - \cos. \tfrac{1}{2}(H+h+d) \cos. \tfrac{1}{2}(H+h-d) \times F$$

$$= \cos.^2 \tfrac{1}{2}(A+a) \left(1 - \frac{\cos. \frac{1}{2}(H+h+d) . \cos. \frac{1}{2}(H+h-d)}{\cos.^2 \frac{1}{2}(A+a)} \times F\right)$$

and, if we make the fraction, on the right-hand side of the equation, $= \sin.^2 \theta$, we shall have

$$\sin.^2 \frac{D}{2} = \cos.^2 \tfrac{1}{2}(A+a) . \cos.^2 \theta,$$

$$\text{and } \sin. \frac{D}{2} = \cos. \tfrac{1}{2}(A+a) . \cos. \theta.$$

Hence, by logarithms, the rule of computation is

1st, $2 . \log. \sin. \theta = \log. \cos. \tfrac{1}{2}(H+h+d) + \log. \cos. \tfrac{1}{2} \pm (H+h-d)$
$+ \log. \cos. A + \log. \cos. a + \text{ar. com. } \log. \cos. H$
$+ \text{ar. com. } \log. \cos. h - 2 \log. \cos. \tfrac{1}{2}(A + a),$

and 2ndly, $\log. \sin. \dfrac{D}{2} = \log. \cos. \tfrac{1}{2}(A + a) + \log. \cos. \theta - 10$†.

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* Cos. $\tfrac{1}{2}(d - H - h)$ if d be $> H + h$.

† This formula of computation is Borda's. If in p. 857, bottom line, instead of substituting for sin. H sin. h, cos. H . cos. h, $-$ cos. $(H + h)$, we substitute cos. $(H-h)-$cos. H . cos. h, we may deduce the formula, which is the basis of Dr. Maskelyne's Rule inserted in the Introduction to Taylor's *Logarithms*, pp. 60, &c.

The other parts (1), (2), &c. p. 856, of the statement * have either already received explanation, in the preceding pages of this Treatise, or are so plain as to need none. We proceed therefore to an Example.

EXAMPLE.

June 5, 1793, about an hour and an half after noon, in 10° 46′ 40″ south latitude, and 149° longitude, *by account* (see p. 800), by means of several observations, it appeared, that

Distance of nearest limbs of ☉ and ☽	83°	26′	46″
Altitude of lowest limb of ☉	48	16	10
Altitude of upper limb of ☽	27	53	30

Here, see (1) p. 856, we must add to the distance, the semi-diameters of the Sun and Moon, taking them from the Nautical Almanack.

The apparent distance of limbs of ☽ and ☉	83°	26′	46″
semi-diameter of ☉	0	15	46
of ☽	0	14	54
Augmentation propor[l]. to altitude, (see p. 657,)	0	0	7
Apparent distance (d) of centres...........	83	57	33

* The distance (see p. 856, bottom line,) between the Moon and a fixed star is easily computed from their latitudes and the difference of their longitudes, the proper formula is

$$\sin.^2 \frac{D}{2} = \sin.^2 \left(\frac{l-l'}{2}\right) + \cos. l . \cos. l' . \sin.^2 \frac{k-k'}{2},$$

(see p. 746: also *Trig*. pp. 170, &c.) l, l', k, k', representing, in this case, the true latitudes and longitudes.

The Moon's latitude and longitude being computed and inserted in the Nautical Almanack, for noon and midnight, the Moon's distances from certain stars are computed, by the above formula, for those times; and, the distances for the intermediate times, at 3^h, 6^h, &c. are determined by *interpolation*, or by the aid of the differential formula. The latitudes and longitudes of the stars, are either to be computed, (see pp. 158, &c.) from their right ascensions and declinations, or to be immediately taken from certain Tables. (See Lalande's Tables, Nautical Almanack 1773, *Connois. des Tems*, an. 12.)

Reduction of the Apparent to the True Altitude. (See [3] p. 856.)

Altitude of Sun's lower limb................	48°	16′	10″
Dip (see p. 824.)........................	−0	4	24
	48	11	46
Semi-diameter...........................	0	15	46
Apparent altitude of Sun's centre (h)..........	48	27	32
Refr. − Par. − correct. for Therm..........	−0	0	43
True alt. of Sun's centre (a)................	48	26	49

Altitude of Moon's upper limb..............	27°	53′	30″
Dip.................................	−0	4	24
	27	49	6
Semi-diameter..........................	0	15	1
Apparent altitude of Moon's centre (H)......	27	34	5
Par. − Refr. + corr. for Therm............	0	46	43
True altitude of Moon's centre (A)..........	28	20	48

Reduction of the Apparent to the True Distance.
(See [5] p. 856, and *Formula*, p. 858.)

d	83° 57′ 33″		
h	48 27 32	ar. co. cos. =	.1783835
H	27 34 5	ar. co. cos. =	.0523390
sum	159 59 10		
$\frac{1}{2}$ sum	79 59 35	..... cos. =	9.2399686
$d - \frac{1}{2}$ sum	3 57 58	..... cos. =	9.9989587
a	48 26 49	..... cos. =	9.8217187
A	28 20 48	..... cos. =	9.9445275
$A + a =$	76 47 37		39.2358960
$\frac{1}{2}(A + a) =$	38 23 48	2 log. cos.	19.7883324
		2)	19.4475636

log. sin. θ = 9.7237818 = log. sin. 31° 57′ 33″

Hence, log. cos. 31 57 53 9.9285875
log. cos. 38 23 48 9.8941662
(10 taken away) 9.8227537 = log. sin. 41° 40′ 27″ $\frac{1}{2}$

$$\therefore \frac{D}{2} = 41^\circ\ 40'\ 27''\tfrac{1}{2},$$

$$\text{and, } D = 83\ \ 20\ \ 55, \text{ nearly.}$$

Time at Greenwich computed. [See (5) p. 856.]

By Nautical Almanack, (p. 70.)

Dist. ☽ from ☉ $\begin{cases} \text{at } 15^h \ldots 83^\circ\ 6'\ 1'' \ldots D = 83^\circ\ 20'\ 55'' \\ \text{at } 18 \ldots 84\ 28\ 26 \ldots \text{ at } 15^h\ 83\ \ 6\ \ 1 \end{cases}$

Increase of dist. in 3^h = 1 22 25   0 14 54

Hence,

$1^\circ\ 22'\ 25'' : 14'\ 54'' :: 3^h$: time corres^g. to the increase of $14'\ 54''$

* Hence, log. 3 = .4771213
log. 894″ = 2.9513375
3.4284588
log. 4945″ = 3.6941663
$\bar{1}$.7342925 = log. 0^h.5425 = log. $32^m\ 33^s$.

Hence, the time at Greenwich = $15^h\ 32^m\ 33^s$.

Time at the Place of Observation computed. [See (6) p. 857, *also*, pp. 795, &c.]

L (Lat.)	10°	$16'$	$40''$	.. cos.	9.9929749
p	113	22	48	.. sin.	9.9627922
a	48	26	49		19.9557671
sum ..	172	6	17		
$\frac{1}{2}$ sum ..	86	3	8.5	cos.	8.8378712
$\frac{1}{2}$ sum $- a$	37	36	19.5	sin.	9.7854864
				(20 added)	38.6233576
					19.9557671
				2)	18.6675905
					9.3337902 = log. sin. $12^\circ\ 27'\ 17''\frac{1}{2}$

* As this is a frequent operation in Nautical Astronomy, it is facilitated by means of Tables of *Proportional Logarithms*, in which log. $3^h = 1$. See Requisite Tables, Tab. XV. also Mendoza's Tables, Tab. XIV.

∴ hour angle (see p. 795, &c.) = $24° \; 54' \; 35''$

(and in time, by Rule, p. 779,) = $1^h \; 39^m \; 38^s.3$.

Hence, see (7) p. 857,

Time at Greenwich, ...	$15^h \; 32^m \; 33^s$
at place of observation	1 39 38.3
Long. from Greenwich *reckoning by the west*	13 52 54.7

∴ longitude *east* of Greenwich $10^h \; 7^m \; 4^s.3$.

We will give a second Example, in which the *lunar distance* is the Moon's distance from a known star.

Example II.

Dec. 14, 1818, at $12^h \; 10^m$, nearly: latitude $36° \; 7'$ N., longitude by account $11^h \; 52^m$, the following observations were made; the eye of the Observer being about 19 feet above the surface of the sea,

Observed Alt. of Regulus.	Observed Alt. of Moon's L. L.	Observed Dist. of Moon's nearest Limb and Star.	
$28° \; 29' \; 17''$	$61° \; 26' \; 12''$	$33° \; 15' \; 25''$	
− 4 18	− 4 18		.... dip of the horizon
	+ 14 56	+ 14 56	☽'s semi-diameter.
(h) $28° \; 24' \; 59''$	(H) $61° \; 36' \; 50''$	(d) $33° \; 30' \; 21''$	
Refⁿ. − 1 45	− 0 31.1		
	Parallax [see below (p)] + 25 40.5		
(a) 28 23 14	(A) 62 2 0, nearly.		

(p) Horizontal Parallax .... $53' \; 59''$,

log. $53' \; 59''$ .... = 3.51041

log. $61° \; 36'$ 9.67726

3.18767

∴ parallax = $1540''.5$

= $25' \; 40''.5$.

Hence, see p. 858,

$$
\begin{array}{rlll}
d = & 33^\circ\ 30'\ 21'' & & \\
h = & 28\ \ 24\ \ 59 & \ldots \text{sec.} & 0.0557579 \\
H = & \underline{61\ \ 36\ \ 50} & \ldots \text{sec.} & 0.3229307 \\
\tfrac{1}{2}\text{sum} = & 61\ \ 46\ \ \ 5 & \ldots \text{cos.} & 9.6748997 \\
\tfrac{1}{2}\text{sum} - d = & 28\ \ 15\ \ 44 & \ldots \text{cos.} & 9.9448723 \\
a = & 28\ \ 23\ \ 14 & \ldots \text{cos.} & 9.9443616 \\
A & \underline{62\ \ \ 2\ \ \ 0} & \ldots \text{cos.} & \underline{9.6711338} \\
 & & & 39.6139560 \\
\tfrac{1}{2}(A+a) & 45\ \ 12\ \ 37 & 2\ \text{cos.} & \underline{19.6957706} \\
 & & 2) & \underline{19.9181854} \\
 & & & 9.9590927\ (\text{log. sin. } 65^\circ\ 31'\ 18'')
\end{array}
$$

$$
\begin{array}{lll}
\text{again, cos. } 65^\circ\ 31'\ 18'' = & 9.6173895 & \\
\text{cos. } 45\ \ 12\ \ 37 & \underline{9.8478853} & \\
 & 9.4652748 & = \text{log. sin. } 16^\circ\ 58'\ 24''.2\,; \\
\end{array}
$$

$$\therefore\ D = 33^\circ\ 56'\ 48''.4.$$

Time at Greenwich (see Nautical Almanack for 1818, p. 140.)

$$
\text{Dist. ☽ from } * \left\{\begin{array}{l} 0^h \ldots 33^\circ\ 58'\ 7'' \\ 3\ \ldots\ \underline{32\ \ 30\ \ 3} \\ \qquad\ \ 1\ \ 28\ \ 4 \end{array}\right.
\qquad
\begin{array}{l} 33^\circ\ 58'\ \ 7'' \\ \underline{33\ \ 56\ \ 48.4} \ldots\ldots (D) \\ \ \ 0\ \ \ 1\ \ 18.6. \end{array}
$$

$$\text{Hence, the time at Greenwich} = 0^h + \frac{1'\ 18''.6}{1^\circ\ 28'\ 4''} \times 3^h = 2^m\ 40^s.6.$$

Time of Observation, at the Place of Observation.

$$
\begin{array}{rlll}
a = & 28^\circ\ 23'\ 14'' & & \\
L = & 36\ \ \ 7\ \ \ 0 & \ldots \text{sec.} & 0.0926862 \\
p = & \underline{77\ \ \ 9\ \ 66} & \ldots \text{cosec.} & 0.0110020 \\
\tfrac{1}{2}\text{sum} = & 70\ \ 49\ \ 40.3 & \ldots \text{cos.} & 9.5164147 \\
\tfrac{1}{2}\text{sum} - a = & 42\ \ 26\ \ 26.3 & \ldots \text{sin.} & \underline{9.8291911} \\
 & & 2) & \underline{19.4492940} \\
 & & & 9.7246470\ \text{log. sin. } 32^\circ\ 2'\ 10''\,;
\end{array}
$$

∴ the horary angle .... $= 64^\circ\ 4'\ 20'' =$	4^h	16^m	$17^s.3$	
But star's right ascension .... $=$	9	58	43.3	
Right Ascension of mid-heaven ...... $=$	5	42	26	
From N. A., the Sun's Æ on the meridian of the place of observation $=$	17	27	12	
Approximate time	12	15	14	
Acceleration	0	2	0.4	
Time at ship	12	13	13.6	
	11	46	46.4	
Time at Greenwich	0	2	40.6	
Longitude	11	49	27	W.

Instead of computing the time from the altitude of the star, we might have computed it from the Moon's altitude, which can be more exactly observed. The computation will be as follows:

☽ 's true alt. (A)	62°	2′	0″		
☽ 's N. P. D. p	63	37	20	cosec.	0.0477480
L	36	7	0	.. sec.	0.0926862
½ sum	80	53	10	.. cos.	9.1997481
½ sum $- A$	18	51	10	.. sin.	9.5093874
				2)	18.8495697
					9.4247848 sin. 15° 25′ 22″.25;

∴ ☽ 's horary angle $= 30^\circ\ 50'\ 44''.5 =$	2^h	3^m	23^s, nearly,	
☽ 's right ascension............	7	45	51	
Right Ascension of mid-heaven	5	42	28	
Sun's right ascension	17	27	12	
	12	15	16	
Acceleration	0	2	0.5	
	12	13	15.5	
	11	46	44.5	
Time at Greenwich	0	2	40.5, nearly,	
Longitude	11	49	25	

The process for finding the longitude from the distance of the Moon from a *star*, similar to the preceding, is, in deducing the true from the observed altitude, somewhat more simple; but, more tedious in the computation of the time from the altitude.

The computation of deriving the time from the star's altitude, it is desirable to supersede, by reason, of the probable errors that will be made in observing the star's altitude*. And it may be superseded, by finding the time and regulating the chronometer by a previous or a subsequent observation of the Sun's altitude: by allowing for the change in longitude (see p. 802, &c.) during the two observations; and then by *computing* the star's altitude, from its north polar distance, the latitude, and the *estimated* time.

The proper formula of computation for this occasion is one that has repeatedly occurred, (see pp. 795, &c.) If L be the latitude, p the north polar distance, h the estimated hour angle and a the altitude, then,

$$\sin. a = \sin. L . \cos. p + \cos. L \sin. p . \cos. h,$$

whence, a may be computed by means of a subsidiary angle. (See *Trig.* pp. 169, &c.)

Hence, the process for finding the longitude, although it does not essentially require the chronometer, is rendered more easy and accurate by its aid.

This is not the sole use of the chronometer. It enables the Observer to use the *mean* of several observed distances of the Moon from a star, or the Sun, instead of a single one. For, he cannot, without error, take the *mean*, except he know the several intervals of time that separate the successive observations. The chronometer enables him to ascertain these intervals.

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* The practical inconvenience of this method, is of the same kind as that which occurs in Dr. Brinkley's method of finding the latitude from the observed altitudes of two known stars: except in the twilight, or by Moonlight, it is very difficult to see the horizon distinctly, when you can see the star. Lacaille was accustomed to use precautions in order to be able to see the horizon.

Thus, in the following observations:

	Time by Watch.	Star's Altitude.	Altitude Moon's Upper Limb.	Dist. Moon's L. from Star.
	13^h 1^m 50^s	43° 0′ 30″	67° 28′ 0″	45° 19′ 45″
	2 25	7 0	67 11 0	19 15
	3 21	14 0	66 59 0	18 45
	4 14	20 30	66 51 0	18 30
	5 11	29 0	66 36. 0	18 15
	6 8	38 0	66 32 0	18 0
Sums	23 9	109	401 37 0	112 30
$\frac{1}{6}$th or means	13 3 52	43 18 10	66 56 10	45 18 45

And, generally, the *elements* of the computation in the *lunar method* are the *means* of several observations, not the results of individual ones.

Since, in Nautical Astronomy, the finding of the longitude is the most important and most difficult operation, several expedients have been devised for facilitating it. *The distance has been cleared**, (see p. 857,) by a formula different, from that which has been given in p. 857, although derived from the same funda-

* M. Delambre has given in Chap. XXXVI. (and there is no great difficulty in the deduction) about 20 different formulæ. The leisure of scientific men cannot be more innocently employed. It is profitably employed when, after comparison, it selects that formula which, sufficiently exact, is the least liable, in its application, to the mistakes of merely practical men: such, as in general, mariners are. But a proper formula once adopted, and invested with its Rules and Tables, ought not hastily to be got rid of. It is no sufficient reason to get rid of it, to be able to supply a method a little more simple, and a little less long. There is no great harm, indeed, in perplexing a mere mathematician. But it is a very mischievous innovation to disturb the technical memory of an old seaman, and to unsettle his familiar rules of computation. Every one, man of science or not, knows, from his own experience, the great value of *fixed* rules, in conducting arithmetical operations.

mental expression. Instead of a logarithmic computation, one proceeding solely by addition, and furnished with appropriate Tables, has been substituted. But, for a satisfactory explanation of the means and artifices, by which, on this occasion, the labour of computation is abridged and expedited, we must refer to the treatises that contain them. (See *Requisite Tables: their explanation and use.* Mendoza's *Treatise on Nautical Astronomy:* Brinkley, *Irish Transactions*, 1808: *Connoissance des Tems* for 1808, and for years 12 and 14: Mackay, *On the Longitude,* Lax's Tables.)

If we wish to reduce, to one of the classes (see p. 837,) the preceding method of finding the longitude, we shall find that it belongs to the second. The principle on which it rests, is, indeed, precisely the same as that which forms the basis of the second method (see p. 841,) of finding the longitude from an occultation; for,

Analogous to the distance D	83°	$20'$	$55''$,	at 1^h	39^m	38^s
is the ☽ 's longitude at Dublin,	67	18	43.3,	at 8	4	51.5
Analogous to the distance	84	28	26,	at 18 (*Greenwich*)		
is the Moon's longitude.....	67	22	26.1,	at 8	37	36.8

(for the Moon's longitude is a species of distance, being the distance of her place referred to the ecliptic from ♈). And the reduction of $84^\circ\ 28'\ 26''$ to $83^\circ\ 20'\ 55''$ by taking away $1^\circ\ 7'\ 31''$, corresponding to $2^h\ 27^m\ 27^s$, is analogous to the reduction of $67^\circ\ 22'\ 26''.1$ to $67^\circ\ 18'\ 43''.3$, by taking away $3'\ 42''.8$, corresponding to $7^m\ 23^s.3$; $1^\circ\ 22'\ 25''$, being, in the former case, the change of the Moon's distance in 3^h and $30'\ 9''.2$, in the latter, the change of the Moon's longitude in 1^h: that is, in other words, the Moon's *horary motion in longitude.*

The problems then of deducing the longitude from an occultation, and from the distance of the Moon from a star, are the same in principle; but the former is more difficult in its process, because, in *clearing* the observation of parallax, it is necessary to compute its resolved parts in the directions of longitude and latitude; whereas, in the latter, the entire effects of parallax, which take place in altitude, are alone considered.

The former, as a practical method of determining the longitude, is exceedingly more accurate than the latter*; because, we are enabled to mark the distance, which is the Moon's semi-diameter, and the corresponding time, which is that either of the immersion or emersion, with much greater precision, than we can measure the distance by means of a sextant, and compute the time from an observed altitude. But, as it has been observed in p. 855, the degree of accuracy does not alone determine the adoption of a method; we are obliged, in finding the longitude at sea, by the exigencies of the case, to rely solely on, what is called technically, the *Lunar Method*.

In finding the longitudes of places at land, circumstances also must determine which of the preceding methods must be adopted. Several have been proposed, not as if they might be indifferently used, but that Observers may select from them, what are suited to their several wants, means, and opportunities. If the Observer, furnished with a telescope and chronometer, wishes readily and speedily to determine the longitude of the place where he is, he may use the method of the eclipses of *Jupiter's* satellites, (see p. 840,) and obtain a result probably within 30 or 40 seconds of the truth. If he has the means of adjusting a telescope to move nearly in the plane of the meridian, the method of the transits of the Moon and of a fixed star, (see p. 856,) will afford a more accurate result, and with an error, perhaps, not exceeding ten seconds. But, if great accuracy be required, and expedition be not, then the Observer must wait for the opportunity of a solar

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* "For the present, I infer, we may take the difference of meridians (Greenwich and Paris) $9^m\ 20^s$, as being within a few seconds of the truth, till some *occultations* of fixed stars by the Moon, already observed, or hereafter to be observed, in favourable circumstances, and carefully calculated, shall enable us to establish it with the *last exactness*." Maskelyne, *On the Latitude and Longitude of Greenwich*, &c. *Phil. Trans.* 1787, p. 186. See also *Phil. Trans.* 1790, p. 230.

eclipse, or, what is better, of an occultation*, and thence compute the longitude †.

The several methods have their peculiar advantages and disadvantages: the last, which is the most accurate, requires computations of considerable length and nicety; the first, probably inaccurate to the extent of ⅛th of a degree, requires scarcely any. The second is more accurate, and may constantly be used, and therefore, on the whole, it is perhaps the readiest and best practical method.

The *Lunar method,* which is the least exact, is yet founded on the most refined theory, and the most complicated calculations. It depends, for its accuracy, entirely on previous computations. We cannot, in applying it, compare, as in the case of an occultation, (pp. 841, &c:) *actual* observations of the same phenomenon, or give accuracy to the result, by *correcting* (see p. 842,) the errors of the Tables. But, the mariner must be guided by the result, such as it comes out at the time of the observation, and which, a few hours after, will have lost all its utility.

In page 849, it was mentioned, that, in a merely theoretical point of view, the longitude ought to be afforded as a result, from

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* An occultation affords a more exact practical result than a solar eclipse, because, in the former, the instant of immersion can be marked with greater precision, than the instant of contact in the latter.

The *recurrence* of occultations may be found as those of eclipses were, p. 730. We must find two numbers in the proportion, or nearly so, of $27^d.321661$ (the Moon's sidereal period) to $6793^d.42118$ (the sidereal revolution of the nodes): which numbers are 17, and 4227: and the period of recurrence is $316^y\ 72^d.1$ $(=4227 \times 27^d.321661)$.

† In speaking of the errors in the determination of the longitude, we have supposed the *mean*, of several observations accurately made with excellent instruments, to be taken. The errors of *single* observations will be much greater than what have been assigned to them. With the first satellite of *Jupiter* it may amount to $3^m\ 44^s$ according to Mr. Short. (See his Paper in the *Phil. Trans.* 1763, p. 167, for determining the difference of longitude between Greenwich and Paris, from the transits of *Mercury* over the Sun's disk).

the separation, during a given interval, of the Sun from a star; but that the slow motion of the former, deprived the method of all practical utility. Now, the material circumstance that confers, what accuracy it possesses, on the *Lunar method*, is the Moon's quick change of place. Were the change greater, the method would be more accurate. For instance, the Moon now moves through 1° in about 2 hours, and therefore, an error of $1'$, in observing and computing her distance, causes an error of 2 minutes of time, or of $30'$ of longitude. But, if she moved through the same space (1°) in $\frac{1}{2}$ hour, then the error of $1'$ would cause only an error of 30^{s} of time, and of $7'\frac{1}{2}$ of longitude.

Hence it follows, that the first satellite which moves round *Jupiter* in less than two days, (see p. 629,) must enable an Observer on that planet to determine, very exactly, the longitude of his station: as exactly, as we can determine the latitude of a place.

CHAP. XLIV.

On the Calendar.

THE Sun naturally regulates the beginnings, ends, and durations of the seasons; and, the calendar is constructed to distribute and arrange the smaller portions of the year.

The calendar divides the year into 12 months, containing, in all, 365 days; now, it is desirable that it should always denote the same parts of the same season by the same days of the same months, that, for instance, the summer and winter solstices, if once happening on the 21st of June and 21st of December, should, ever after, be reckoned to happen on the same days; that, the date of the Sun's entering the equinox, the natural commencement of spring, should, if once, be always on the 20th of March. For thus, the labours of agriculture, which really depend on the situation of the Sun in the heavens, would be simply and truly regulated by the calendar.

This would happen, if the civil year of 365 days were equal to the astronomical; but, (see p. 529, &c.) the latter is greater; therefore, if the calendar should invariably distribute the year into 365 days, it would fall into this kind of confusion; that, in progress of time, and successively, the vernal equinox would happen on every day of the civil year. Let us examine this more nearly.

Suppose the excess of the astronomical year above the civil to be exactly 6 hours, and, on the noon of March 20th of a certain year, the Sun to be in the equinoctial point; then, after the lapse of a civil year of 365 days, the Sun would be on the meridian, but not in the equinoctial point; it would be to the west of that

point; and would have to move 6 hours in order to reach it, and to complete (see pp. 197, &c.) the astronomical or tropical year.

At the completions of a second, and a third civil year, the Sun would be still more and more remote from the equinoctial point: and would be obliged to move, respectively, for 12 and 18 hours, before he could rejoin it, and complete the astronomical year.

At the completion of a fourth civil year, the Sun would be more distant, than on the two preceding ones, from the equinoctial point. In order to rejoin it, and to complete the astronomical year, he must move for 24 hours, that is, for *one whole day*. In other words, the astronomical year would not be completed till the beginning of the next astronomical day; till, in civil reckoning, the *noon of March* 21st.

At the end of four more common civil years, the Sun would be in the equinox on the noon of March 22. At the ends of 8 and 64 years, on March 23, and April 6, respectively; at the end of 736 years, the Sun would be in the vernal equinox on September 20. And, in a period of about 1508 years, the Sun would have been in every sign of the Zodiac on the same day of the calendar, and in the same sign on every day.

If the excess of the astronomical above the civil year, were really, what we have supposed it to be, 6 hours, this confusion of the calendar might be, most easily, avoided. It would be necessary merely to make every fourth civil year to consist of 366 days; and, for that purpose, to interpose, or to *intercalate* a day in a month previous to March. By this *intercalation* what would have been March 21st is called March 20th; and, accordingly, the Sun would be still in the equinox on the same day of the month.

This mode of correcting the calendar was adopted by Julius Cæsar. The fourth year into which the intercalary day is introduced was called *Bissextile* *: it is now frequently called the *Leap*

* The *Bissextus* dies ante Calendas, being the intercalated day in the Julian Calendar.

year. The correction is called the *Julian correction*, and the length of a mean Julian year is equal to $365^d.25$.

If the astronomical year (see p. 529,) be equal to $365^d.242264$, it is less than the mean Julian by $0^d.007736$. The *Julian* correction. therefore, itself needs a correction. The calendar, regulated by it, would, in progress of time, become erroneous, and would require *reformation*.

The intercalation of the Julian correction being too great, its effect would be to *antedate* the happening of the equinox. Thus, (to return to the old illustration) the Sun, at the completion of the fourth civil year, now the Bissextile, would have passed the equinoctial point, by a time equal to four times $0^d.007736$: at the end of the next Bissextile, by eight times $0^d.007736$: at the end of 129 years, nearly by one day. In other words, the Sun would have been in the equinoctial point 24 *hours previously*, or on the *noon of March* 19th.

In the lapse of ages, this error would continue and be increased. Its accumulation in 1292 years would amount, nearly, to 10 days, and then, the vernal equinox would be reckoned to happen on March 10th.

The error into which the calendar had fallen, and would continue to fall, was noticed by Pope Gregory in 1582. At his time, the length of the year was known to greater precision, than at the time of Julius Cæsar. It was supposed equal to $365^d\ 5^h\ 49^m\ 16^s.23$. Gregory, desirous that the vernal equinox should be reckoned on or near March 21st, (on which day it happened in the year 325, when the Council of Nice was held,) ordered that the day succeeding the 4th of October 1582, instead of being called the 5th, should be called the 15th; thus, suppressing 10 days, which, in the interval between the years 325 and 1582, represented, nearly, the accumulation of error arising from the *excessive intercalation of the Julian correction*.

This act *reformed* the calendar: in order to *correct* it in future ages, it was prescribed that, at certain convenient periods, the intercalary day of the Julian correction should be omitted. Thus, the centenary years, 1700, 1800, 1900, are (as every

year divisible by 4 is) according to the Julian correction, Bissextiles, but on these it was ordered that the intercalary day *should not be inserted:* inserted again in 2000, but not inserted in 2100, 2200, 2300; and so on for succeeding centuries *.

This is a most simple mode of regulating the calendar. It corrects the insufficiency of the Julian correction by omitting, in the space of 400 years, 3 intercalary days. And, it is easy to estimate the degree of its accuracy. For, the real error of the Julian correction is $0^d.007736$ in 1 year, consequently, $4 \times 0^d.7736$, or $3^d.0944$ in 400 years. Consequently, $0^d 0944$, or, $2^h\ 15^m\ 56^s.16$ in 400 years, or 1 day in 4237 years is the measure of the degree of inaccuracy in the Gregorian correction. Against such, it perhaps, is not worth the while to make any formal provision in the mode of regulating the calendar.

The calendar may be thus examined and regulated, without the aid of mathematical processes and formulæ. Yet, on this subject, the method of *continued Fractions* † is frequently

* M. Delambre proposed to keep the calendar correct on this principle. Assuming the length of the year to be equal to $365^d.24\frac{2}{9}$, in 9 years the excess above the common civil year would be $24 \times 9 + 2$, or $2^d.18$

in 450 years 109
in 900...218
in 3600..872

According to the Julian correction there would be in 3600 years (3600 divided by 4 gives 900,) 900 intercalations, or 900 Bissextiles, too many by 28.

The Gregorian calendar casts out 27; in order, then, to cast out the 28th, and to keep the calendar right, it is merely necessary to make the year 3600 and its multiples common years.

† Since the excess of the tropical year above the civil is $0^d.242264$, the exact intercalation is that of 242264 days, in 1000000 years. But, since this intercalation would be of no practical use, we must find numbers nearly in the ratio of 242264 to 1000000: which may be effected by the method of continued fractions, as in pages 279, 280, &c. See on this subject, Euler's *Algebra.* Addition, pp. 426, &c. edit. 1774.

employed. This, however, is to use an instrument too *fine* for the occasion. The results have a degree of exactness, beyond what we require, or can practically avail ourselves of. The only thing, in the correction of the calendar, that requires a high degree of mathematical science, is the determination of the length of the astronomical year. Had this been known, to a greater exactness, by the Astronomers of the time of Julius Cæsar, the Julian correction would, probably, have superseded the necessity of the Gregorian.